Unit 4: Inference: Conclusions with Confidence

Chapter 10: Introduction to Inference

10.1A: Estimating with Confidence

Statistical Confidence: - the repeatability and the accuracy of a sampling proportion from a large population.
- is based on the central limit theorem and the resulting population will be normal.

Confidence Intervals: - consists of a symmetrical area around the centre, and indicates the mean (μ) along the margin of error.

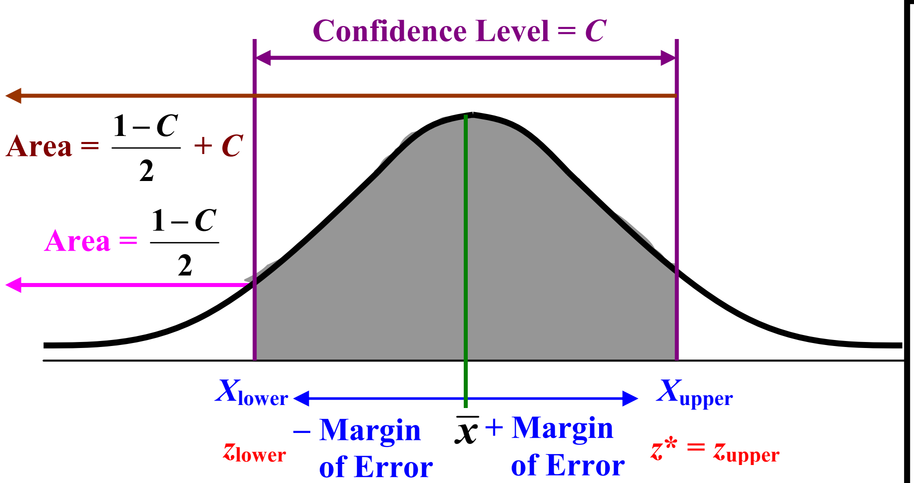
Margin of Error (***m***): - the amount of accuracy around the mean.

Confidence Level: - the level of assurance from a statistical report as represented by the symmetrical area around the centre.
- always expressed in percent, but can be expressed in fraction or decimal (95% confidence, or 19 times out of 20, or $C = 0.95$ – commonly known as **Level *C***).

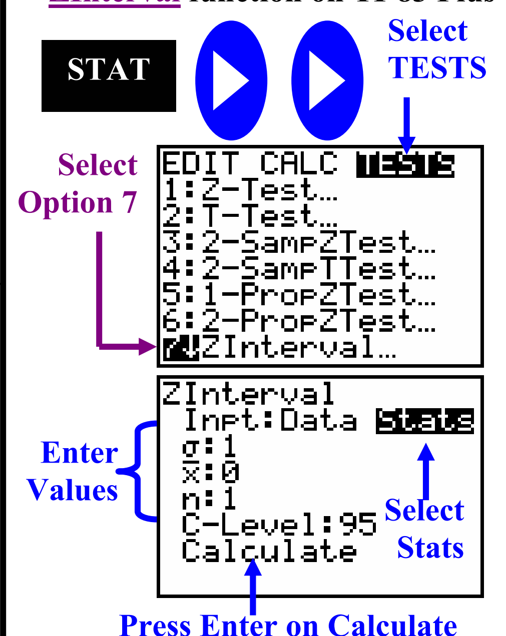
Critical Value (z^*): - the z-score needed on the upper side of the mean to achieve the confidence level.

Confidence Intervals ($\bar{x}$ ± Margin of Error) at Level *C*

Margin of Error (m) = Critical Value (z^*) × Standard Deviation of Statistic (σ_x or $\sigma_{\hat{p}}$ or $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$)

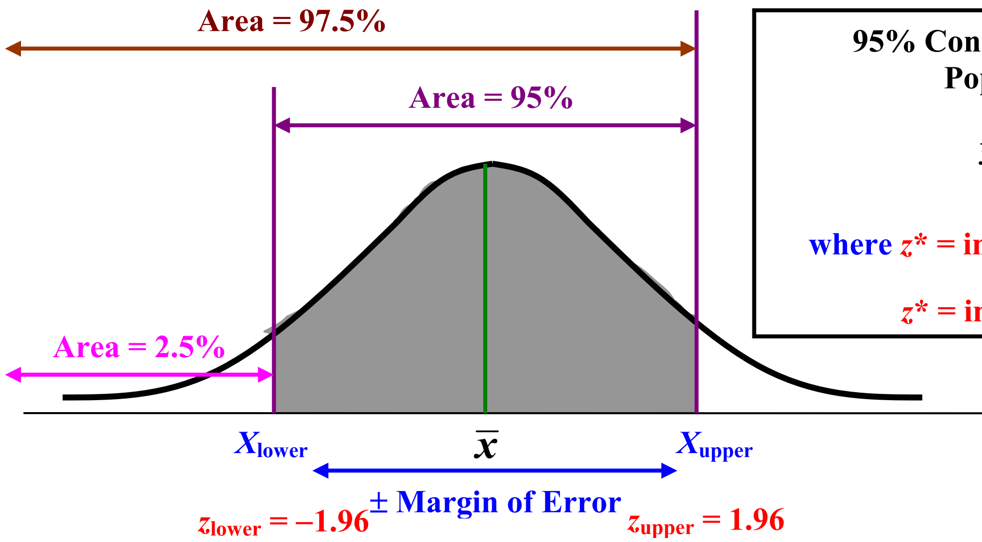


Confidence Interval with ZInterval function on TI-83 Plus



Confidence Interval for a Population Mean

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} \quad \text{where } z^* = \text{invNorm}\left(\frac{1-C}{2} + C\right) \text{ and } m = z^* \frac{\sigma}{\sqrt{n}}$$



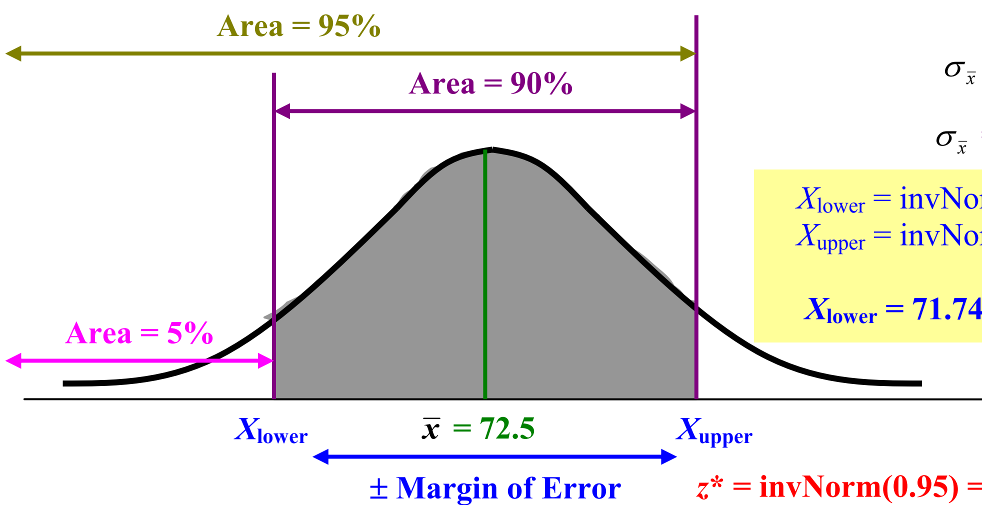
95% Confidence Interval for a Population Mean

$$\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}$$

where $z^* = \text{invNorm}\left(\frac{1-0.95}{2} + 0.95\right)$

$z^* = \text{invNorm}(0.975) = 1.96$

Example 1: Taken from a large population, a sample size of 130 has $\bar{x} = 72.5$ and $\sigma = 5.24$, draw the distribution that indicates a 90% confidence level and state the confidence interval. Report your final answer in complete sentences.



$z^* = \text{invNorm}(0.95) = 1.645$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{5.24}{\sqrt{130}}$$

$$\sigma_{\bar{x}} = 0.4595784021$$

$X_{lower} = \text{invNorm}(0.05, 72.5, 0.4595784021)$
$X_{upper} = \text{invNorm}(0.95, 72.5, 0.4595784021)$

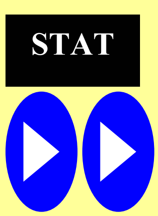
$X_{lower} = 71.744$ $\quad X_{upper} = 73.256$

Margin of Error $= \pm z^* \sigma_{\bar{x}}$
$= \pm 1.645 \times 0.45958$

Margin of Error = ± 0.756

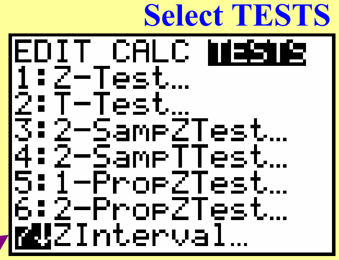
We can say that we are 90% confident that the scores are in the range of 72.5 ± 0.756 from a sample size of 130.

Using TI-83 Plus Calculator

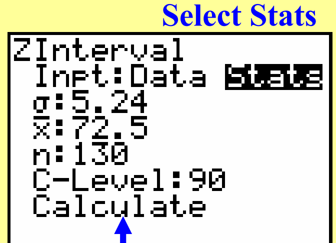


Select Option 7

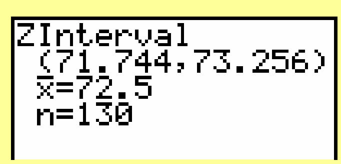
Select TESTS



Select Stats



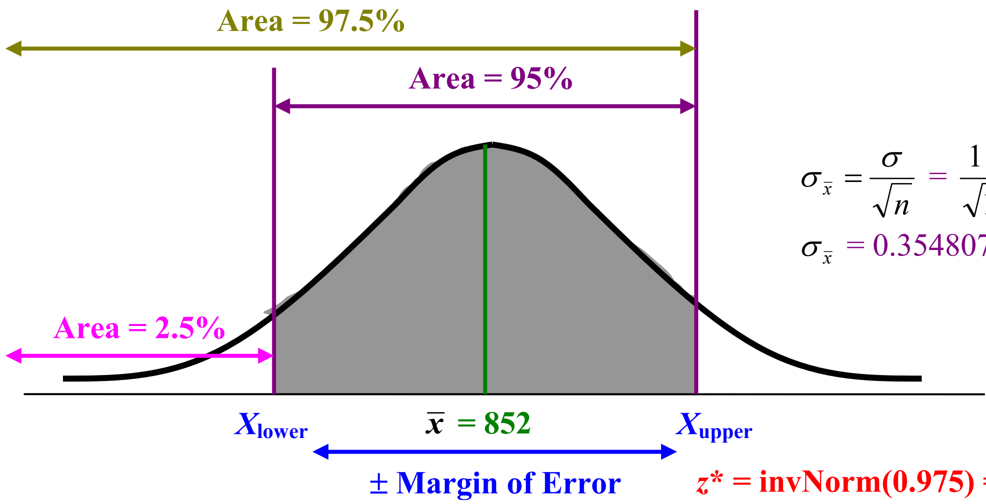
Press Enter on Calculate



$m = \pm(73.256 - 72.5)$

$m = \pm 0.756$

Example 2: From a large population, a random survey of 1000 people, 852 of them believe that the government should regulate the electricity industry. If the standard deviation, σ = 11.22, draw the distribution that indicates 95% confidence level and state the confidence interval. Report your final answer in complete sentences.



$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{11.22}{\sqrt{1000}}$$

$$\sigma_{\bar{x}} = 0.3548075535$$

z^* = invNorm(0.975) = 1.96

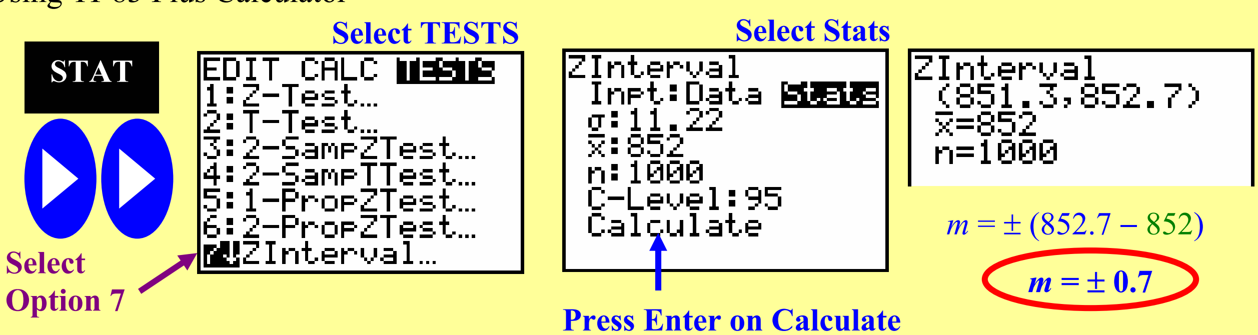
X_{lower} = invNorm(0.025, 852, 0.3548075535)
X_{upper} = invNorm(0.975, 852, 0.3548075535)

X_{lower} = 851.3 **X_{upper} = 852.7**

Margin of Error = $\pm z^* \sigma_{\bar{x}}$
= ± 1.96 × 0.35481

Margin of Error = ± 0.7

Using TI-83 Plus Calculator



$m = \pm (852.7 - 852)$

$\boldsymbol{m = \pm 0.7}$

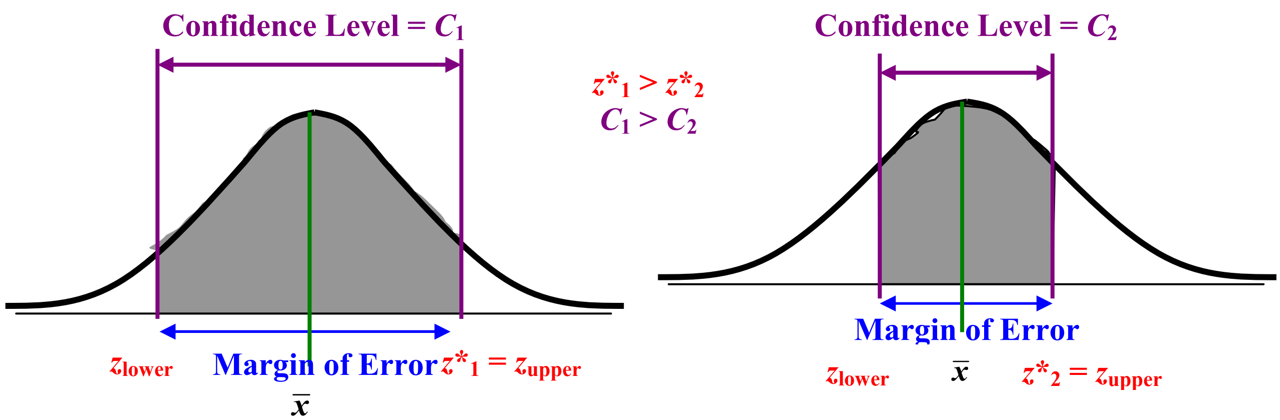
We can say that we are 95% confident that the survey is in the range of 852 ± 0.7 from a sample size of 1000.

10.1A Assignment
pg. 512–513 #10.1 and 10.3;
pg. 518–519 #10.5 and 10.7

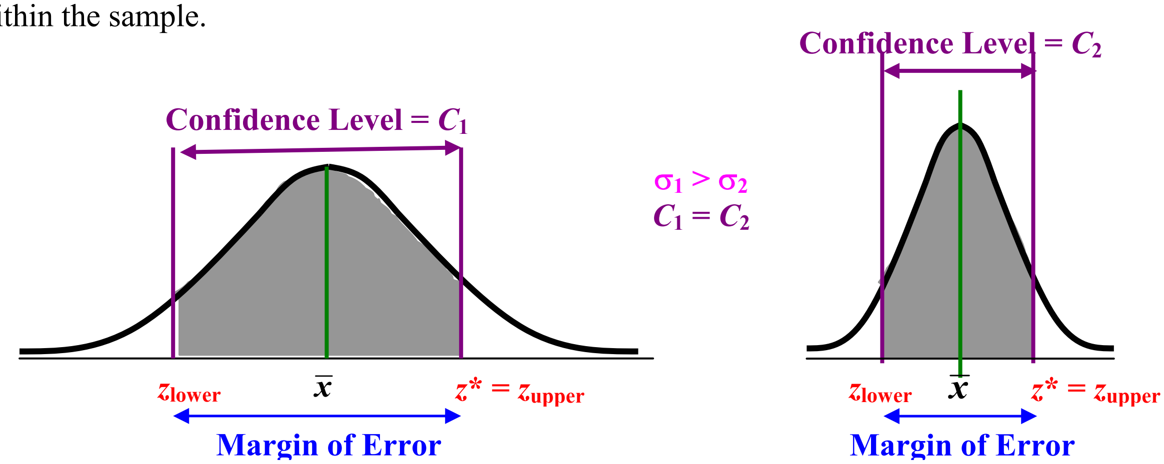
10.1B: Behaviours of Confidence Interval and Sample Size

Behaviours of Confidence Interval and Sample Size

1. **A decrease in z^* lowers the margin of error (increasing accuracy), but the level C decreases (less reliable).** This is because the area of the curve gets smaller as $\pm z$ from the mean shrinks.



2. **As σ decreases, so will the margin of error (increasing accuracy)** because there is less variation within the sample.



3. **As n increases, the margin of error decreases (increasing accuracy).** This is due to the law of large numbers – as n gets larger, the sample becomes the population. $m\downarrow = z^*\dfrac{\sigma}{\sqrt{n}\uparrow}$

Sample Size for Desired Margin of Error: - to calculate the **sample size (n) needed to yield a specific margin of error (m), we have to know the standard deviation of the population (σ)** and the **confidence interval** (so we can find z^*).

Sample Size for Desired Margin of Error
$z^*\dfrac{\sigma}{\sqrt{n}} \leq m$ or $n \geq \left(z^*\dfrac{\sigma}{m}\right)^2$

Example 1: For a margin of error of ± 0.02 with 95% confidence interval, what will be the sample size needed if the standard deviation of the population is 0.0325?

$m = 0.02$ $\sigma = 0.0325$
$C = 0.95$
$z^* = \text{invNorm}(0.975) = 1.95996$
$n = ?$

$$n \geq \left(z^* \frac{\sigma}{m}\right)^2 \quad n \geq \left(1.95996 \times \frac{0.0325}{0.02}\right)^2 \quad n \geq 10.144$$

$n = 11$ (always ROUND UP!)

Cautious Notes about Confidence Interval

1. The data **MUST be a Simple Random Sample of a Population**.
2. The Confidence Interval formula is **ONLY appropriate for Simple Random Sample Design**. Other more complex sampling methods like multistage and stratified Samplings have confidence intervals using different formulas.
3. The **Standard Deviation of the Population, σ,** **and the** **Sample Size, *n***, **must be known**.
4. **Outliers can greatly affect confidence interval** due to its strong effect on the mean.
5. **Small Sample Size couple with Non-Normal Distribution** can make confidence interval **different than the true *C*.**
6. **Do NOT use the word "probability" to describe confidence interval**. **The confidence interval, *C*, describes only the PROPORTION of repeated samples that fall within the margin of error**.

10.1B Assignment
pg. 521–522 #10.9 and 10.11;
pg. 523–524 #10.13; pg. 526 #10.17;
pg. 528–529 #10.19 and 10.21

10.2A: Tests of Significance

Significance Test: - a test that asks the question, "**is the Sampling Mean ($\bar{x}$) of some Statistical Significance from the Population Mean (μ_0 – population mean being tested against)?**"

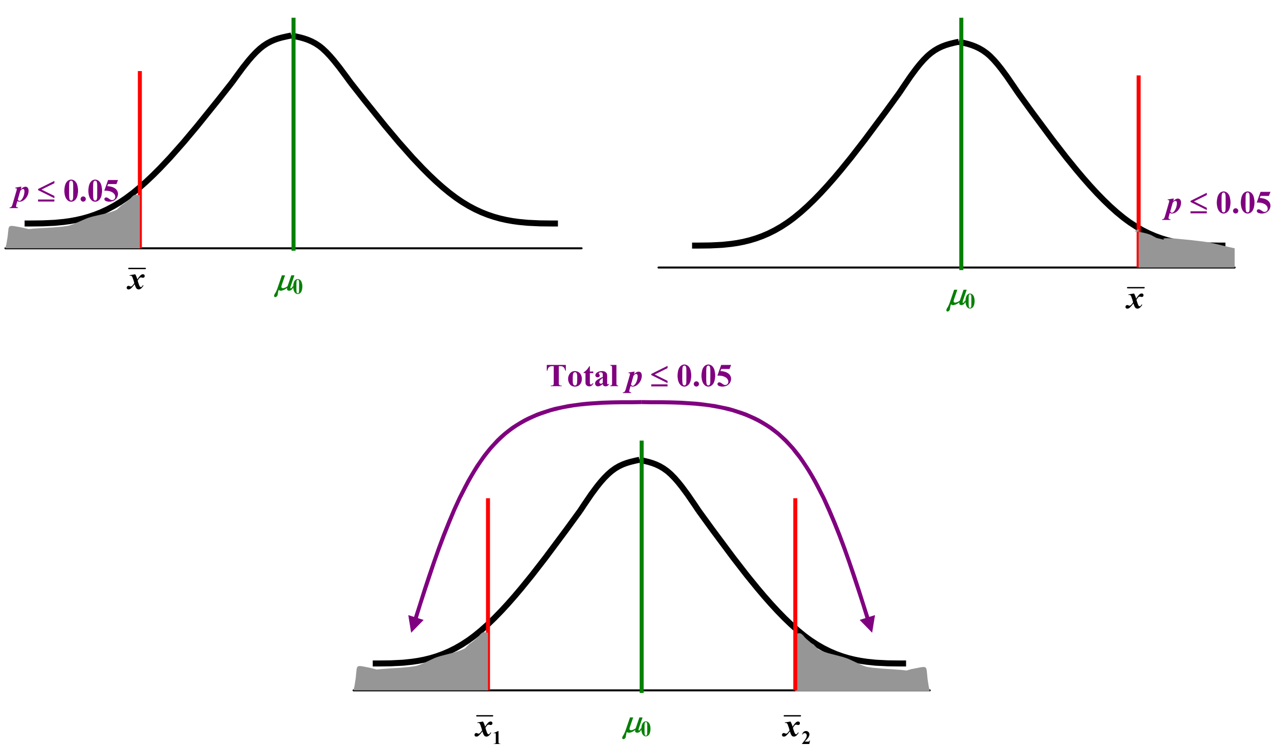
Statistical Significant: - when the **Sampling Mean is Far Enough Away from the Population Mean** that we can **conclude the Sampling result is NOT Reflective of the Population.** (It is likely due to some other reason for the difference other than chance alone.)

Test Statistics: - the sampling statistics (usually the **Sampling Mean** and **Sampling Standard Deviation**), that is being tested for statistical significance against a population parameter (usually the Population Mean).

Example: The male population of United States has a average life expectancy of 79 years of age in 2002. A survey of 30 male fire fighters that have passed away in 2002 has a mean life expectancy of 65 years of age. Is the mean life expectancy of fire fighters statistically significant from that of the country's population?

***P*-Value (*p*)**: - the probability (area under the normal curve) between the Sampling Mean and positive or negative infinity.
- **when *P*-Value is LESS than a Statistically Significant Level (α) – we usually set $\alpha \leq 0.05$, then the sampling mean is deemed statistically significant from the population mean.**

Normal Curves describing Statistical Significance



Three Steps of the Significance Test:

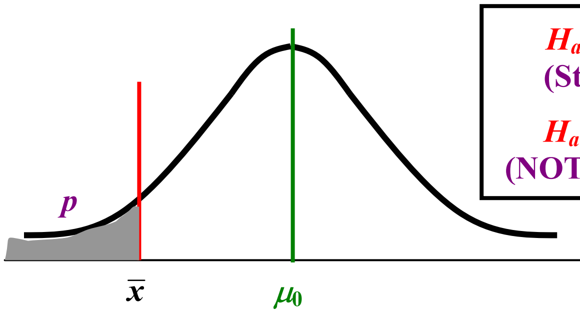
1. **Stating Hypothesis:**

   a. **Null Hypothesis** (H_0): - the hypothesis that assumes the **sampling mean** **is NOT Significantly Away from** **the population mean**. (H_0: $\mu = \mu_0$)
   - that any difference between the sampling mean and the population mean is just by chance.

H_0 is True when $p > 0.05$ (NOT Statistically Significant)	H_0 is False when $p \le 0.05$ (Statistically Significant)

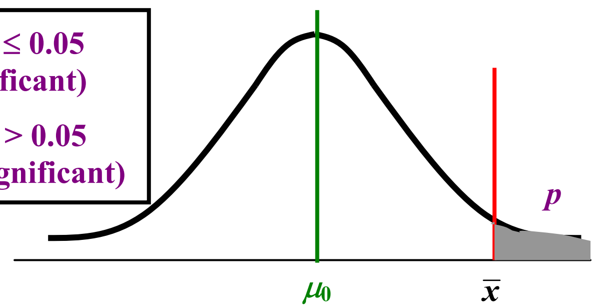
   b. **Alternative Hypothesis** (H_a): - the hypothesis that assumes the **sampling mean** **IS Significantly Away from** **the population mean**. (H_a: $\mu > \mu_0$, or H_a: $\mu < \mu_0$, or H_a: $\mu \ne \mu_0$)

   i. **One-Sided Alternative**: - when the alternative hypothesis describes the sampling means that are greater than or less than the population's mean. (H_a: $\mu > \mu_0$, or H_a: $\mu < \mu_0$)



H_a is True when $p \le 0.05$
(Statistically Significant)

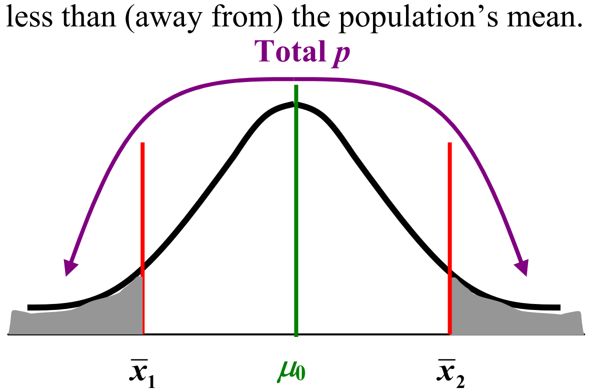
H_a is False when $p > 0.05$
(NOT Statistically Significant)



   ii. **Two-Sided Alternative**: - when the alternative hypothesis describes the sampling means that are greater than and less than (away from) the population's mean. (H_a: $\mu \ne \mu_0$)

H_a is True when Total $p \le 0.05$
(Statistically Significant)

H_a is False when Total $p > 0.05$
(NOT Statistically Significant)



2. **Test Statistics**: - sometimes the individual scores are given, so the calculation of the sampling mean ($\bar{x}$) becomes necessary.
   - in any event, we will need sampling standard deviation, $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

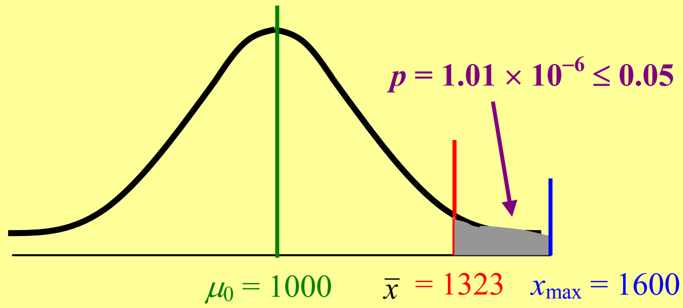
3. ***P*-Value**: - calculate ***P*-Value** by using **normalcdf function** in TI-83 Plus or looking it up **from the Table**.

Example 1: The mean SAT scores last year was 1000 with a standard deviation of 152. Five students wrote the SAT last year and its mean was 1323 with the maximum score being 1600. Is this good evidence that the five students scores were well above average of the population? (State the hypothesis. Calculate the test statistics and *P*-value. Draw a Normal curve to supplement your conclusion.)

$\mu_0 = 1000$ $n = 5$
$\sigma = 152$ $\bar{x} = 1323$
$x_{max} = 1600$

1. Stating Hypothesis

H_0: $\mu = 1000$
H_a: $\mu > 1000$



2. Calculate **Test Statistics**

$\bar{x} = 1323$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{152}{\sqrt{5}}$$

$\sigma_{\bar{x}} = 67.97646652$

3. Calculate ***P*-Value**

p = normalcdf(1323, 1600, 1000, 67.97646652)

$p = 1.01 \times 10^{-6}$

Since $p \leq 0.05$, H_a is true and H_0 is false. Therefore, the five students' scores on the SAT last year were significantly above the population mean.

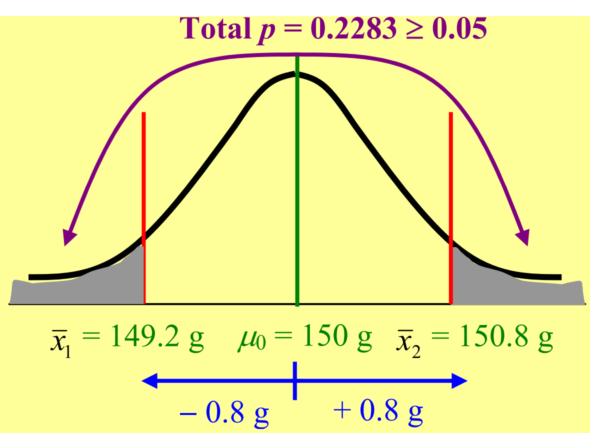
Example 2: A candy bar company makes a product that has a net mean weight of 150 g with a standard deviation of 2.1 g. The quality controller randomly selected 10 candy bars throughout the day and measured their weights shown below. Do the sampling data provide sufficient evidence that the assembly line is not producing the required weight of 150 g per candy bar?

148 g, 155 g, 149 g, 154 g, 155 g, 147 g, 146 g, 148 g, 150 g, 156 g

$\mu_0 = 150$ g
$\sigma = 2.1$ g
$n = 10$

1. Stating Hypothesis

H_0: $\mu = 150$
H_a: $\mu \neq 150$



2. Calculate **Test Statistics**

$\bar{x} = 150.8$ (from 10 samples)

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.1}{\sqrt{10}}$$

$\sigma_{\bar{x}} = 0.6640783086$

3. Calculate ***P*-Value**

p = 1 − normalcdf(149.2, 150.8, 150, 0.6640783086)

$p = 0.2283$

Since $p \geq 0.05$, H_a is false and H_0 is true. Therefore, the five students' scores on the SAT last year were significantly above the population mean.

10.2A Assignment

pg. 536 #10.27; pg. 539–540 #10.29 and 10.31; pg. 542 #10.33 and 10.35

10.2B: Test for Population Mean, Fixed Significance Level and Confidence Intervals

One-Sample z-Test: - a significance test comparing the sample mean from a single random variable after it has converted to a z-score with the population mean as $z = 0$.

One-Sample z-test statistics ($z_{\bar{x}}$): - when the sample mean from a single variable is converted to a z-score for the comparison with the population mean using the sample standard deviation.

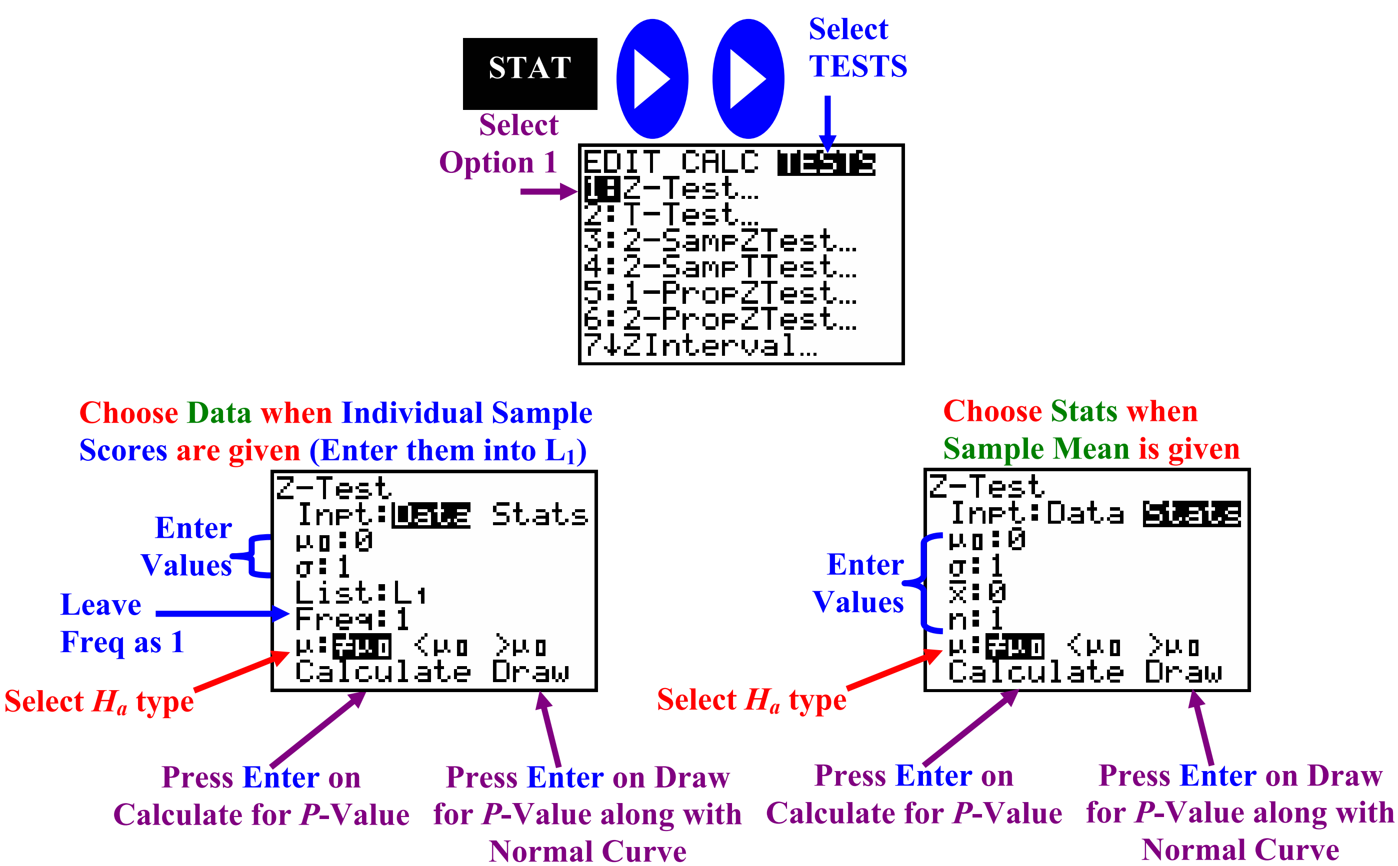
One-Sample z-Test Statistics

$$z_{\bar{x}} = \frac{\bar{x} - \mu_0}{\left(\sigma / \sqrt{n}\right)} = \frac{\sqrt{n}\left(\bar{x} - \mu_0\right)}{\sigma}$$

Three Steps of One-Sample z-Test for a Population Mean:

1. **State the Hypothesis (H_0: $\mu = \mu_0$ and H_a).**
2. **Calculate the One-Sample z-test statistics.**
3. **Determine the P-Value.**

Z-Test function on TI-83 Plus



Fixed Significance Level One-Sample *z*-Tests for a Population Mean

Given a fixed significance level, α, the CONCLUSION can be drawn after *z*-test is performed.

1. **If $p \leq \alpha$, then H_a is True (Statistically Significant)**
2. **If $p > \alpha$, then H_0 is True (NOT Statistically Significant)**

Example 1: A recent news article stated that 48% of dogs and cats kept as pets are overweight. It referenced the finding from the *Journal of American Veterinary Medical Association* along of the standard deviation of 5.87%. After reading the article, a local veterinarian randomly sampled 20 pets in her care and found that 9 of them were overweight. Using the significance level of 0.02, does this evidence suggest the pets under her care are significantly healthier than that of the population?

$\mu_0 = 0.48$ $\quad n = 20$

$\sigma = 0.0587$ $\quad \alpha = 0.02$

$\bar{x} = \dfrac{9}{20} = 0.45$

1. Stating Hypothesis

H_0: $\mu = 0.48$

H_a: $\mu < 0.48$

2. Calculate *z*-test statistics

$$z = \frac{\bar{x} - \mu_0}{\left(\sigma/\sqrt{n}\right)} = \frac{(0.45 - 0.48)}{\left(0.0587/\sqrt{20}\right)}$$

$z = -2.285589074$

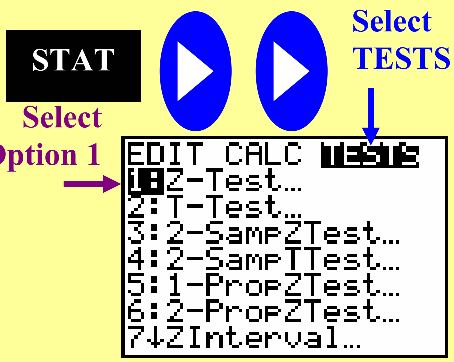
3. Calculate *P*-Value

$p = \text{normalcdf}(-1 \times 10^{99}, -2.285589074)$

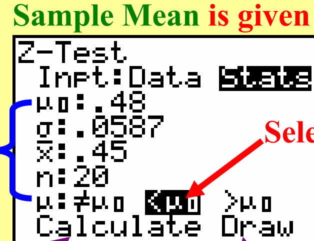
$p = 0.01114$

Since p (0.01114) $\leq \alpha$ (0.02), H_a is true and H_0 is false. Therefore, the pets under the local veterinarian's care are healthier than that of the population.

Using *Z*Test of the TI-83 Plus Calculator,



Choose Stats when Sample Mean is given



Enter Values

Select H_a type

Press Enter on Calculate for *P*-Value

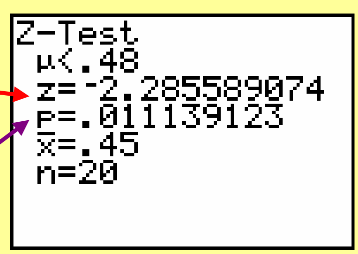
Press Enter on Draw for *P*-Value along with Normal Curve

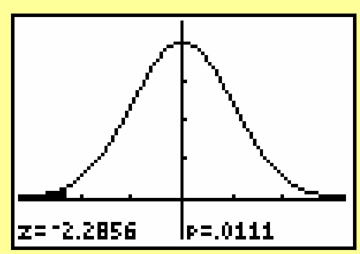
$$z = \frac{\bar{x} - \mu_0}{\left(\sigma/\sqrt{n}\right)} = \frac{(0.45 - 0.48)}{\left(0.0587/\sqrt{20}\right)}$$

$z = -2.285589074$

$p = \text{normalcdf}(-1 \times 10^{99}, -2.285589074)$

$p = 0.01114$





Example 2: A leading auto manufacturer claimed that a popular version of its mini-van was available in California for an average price of \$24,000 with a standard deviation of \$800. A consumer group doubted the report and survey 10 recent purchase of the min-van in order to dispute the claim. The purchase prices were as follows

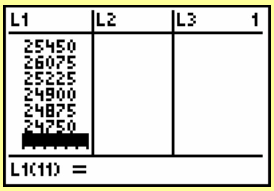
\$24,200	\$23,895	\$23,875	\$24,750	\$25,450
\$26,075	\$25,225	\$24,900	\$24,875	\$24,750

Using the significance level of 0.01, does this evidence suggest the sample taken by the consumer group is significantly different than the manufacturer's claim?

We have to enter the individual samples in L_1 of the Stats Editor

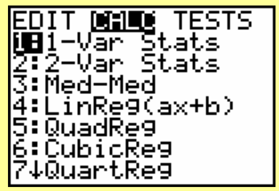
STAT

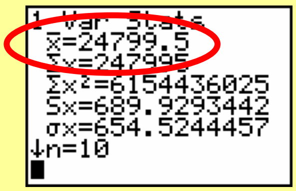
ENTER



Then, we run 1-Var Stats to obtain the Sample Mean, $\bar{x}$.

STAT ▶ **ENTER**





μ_0 = \$24,000
σ = \$800
$\bar{x}$ = \$24,799.50
n = 10
α = 0.01

1. Stating Hypothesis

H_0: μ = \$24,000
H_a: $\mu \neq$ \$24,000

2. Calculate **z-test statistics**

$$z = \frac{\bar{x} - \mu_0}{\left(\sigma / \sqrt{n}\right)} = \frac{(\$24{,}799.50 - \$24{,}000)}{\left(\$800 / \sqrt{10}\right)}$$

$$z = \pm 3.160301237$$

(± because this is a two-sided alternative – H_a: $\mu \neq \mu_0$)

3. Calculate ***P*-Value**

$$p = 1 - \text{normalcdf}\,(-3.160301237,\ 3.160301237)$$

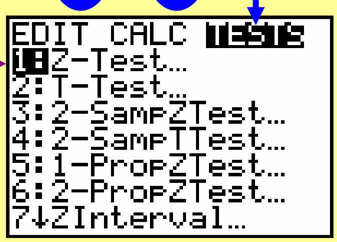
$$p = 0.001576$$

Since p (0.001576) ≤ α (0.01), H_a is true and H_0 is false. Therefore, the manufacturer's claim that the mini-van has a mean price of \$24,000 was significantly undervalued.

Using *Z*Test of the TI-83 Plus Calculator,

STAT ▶ ▶ **Select TESTS**

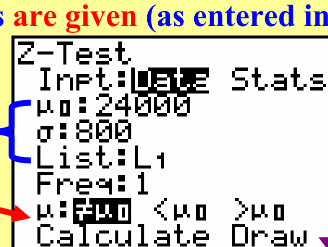
Select Option 1



Choose Data when Individual Sample Scores are given (as entered into L_1)

Enter Values

Select H_a type



Press Enter on Calculate for *P*-Value

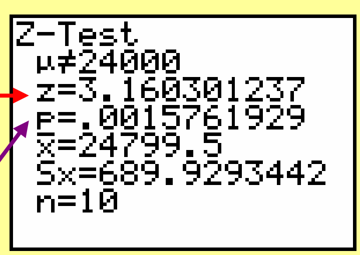
Press Enter on Draw for *P*-Value along with Normal Curve

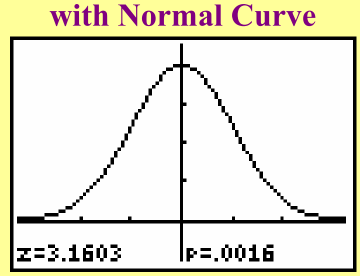
$$z = \frac{\bar{x} - \mu_0}{\left(\sigma / \sqrt{n}\right)} = \frac{(\$24{,}799.50 - \$24{,}000)}{\left(\$800 / \sqrt{10}\right)}$$

$$z = -2.285589074$$

$$p = 1 - \text{normalcdf}\,(-3.160301237,\ 3.160301237)$$

$$p = 0.0015761929$$



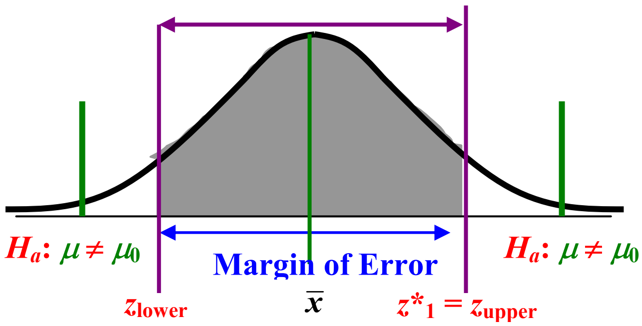


z-Confidence Intervals with One-Sample Two-Sided z-Tests

A Confidence Interval of a Sample can be tested for Significance (Level $C = 1 - \alpha$). The CONCLUSION can then be drawn.

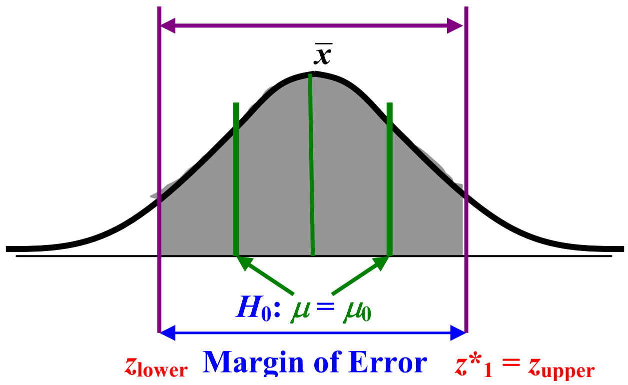
1. **If $\mu_0 < (\bar{x} - m)$ or $\mu_0 > (\bar{x} + m)$, then H_a is True (Statistically Significant) as μ_0 falls OUTSIDE Level C of the sample.**

Confidence Level = $C = 1 - \alpha$



2. **If $(\bar{x} - m) < \mu_0 < (\bar{x} + m)$, then H_0 is True (NOT Statistically Significant) as μ_0 falls WITHIN Level C of the sample.**

Confidence Level = $C = 1 - \alpha$



Example 3: A drug company sampled 30 pills from 30 different bottles and have found the dosage has a confidence interval of 148 mg ± 4 mg at Level $C = 0.98$. The dosage for each pill is labelled 150 mg. Are the pills sampled have a significantly different dosage than the label?

Confidence Interval at 148 mg ± 4 mg has a range of (144 mg, 152 mg) at Level $C = 0.98$ ($\alpha = 1 - C = 0.02$). Since $\mu_0 = 150$ mg is WITHIN this range, the sample of 30 pills is NOT Significantly Different at level $\alpha = 0.02$ than what is stated on the label.

10.2B Assignment

pg. 549 #10.39; pg. 553 #10.41; pg. 555 #10.44; pg. 557–559 #10.47, 10.49, 10.53. 10.55 and 10.57

10.3: Using Significance Tests

Purpose of Calculating *P*-Value: - to provide evidence **against** the **null hypothesis** (H_0).

Purpose of Setting a Level of Significance (α): - to make decision or take action **if the evidence reaches a certain pre-set standard**.
- **CANNOT be use ONLY to describe the Strength of the evidence**, other descriptions such correlation coefficient, confidence interval or *P*-Value must also be present to support the case.

Choosing a Level of Significance

1. The tradition significance level is at α = 0.05. However, we can choose level lower than 0.05 depending on the situation.

a. **Plausibility of H_0**: - the **stronger the assumption, the smaller α** needs to be to provide evidence for H_a.

b. **Consequences of Rejecting H_0**: - the **more serious are the consequences for accepting H_a, the smaller α** is needed to convince the case for H_a.

2. Because choosing α can be subjective, ***P*-Value should always be stated** to let others decide other α.

3. **The strength of the evidence lies in the *P*-Value. The judgement of "significant" versus "insignificant" depends on the setting of α level by an individual**.

4. **Statistical Significance is NOT the same as Practical Significance**. **Large Sample Size** (means smaller sample standard deviation - $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$) **can cause tiny deviations from the null hypothesis significant**.

5. **Low Significance Level does NOT imply a definite strong association between two variables**. It only means there is strong evidence of some association. In other words, ***P*-Values can merely "Suggestive", and NOT Conclusive.**

6. **Beware of Outliers, which can affect the means of the population and sample**.

7. **Always provide Confidence Interval as well as the *P*-Value**.

8. **Significance test gives no meaning when surveys are badly designed surveys** because they produce invalid results.

    Example: The ***Hawthorne Effect*** results in more desirable outcome due to the subjects' knowledge that they are being tested and measured against a known standard.

9. **Multiple Analyses (testing many hypotheses simultaneously) without any controlled variables gives no meaning to significance test**. This is because you can only evaluate evidence between the population and sample mean of one variable at a time.

10. **Real evidence can be taken seriously if and only if a hypothesis is stated, a reliable and randomized survey or controlled experiment is carried out, and finally the data is tested for significance**.

10.3 Assignment
pg. 563 #10.59; pg. 565 #10.61; pg. 567 #10.63 and 10.65

10.4: Inference as Decisions

Acceptance Sampling: - the purpose and the end result of a sampling is make a decision to either accept or reject H_0.
- a **significance level is decided BEFORE** the sampling is carried out. (This is very different than simply stating the P-value and let the user of the statistics decide on the significance level **AFTER** the sampling.)

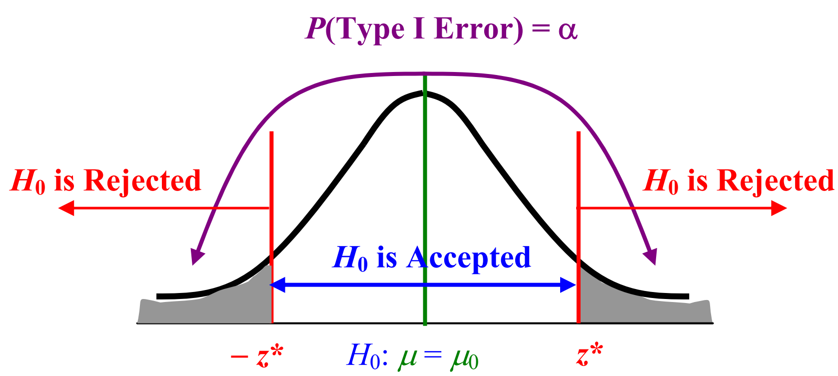
Error Probabilities: - the probabilities of error made rejecting one hypothesis over another.
- most commonly result from the inherit inaccuracy common in all measuring devices.

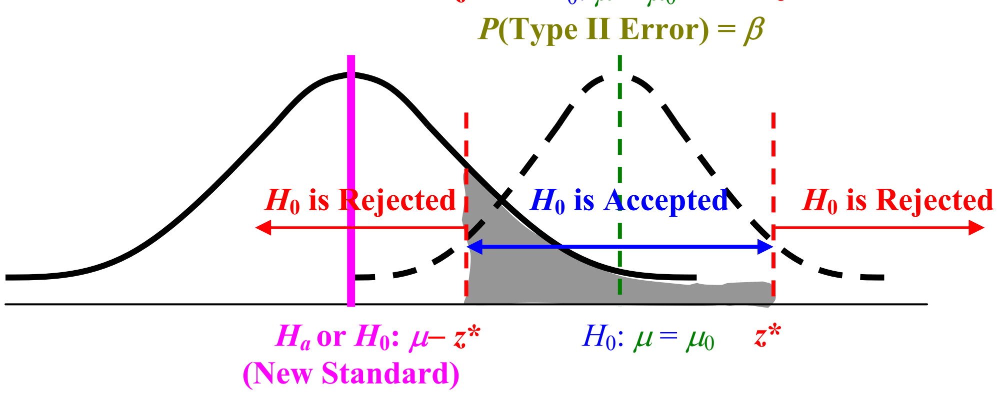
1. **Type I Error** (α): - an error made when **we reject H_0 or accept H_a** when **in fact H_0 is true** (sometimes refer to as ***"false negative"***).
   - **equals to the significance level of any fixed level tests**.

2. **Type II Error** (β): - an error made when **we accept H_0 or reject H_a** when **in fact H_a is true** (sometimes refer to as ***"false positive"***).
   - a **new standard of acceptance (H_a or updated H_0) is usually set**. It is then **measured against the pre-set significance level of the old H_0**.

Actual Truth about Population

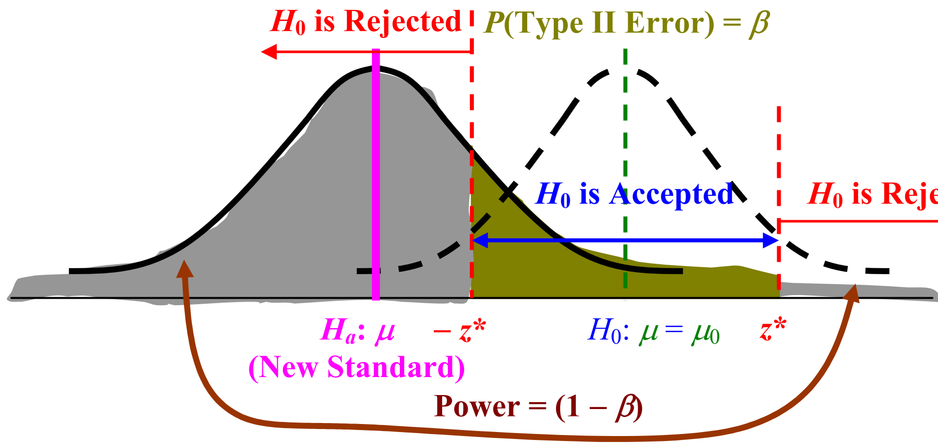
Decision made from Sample		H_0 True	H_a True
	Reject H_0 / Accept H_a	Type I Error	Correct Decision
	Accept H_0 / Reject H_a	Correct Decision	Type II Error





Power: - the probability that a fixed level α significance test will reject H_0 when a particular alternative value of the parameter is true (supposing H_a is correct).

- **the complement of a type II error (1 – β).**
- a **high probability of power means the test is NOT sensitive enough to detect the H_a case**.



Example 1: There had been many controversies regarding the accuracy of polygraph tests used in criminal investigations. A polygraph typically measures five bodily functions. They are thoracic respiration, abdominal respiration, blood pressure / pulse rate, muscular movement / pressure, and galvanic skin response. The magnitudes of these responses presumably indicate whether the subject is telling the truth. Five experienced polygraph examiners were given a set of 40 records, 20 from innocent subjects and 20 from guilty ones. Each subject had been asked five questions. On the basis of each question, the examiner was to make a summary judgement: "innocent" or "guilty". The results are shown below:

		Actual Status	
		Innocent	**Guilty**
Examiner's Decision	**Innocent**	94	11
	Guilty	6	89

Describe the Type I and Type II errors in this context. What are the their respective error probabilities?

We assume that people are asked to take a polygraph test when they claim they are innocent but there is a reason to believe they are guilty. Hence H_0 is true when the subject is Innocent and H_a is true when the subject is Guilty.

Type I Error: When examiners decide that subjects are guilty when in fact they are innocent.
Type II Error: When examiners decide that subjects are innocent when in fact they are guilty.

$$P(\text{Type I Error}) = \frac{\text{Guilty Cases as decided by Test} \mid \text{Actually Inncoent}}{\text{All Cases Actually Innocent}} = \frac{6}{(94+6)}$$

P(Type I Error) = 0.06

$$P(\text{Type II Error}) = \frac{\text{Innocent Cases as decided by Test} \mid \text{Actually Guilty}}{\text{All Cases Actually Guilty}} = \frac{11}{(11+89)}$$

P(Type II Error) = 0.11

Example 2: A local school district has a mean IQ test score of 108 with a standard deviation of 15. A sample of 10 students' IQ test scores from a particular school is taken.

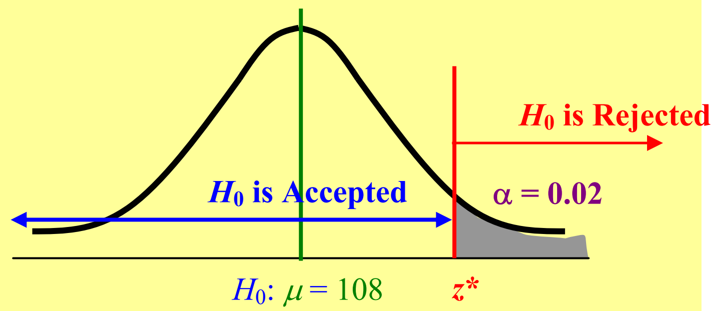
a. A high school counsellor would like to test at the significance level of 0.02 the proportion of the students above the mean. What is the rule for rejecting H_0 in terms of z-statistic?
b. Describe Type I error and find the error probability.
c. The counsellor later learned that the population mean of a much larger population is 100. Describe Type II error and find the error probability when $\mu = 100$.
d. Find the power against $\mu = 100$ and evaluate the test sensitivity for H_a.

a. Rule for Rejecting H_0 in terms of z-statistic

$\mu_0 = 108$
$\sigma = 15$
$n = 10$
$\alpha = 0.02$

1. Stating Hypothesis

H_0: $\mu = 108$
H_a: $\mu > 108$



2. Find critical value (z*) using α

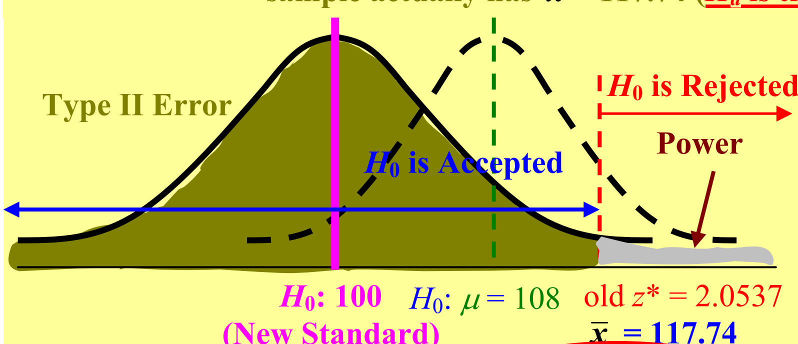
$z^* = \text{invNorm}(1 - 0.02) = 2.053748911$

3. **State the z-test statistic** that we will reject H_0

Reject H_0 when $z > z^*$: Since $z = \dfrac{\bar{x} - \mu_0}{\left(\sigma/\sqrt{n}\right)} = \dfrac{(\bar{x} - 108)}{\left(15/\sqrt{10}\right)}$, **we reject H_0 when** $\dfrac{(\bar{x} - 108)}{\left(15/\sqrt{10}\right)} \geq 2.053748911$

b. Type I Error: When the counsellor believes that H_a is true (z-statistic is above z^*, $z \geq 2.0537$) but the sample actually has $z < 2.0537$ (H_0 is true). **P(Type I Error) = α = 0.02**

c. Type II Error: When the counsellor believes that H_0 is true ($\bar{x} \leq 117.74$ with $\mu = 100$) but the sample actually has $\bar{x} > 117.74$ (H_a is true).



Raw Score of z^*:

$$\frac{(\bar{x} - 108)}{\left(15/\sqrt{10}\right)} = 2.053748911$$

$$\bar{x} = (2.053748911)\left(15/\sqrt{10}\right) + 108$$

$$\bar{x} = 117.7417864$$

P(Type II Error) = normalcdf(-1×10^{99}, 117.74, 100, $15/\sqrt{10}$)

P(Type II Error) = β = 0.99991

d. Power = $1 - \beta = 1 - 0.99991$ **Power = 0.00009**

Since Power is really small, the test is very sensitive for H_a.

10.4 Assignment
pg. 572–573 #10.66 to 10.68; pg. 575–576 #10.69 and 10.71; pg. 578 #10.73

Chapter 10 Review
pg. 580–582 #10.77 to 10.79, 10.81, 10.83 and 10.85

Chapter 11: Inference for Distributions

11.1A: Inference for the Mean of a Population

Assumptions for Inference about a Mean:

1. The data from our survey is from a **Simple Random Sample of size n from a Normal Population**.
2. **In reality, we do NOT know the exact mean (μ) and standard deviation (σ) of the population**. This is because to find the exact population mean and standard deviation requires one to do a census of the entire population. If the population is large, then it becomes an extremely difficult and time-consuming task. Therefore, any population mean and standard deviation stated as reference are either theoretical (as in the case of normal approximation to a binomial distribution), or from previous surveys taken.

Standard Error: - the standard deviation of a statistics as **estimated** from the sample data.
- instead of $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$, **standard error** = $\frac{s}{\sqrt{n}}$

***t*–Distribution**: - **the distribution as using standard error** as the standard deviation which is defined the **degree of freedom**.
- like the ***z*-distribution** (Normal Distribution) with actual or pretend population mean and standard deviation (***z* stands for *z*-score**).

Degrees of Freedom (*df*): - equal to **(n – 1)**. It is the same degree of freedom in the standard deviation formula, $s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{(n-1)}}$.
- measures the number of ways to compare with the sample mean based on the size of the sample.

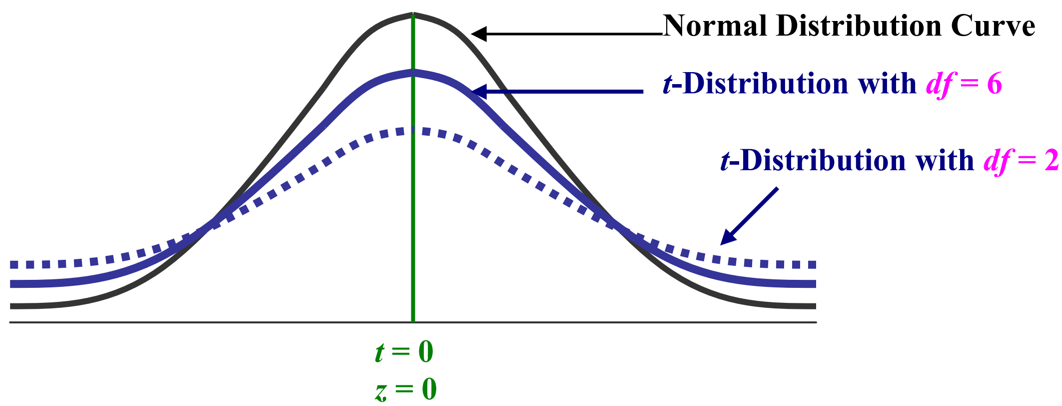
One-Sample *t*-Statistic: - the *t*-distribution of a sample using one random variable.

One-Sample *t*-Statistics

$$t_{\bar{x}} = \frac{\bar{x} - \mu}{\left(s_{\bar{x}} / \sqrt{n}\right)}$$ **with (n – 1) degrees of freedom (*df*)**

Characteristics of *t*-Distribution:

1. It has a **similar shape to the Standard Normal Curve** (**symmetric about 0 and bell-shaped**).
2. The **Spread is bigger than the Standard Normal Curve.** The **probability** (area under the curve) is **bigger at the tails and less at the center** compared to the normal distribution.
3. As the **degree of freedom increases, the shape of the *t*-distribution approaches the normal curve**. This is because of the law of large numbers, where the sample mean and sample standard deviation will approach the true population mean and standard deviation.

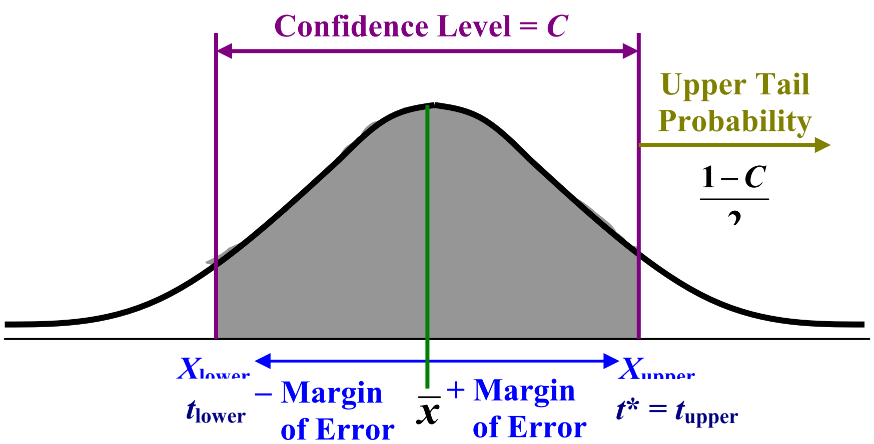


<u>*t*-Confidence Interval</u>: - as in confidence level of *z*-distribution, this is **an interval that shows the level of assurance** from a statistical report **as represented by the symmetrical area around the centre of a *t*-distribution**.

<u>Critical Value</u> (t^*): - the **<u>*t*-score</u>** needed on the upper side of the mean to achieve the confidence level.

One-Sample *t*-Confidence Intervals ($\bar{x}$ ± Margin of Error) at Level *C*

Margin of Error (*m*) = Critical Value (t^*) × Standard Error of Statistic ($\sigma_{\bar{x}} = \frac{s_{\bar{x}}}{\sqrt{n}}$)



One-Sample Confidence Interval for a Population Mean

$$\bar{x} \pm t^* \frac{s_{\bar{x}}}{\sqrt{n}} \quad \text{where } m = t^* \frac{s_{\bar{x}}}{\sqrt{n}}$$

***Note*: DO <u>NOT</u> use invNorm$\left(\frac{1-C}{2} + C\right)$ to find t^***

(Must use the Table on the next page to find t^* from Level *C* or Upper Tail Probability and *df*)

One-Sample Confidence Interval (<u>*T*Interval</u>) function on TI-83 Plus

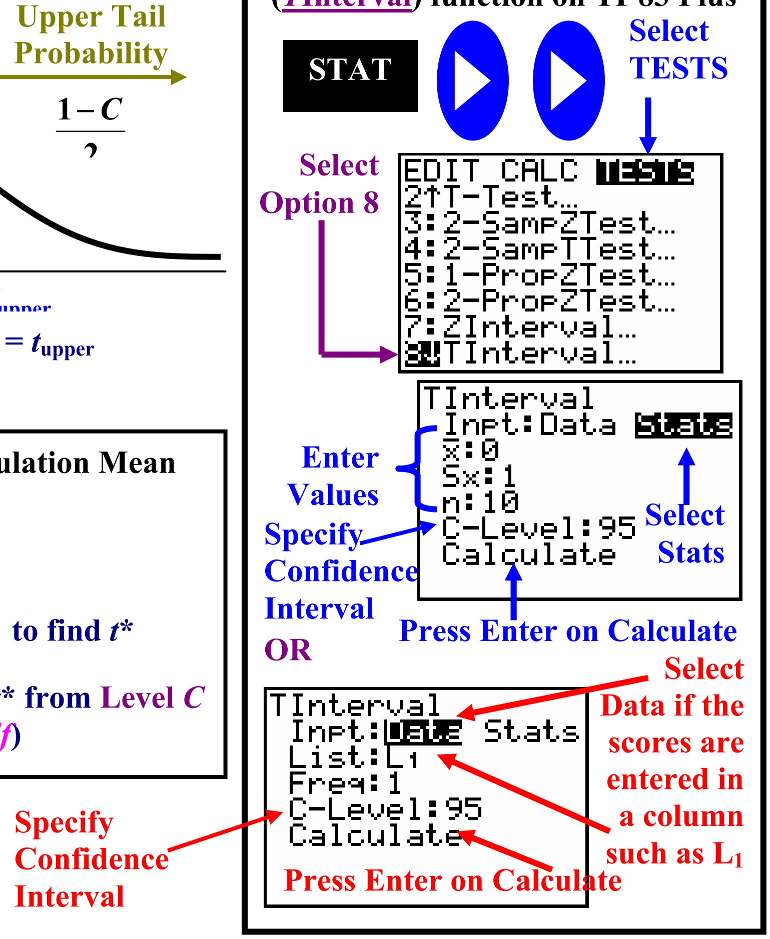


Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.

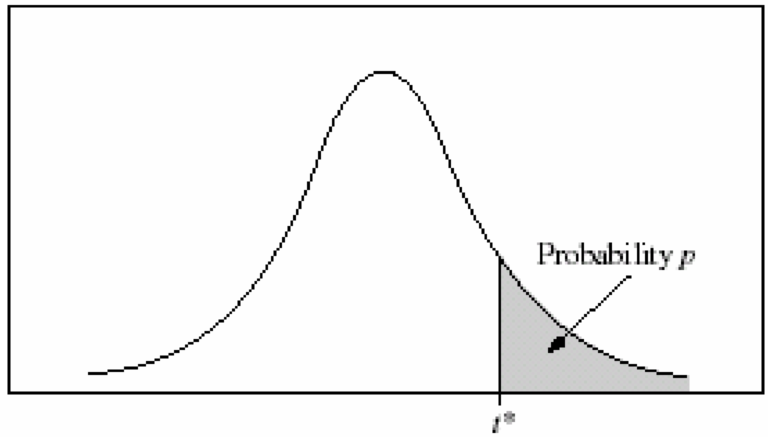


Table B t distribution critical values

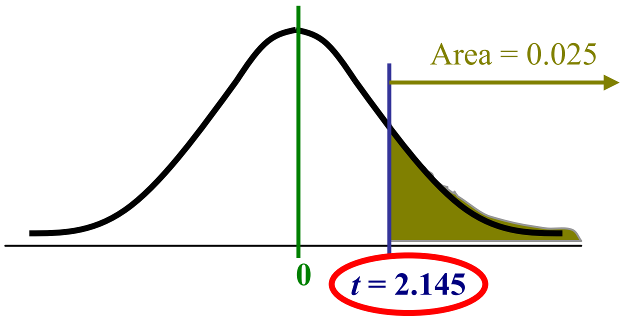
df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

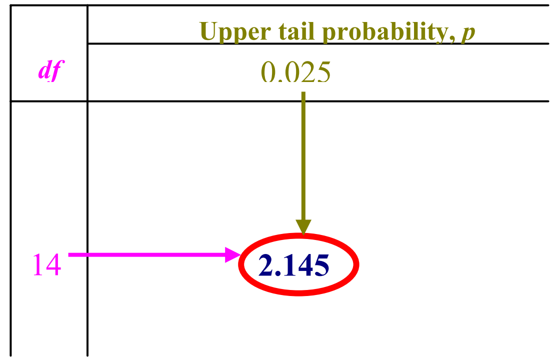
Using the *t*-Distribution Critical Values Table:

1. Converting Upper Tail Probability (Area RIGHT of the *t*-score boundary) to *t*-score

- **a.** Look up the ***df* (degree of freedom) from the row heading**.
- **b.** Look up the **upper tail probability, *p*, from the column heading**.
- **c.** Follow that row and column to find the ***t*-score**.

Example: Find t when $p = 0.025$ and $n = 15$ ($df = 14$).

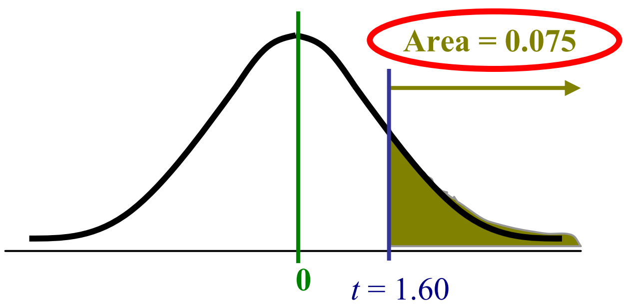


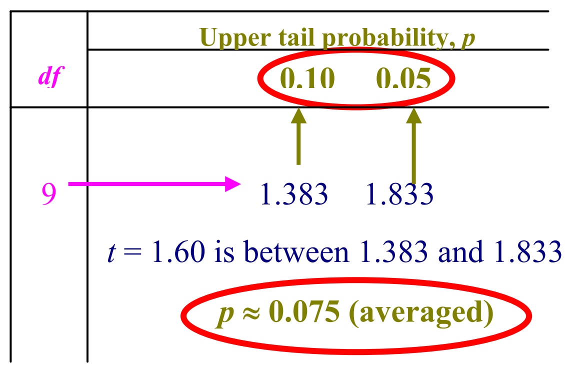


2. Converting *t*-score back to Upper Tail Probability (Area Right of the *t*-score boundary)

- **a.** Along the row of *df* (degree of freedom), look up the **closest *t*-score(s)** from **INSIDE** the table.
- **b.** Follow that column(s) back UP to the heading and locate the corresponding **upper tail probability**.
- **c.** May have to average the probabilities or guessed if the t-score used on the table is not exact.

Example: Find $P(t \geq 1.60)$ when $n = 10$ ($df = 9$)

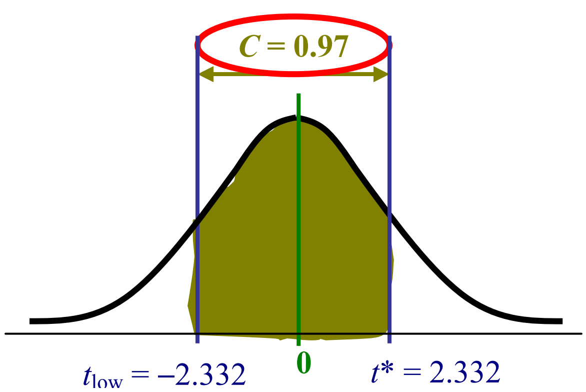


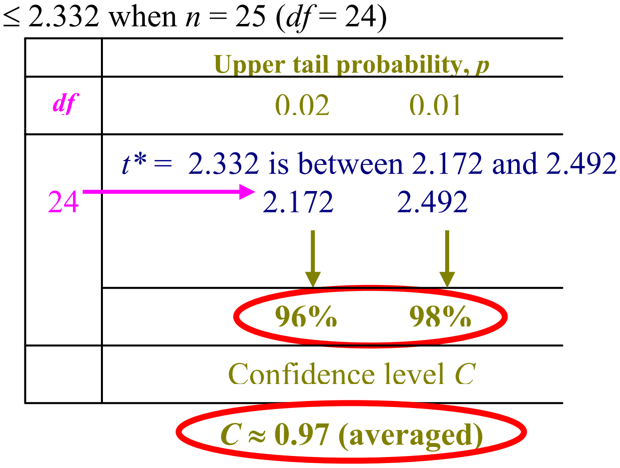


3. Converting t^* (critical *t*-score) back to Confidence Level *C* (Area Within the | *t* | boundary)

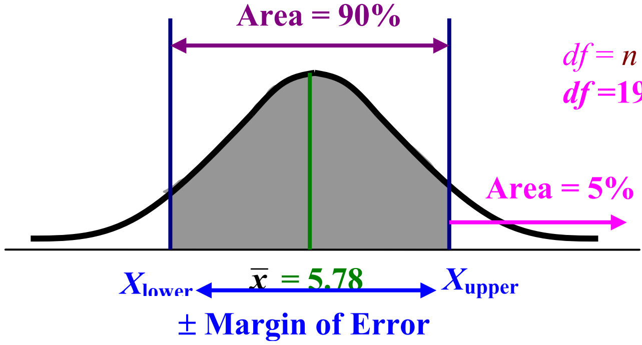
- **a.** Along the row of *df* (degree of freedom), look up the **closest t^*(s)** from **INSIDE** the table.
- **b.** Follow that column(s) back DOWN to the heading and locate the corresponding **Confidence Level**.
- **c.** May have to average the confidence levels or guessed if the t^* used on the table is not exact.

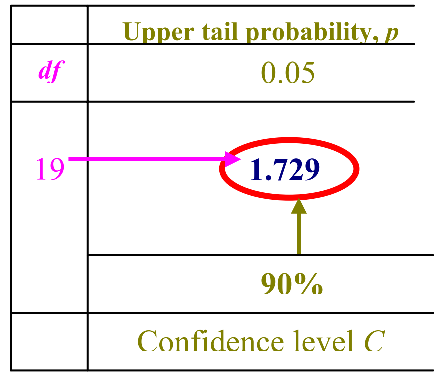
Example: Find Confidence Level, C, when $-2.332 \leq t \leq 2.332$ when $n = 25$ ($df = 24$)





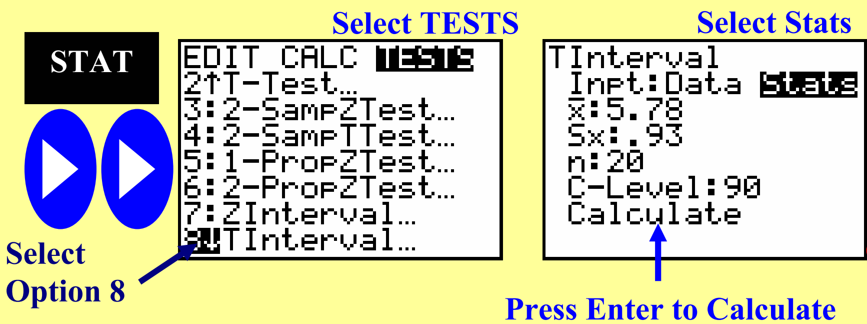
Example 1: Suppose a sample of size 20 from a normal distribution gives $\bar{x}$ = 5.78 and $s_{\bar{x}}$ = 0.93. Find the 90% *t*-confidence interval.

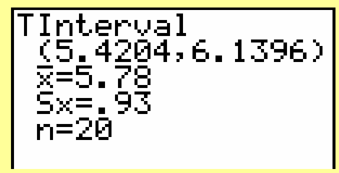




Confidence Interval = $\bar{x} \pm t^* \frac{s_{\bar{x}}}{\sqrt{n}} = 5.78 \pm 1.729 \times \frac{0.93}{\sqrt{20}}$ **Confidence Interval = 5.78 ± 0.3596**

Using TI-83 Plus Calculator





$m = \pm (6.1396 - 5.78) = \pm$ **0.3596**

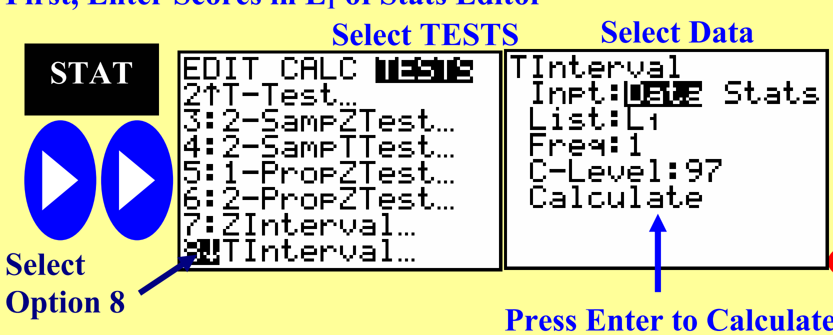
Confidence Interval = 5.78 ± 0.3596

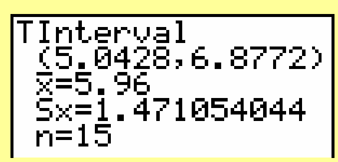
We can say that with 90% confidence that the scores are in the range of 5.78 ± 0.3596 from a sample size of 20.

Example 2: Staffs from 15 different hospitals participated in a surveillance program to monitor the number of patients experiencing adverse reactions to prescribed medication. The percentages for the 15 hospitals are listed below. Find the 97% *t*-confidence interval for the percentage of patients that developed adverse reactions with prescribed medication.

5.8 5.3 4.5 3.9 4.6 5.4 7.9 8.2 6.9 5.7 4.6 6.3 8.4 4.6 7.3

First, Enter Scores in L_1 of Stats Editor





$m = \pm (6.8772 - 5.96) = \pm$ **0.9172**

Confidence Interval =(5.96 ± 0.9172)%

We can say with 97% confidence that there are 5.96% ± 0.9172% of the patients that will develop adverse reactions with prescribed medication from a sample size of 15.

One-Sample *t*-Test: - a significance test comparing the sample mean after it has converted to a *t*-score with the population mean (μ_0) as $t = 0$.

Three Steps of One-Sample *t*-Test for a Population Mean:

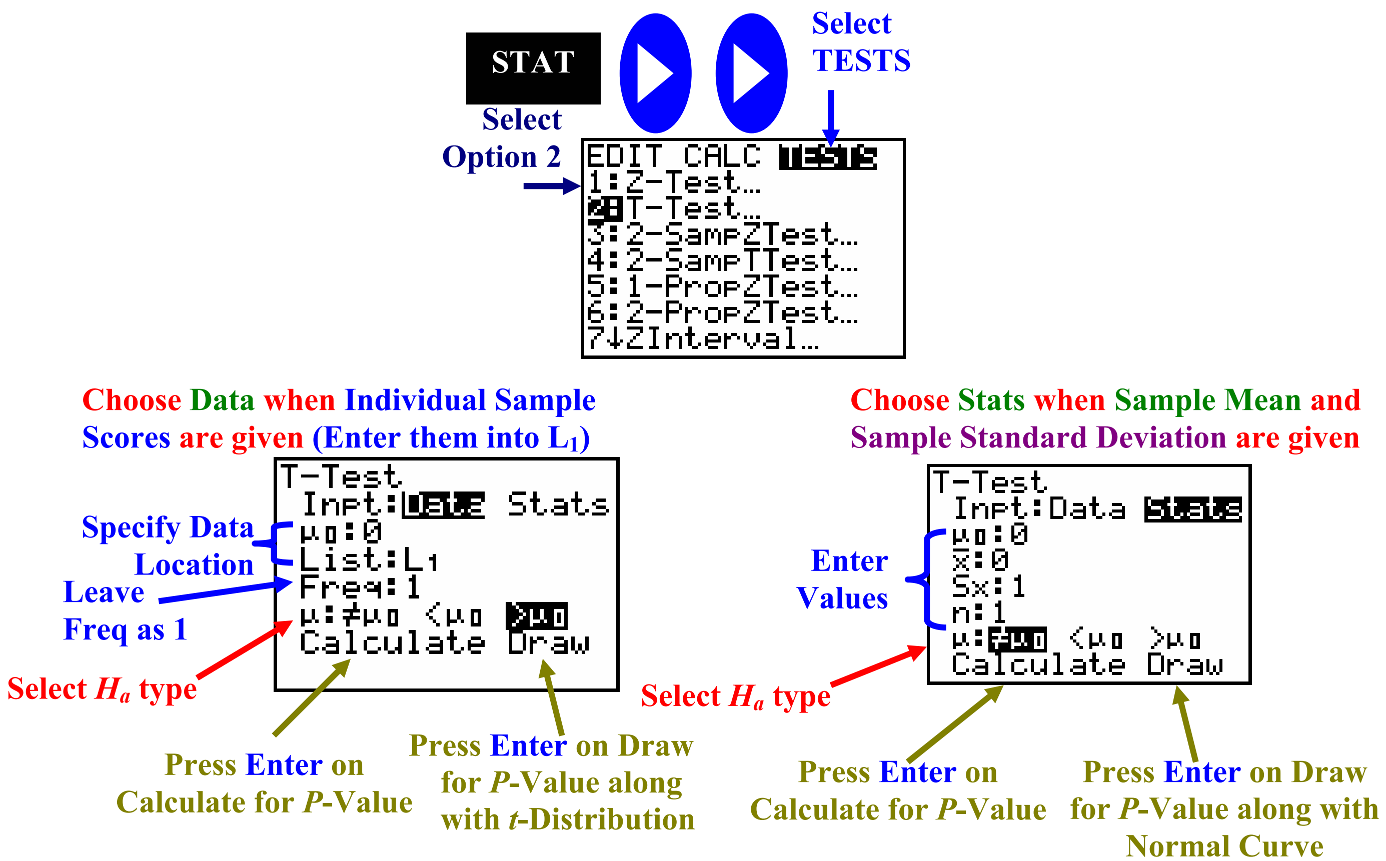
1. **State the Hypothesis (H_0: $\mu = \mu_0$ and H_a).**
2. **Calculate the One-Sample *t*-test statistics.**
3. **Determine the *P*-Value.**

One-Sample *t*-Test Statistics

$$t_{\bar{x}} = \frac{\bar{x} - \mu_0}{\left(s_{\bar{x}} / \sqrt{n}\right)} = \frac{\sqrt{n}\left(\bar{x} - \mu_0\right)}{s_{\bar{x}}} \text{ with}$$

$(n - 1)$ degrees of freedom (*df*)

T-Test function on TI-83 Plus



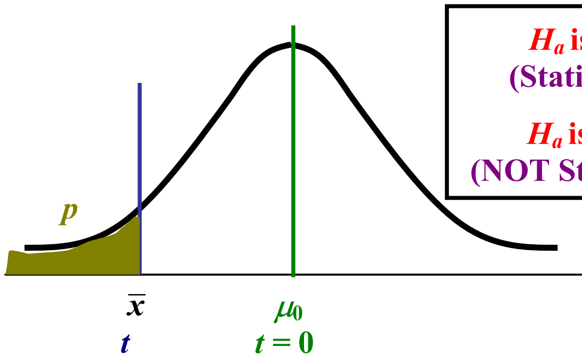
Fixed Significance Level One-Sample *t*-Tests for a Population Mean

Given a fixed significance level, α, the CONCLUSION can be drawn after *t*-test is performed.

1. **If $p \leq \alpha$, then H_a is True (Statistically Significant)**
2. **If $p > \alpha$, then H_0 is True (NOT Statistically Significant)**

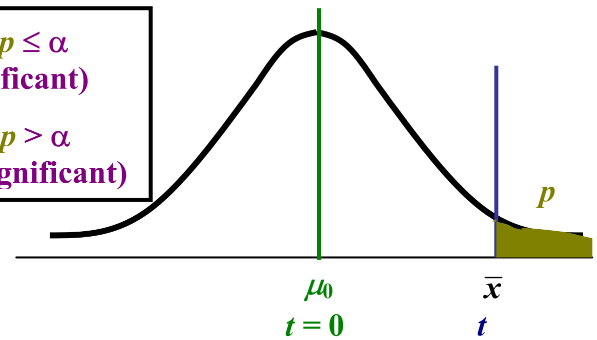
One Sample One-Sided t-Tests

A Sample can be tested for Significance Level as α = tail probability. The CONCLUSION can then be drawn. (H_a: $\mu > \mu_0$, or H_a: $\mu < \mu_0$)



H_a is True when $p \leq \alpha$
(Statistically Significant)

H_a is False when $p > \alpha$
(NOT Statistically Significant)

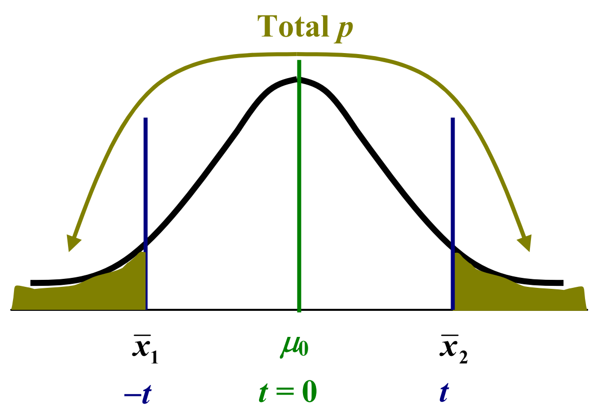


One Sample Two-Sided t-Tests

A Sample can be tested for Significance Level as α = combined tail probabilities. The CONCLUSION can then be drawn. (H_a: $\mu \neq \mu_0$)

H_a is True when Total $p \leq \alpha$
(Statistically Significant)

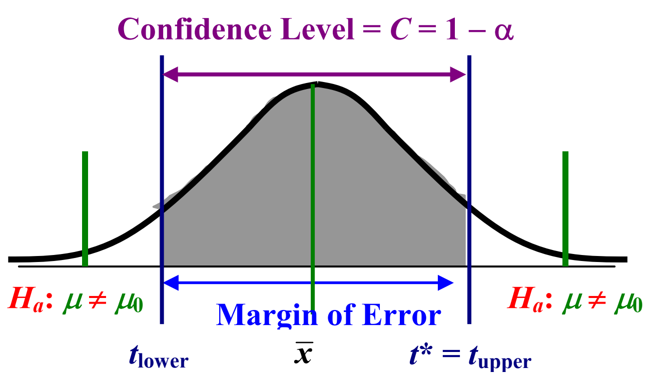
H_a is False when Total $p > \alpha$
(NOT Statistically Significant)

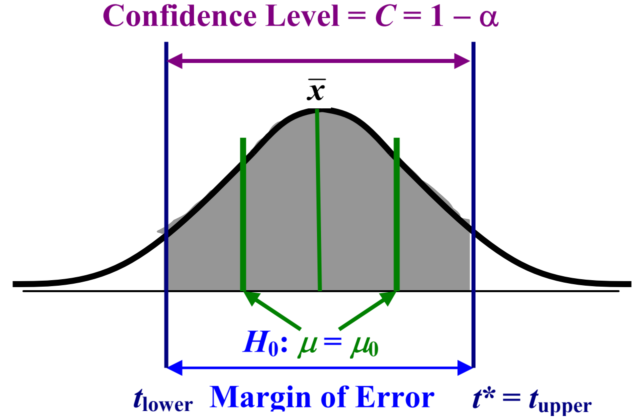


t-Confidence Intervals with One Sample Two-Sided t-Tests

A Confidence Interval of a Sample can be tested for Significance (Level $C = 1 - \alpha$). The CONCLUSION can then be drawn.

1. **If $\mu_0 < (\bar{x} - m)$ or $\mu_0 > (\bar{x} + m)$, then H_a is True (Statistically Significant) as μ_0 falls OUTSIDE Level C of the sample.**
2. **If $(\bar{x} - m) < \mu_0 < (\bar{x} + m)$, then H_0 is True (NOT Statistically Significant) as μ_0 falls WITHIN Level C of the sample.**





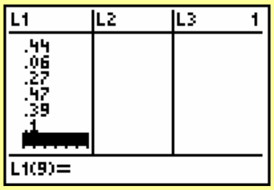
Example 3: Lead content measurements (in mg/L) taken at eight California wineries are listed below. The recommended maximum level of lead set by the US government is 0.20 mg/L. Can we conclude that wineries, on the average, exceed the government limit with a significance level of 0.05?

0.09 0.22 0.44 0.06 0.27 0.47 0.39 0.10

We have to enter the individual samples in L_1 of the Stats Editor

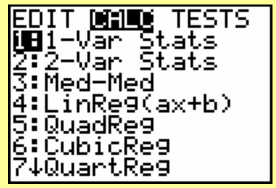
STAT

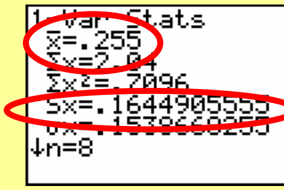
ENTER



Then, we run 1-Var Stats to obtain the Sample Mean, $\bar{x}$.

STAT ▶ ENTER





$\mu_0 = 0.20$ mg/L
$\bar{x} = 0.255$ mg/L
$s_{\bar{x}} = 0.1644905555$
$n = 10$ $\alpha = 0.05$

1. Stating Hypothesis

H_0: $\mu = 0.20$ mg/L
H_a: $\mu > 0.20$ mg/L

2. Calculate *t*-test statistics

$$t = \frac{\bar{x} - \mu_0}{\left(s_{\bar{x}}/\sqrt{n}\right)} = \frac{(0.255 - 0.20)}{\left(0.1644905555/\sqrt{8}\right)}$$

$$t = 0.9457290201$$

3. Fine *P*-Value from *t*-Distribution Critical Value Table using $df = 7$, $(n - 1)$ where $n = 8$

$p \approx 0.19$

(t is slightly lower than the average of the two p values. Therefore, it is closer to p as 0.20)

	Upper tail probability, *p*	
df	0.20	0.10
7	0.896	1.119

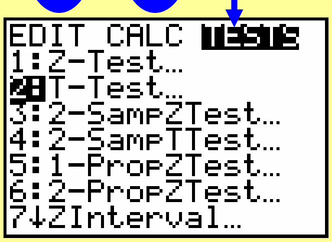
$t =$ 0.94573 is between 0.896 and 1.119

Since p (0.19) > α (0.05), H_0 is true and H_a is false. Hence, the mean lead level (0.255 mg/L) of the 8 California wineries sampled was <u>not</u> significantly higher than the U.S. standard of 0.20 mg/L.

Or Using *T*-Test of the TI-83 Plus Calculator,

STAT ▶ ▶ **Select TESTS**

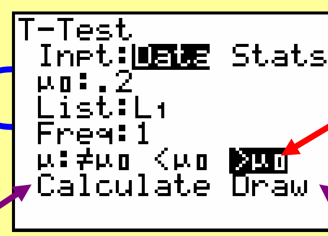
Select Option 2



Choose Data when Individual Sample Scores are given (as entered into L_1)

Enter Values

Select H_a type



Press Enter on Calculate for *P*-Value

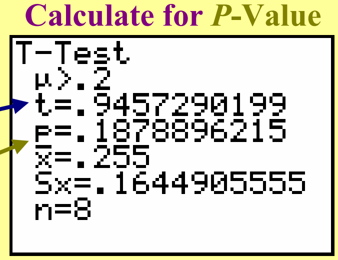
Press Enter on Draw for *P*-Value along with *t*-Curve

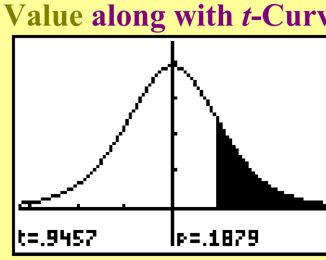
$$t = \frac{\bar{x} - \mu_0}{\left(s_{\bar{x}}/\sqrt{n}\right)} = \frac{(0.255 - 0.20)}{\left(0.1644905555/\sqrt{8}\right)}$$

$$t = 0.9457290201$$

$$p = 0.1878896215$$

(very close to 0.19 we got from the Table)





Again, p (0.1879) > α (0.05), H_0 is true and H_a is false. Therefore, not significantly high enough compare to government standard.

Data from Computer Assisted Program Minitab on Example 3

TEST OF MU = 0.200 VS MU G. T. 0.200 (Test of $\mu_0 = 0.200$ vs. $\mu_0 > 0.200$)

	n	$\bar{x}$	$s_{\bar{x}}$	Standard Error	t	p-Value
	N	MEAN	STDEV	SE MEA	T	P VALUE
lead	8	0.255	0.164	0.058	0.946	0.188

Example 4: *The Statistical Abstract of the United States* in 2000 reported that the average stay for delivering baby was 3.2 days. At Manoa Community Hospital in Honolulu, records show that a random sample of 10 women delivering babies has a sample mean stay of 2.6 days and a sample standard deviation of 0.82. Using a 5% level of significance to test the claim the population average length of stay at Monoa Community Hospital is different than the national average.

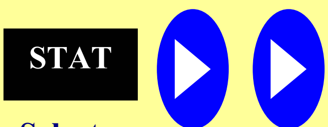
$\mu_0 = 3.2$ days $\quad \bar{x} = 2.6$ days $\quad s_{\bar{x}} = 0.82$

$n = 10 \quad \alpha = 0.05$

State Hypothesis

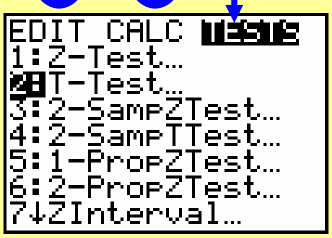
H_0: $\mu = 3.2$ days $\quad H_a$: $\mu \neq 3.2$ days

Using *T*-Test of the TI-83 Plus Calculator,

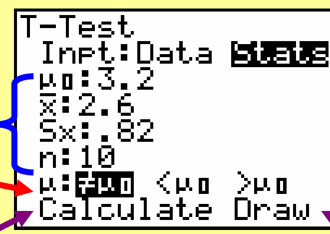


Select TESTS

Select Option 2



Choose Stats when Sample Mean and Sample Standard Deviation are given



Enter Values

Select H_a type

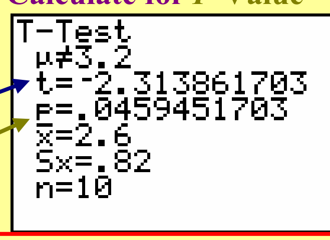
Press Enter on Calculate for *P*-Value

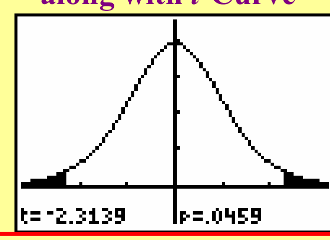
Press Enter on Draw for *P*-Value along with *t*-Curve

$$|t| = \frac{\bar{x} - \mu_0}{\left(s_{\bar{x}}/\sqrt{n}\right)} = \frac{(2.6 - 3.2)}{\left(0.82/\sqrt{10}\right)} \quad \text{(two sided)}$$

$t = \pm 2.313861703$

$p = 0.045951703$





Since p (0.04595) < α (0.05), H_a is true and H_0 is false. Hence, the mean days of stay after delivering babies at Manoa Community Hospital (2.6 days) of the 10 patients sampled was significantly different than the national average of 3.2 days.

Matched Pairs *t*-Procedure:
- a *t*-procedure can be applied to matched pairs design (where a treatment are given to subjects with before and after measurements taken).
- the raw data would be the difference between the "after" measurement and the "before" measurement.

Example: An author of a "Fast Mental Math" book tested a small number of subjects before they read her book and after they have learned her "fast" way of mental math. The differences between the test scores are analyzed with the **one-sample *t*-distribution**.

Example 5: ABC Driving School claims that students who received their instructions do significantly better on their driving test. A sample of 16 students was tested before and after instructions were given by the school. Evaluate the claim made by the school at the 5% significance level.

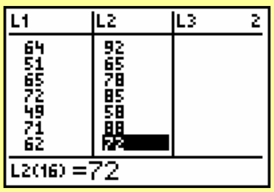
Student #	Before Instruction	After Instruction	Student #	Before Instruction	After Instruction
1	45	60	9	52	84
2	55	72	10	64	92
3	89	92	11	51	65
4	51	62	12	65	78
5	22	35	13	72	85
6	68	71	14	49	58
7	99	100	15	71	88
8	45	49	16	62	72

Since this is a before and after scenario and each subject is tested twice, we consider it as a matched pairs design. Hence, the difference in the scores must be obtained for the one-sample *t*-procedure.

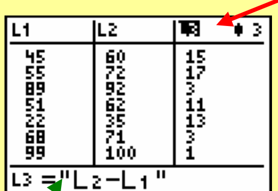
1. We have to enter the individual samples in L_1 and L_2 of the Stats Editor

STAT

ENTER



Enter L_3 = "$L_2 - L_1$"



Cursor must be on the column heading before entering formula

" must be used to enter formula

ALPHA

MEM "

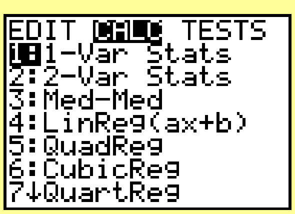
+

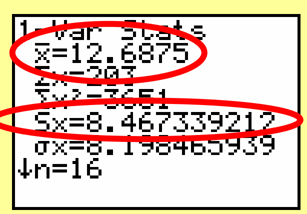
Student #	Difference in Scores (After – Before)	Student #	Difference in Scores (After – Before)
1	+15	9	+32
2	+17	10	+28
3	+3	11	+14
4	+11	12	+13
5	+13	13	+13
6	+3	14	+9
7	+1	15	+17
8	+4	16	+10

Then, we run 1-Var Stats L_3 to obtain the Sample Mean of the difference, $\bar{x}_D$.

STAT

ENTER





$\mu_0 = 0$ (No Improvement)
$\bar{x} = 12.6875$
$s_{\bar{x}} = 8.467339212$
$n = 16$ $\alpha = 0.05$

2. Stating Hypothesis

H_0: $\mu = 0$
H_a: $\mu > 0$

3. Calculate ***t*-test statistics**

$$t = \frac{\bar{x} - \mu_0}{\left(s_{\bar{x}} / \sqrt{n}\right)} = \frac{(12.6875 - 0)}{\left(8.467339212 / \sqrt{16}\right)}$$

$$t = 5.993618388$$

4. Fine ***P*-Value from *t*-Distribution Critical Value Table using $df = 15$, $(n - 1)$ where $n = 15$**

$p << 0.0005$

	Upper tail probability, *p*
df	0.0005
15	4.073

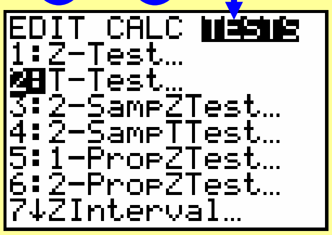
$t =$ 5.9936 is greater than 4.073

Since $p << 0.0005 < \alpha$ (0.05), H_0 is false and H_a is true. Hence, <u>ABC Driving School can claim their instructions significantly improve driving test scores</u>.

Or Using *T*-Test of the TI-83 Plus Calculator,

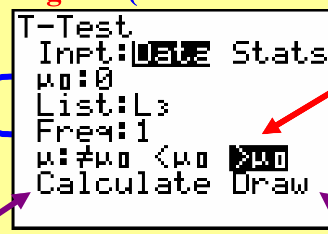
Select TESTS

Select Option 2



Choose Data when Individual Sample Scores are given (as entered into L_1)

Enter Values



Select H_a type

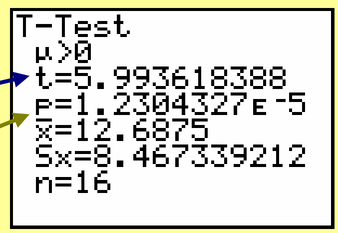
Press Enter on Calculate for *P*-Value

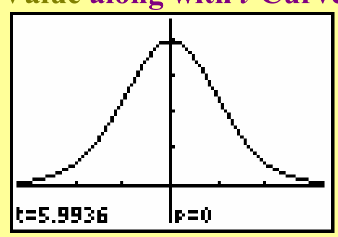
Press Enter on Draw for *P*-Value along with *t*-Curve

$$t = \frac{\bar{x} - \mu_0}{\left(s_{\bar{x}}/\sqrt{n}\right)} = \frac{(12.6875 - 0)}{\left(8.467339212/\sqrt{16}\right)}$$

$t = 5.99361838$

$p = 1.2304327 \times 10^{-5}$





Again, p (1.23×10^{-5}) $< \alpha$ (0.05), H_0 is false and H_a is true. Therefore, the driving test score is significantly higher after the instructions of the ABC Driving school.

<u>11.1A Assignment</u>

pg. 590–591 #11.1, 11.3, and 11.5; pg. 597–598 #11.7, 11.9 and 11.11; pg. 604 #11.13

11.1B: Characteristics of *t*-Procedures

Robust Procedure: - the ability of a procedure such that the sampling distribution of a test statistic remains relatively unaffected by violations of its underlying assumption.

Characteristics of *t*-Procedure:

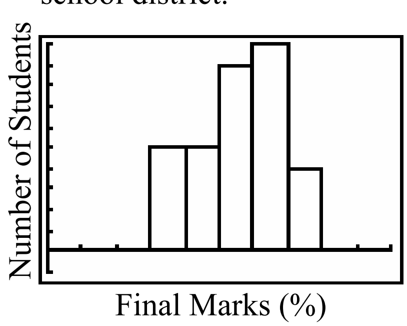
1. **Outliers can greatly affect *t*-test statistic due to the strong effect they have on $\bar{x}$ and $s_{\bar{x}}$.**
   Therefore, it is very important to check for outliers by plotting a histogram or modified box-plot.
2. ***t*-procedures are fairly robust against non-normality (skewness)** of a population when there are no outliers. This property against non-normal population can be enhanced by larger sample size (law of large numbers and central limit theorem).

Guidelines of Using the *t*-Procedure:

1. We **must assume that the sample is from a Simple Random Survey of a large population**. This is a primary assumption over the possible non-normality of the population.
2. **When $n < 15$, only use *t*-procedure if the data have no outliers and very normal.**
3. **When $n \geq 15$, we may use *t*-procedure if the data have no outliers and somewhat normal (slightly skewed).**
4. **When $n \geq 40$, we can use *t*-procedure if the data have no outliers and non-normal (any degree of skewness is fine).**

Example 1: Evaluate if the *t*-procedure can be utilized in the following situations.

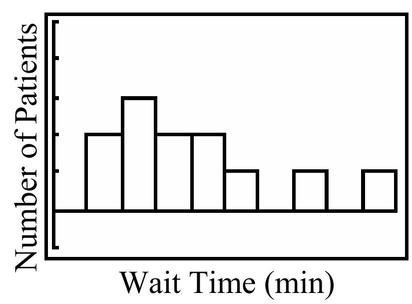
a. The final marks of 33 Algebra II students in a school district.



x: [0, 100%, 10%]
y: [−5, 10, 1]

We can use the *t*-procedure because even it is slightly skewed, there are no outliers and $n \geq 15$.

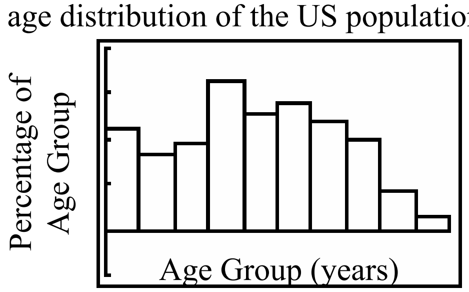
b. A sample wait time of 12 patients from a doctor's office is recorded.



x: [0, 20 min, 2 min]
y: [−1, 5, 1]

We cannot use the *t*-procedure because this distribution contains outliers.

c. The age distribution of the US population in 1975.

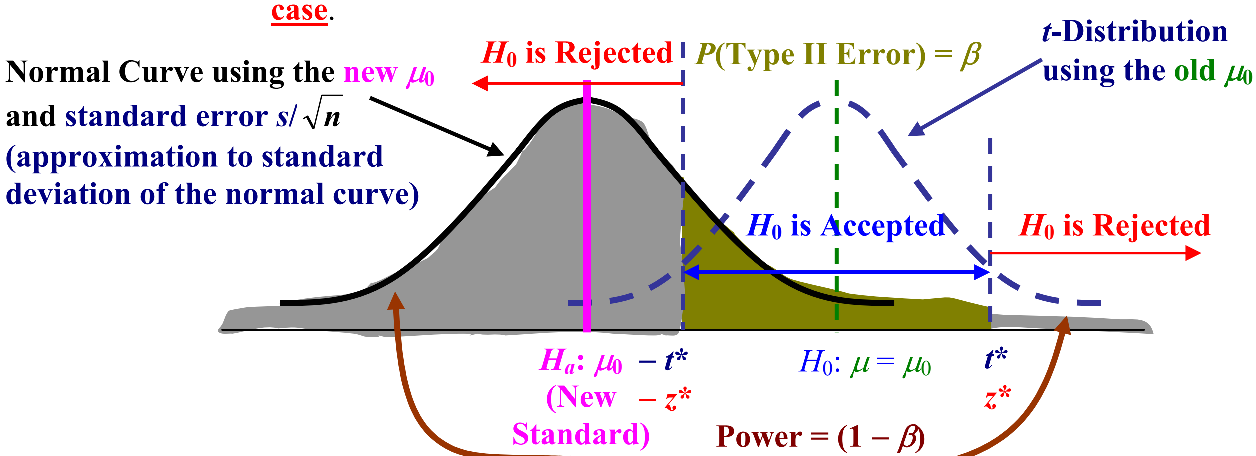


x: [0, 100, 10]
y: [−5, 20, 5]

We cannot use the *t*-procedure because this is a population distribution, and not a sample.

Power of *t*-Test: - the probability that a fixed level α significance test will reject H_0 when a particular alternative value of the parameter is true (supposing H_a is correct).

- **the complement of a type II error ($1 - \beta$).**
- a **high probability of power means the test is NOT sensitive enough to detect the H_a case**.



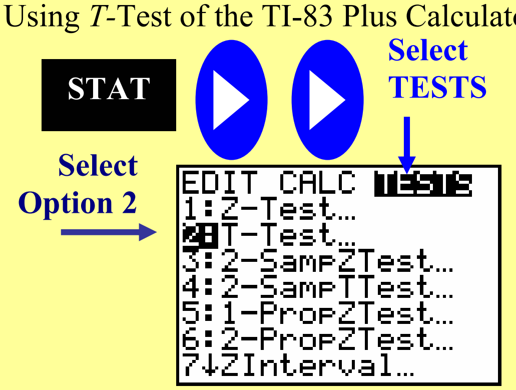
Example 2: The samples mean percentage of caesarean births in 1999 for 20 states is 26.095% with a standard deviation of 7.087%. The national average of caesarean births in 1999 was 25%.

a. Conduct a hypothesis *t*-test with a significance level of 0.20 that the sample of 20 states has a higher average percentage of caesarean births than the national average.
b. An international average of caesarean births among all the western European countries was found to be 19.3%. Write the rule of rejecting H_0 in terms of *t*-statistics.
c. Describe the Type I error and find the error probability.
d. Calculate the Type II error probability and the power against this alternate parameter.

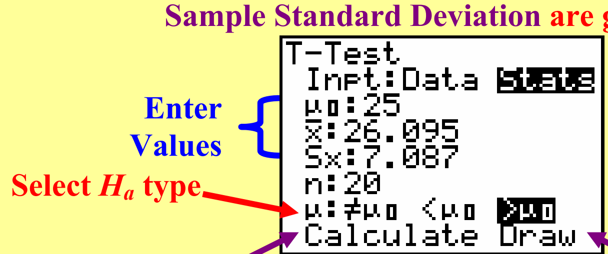
a. $\mu_0 = 25\%$ $\bar{x} = 26.095\%$ $s_{\bar{x}} = 7.087\%$ $n = 20$ $\alpha = 0.20$

State Hypothesis
H_0: $\mu = 25\%$ H_a: $\mu > 25\%$

Using *T*-Test of the TI-83 Plus Calculator,



Choose Stats when Sample Mean and Sample Standard Deviation are given

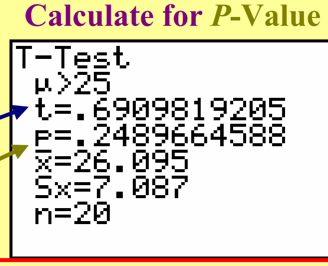


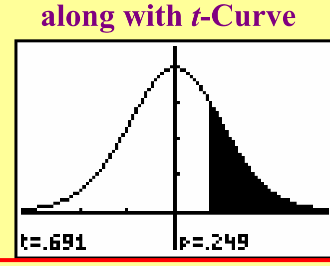
Press Enter on Calculate for *P*-Value **Press Enter on Draw for *P*-Value along with *t*-Curve**

$$t = \frac{\bar{x} - \mu_0}{\left(s_{\bar{x}}/\sqrt{n}\right)} = \frac{(26.095 - 25)}{\left(7.087/\sqrt{20}\right)}$$

$t = 0.6909819205$

$p = 0.2489664588$





Since p (0.24897) > α (0.20), H_0 is true and H_a is false. Hence, the mean average caesarean birth (26.095%) of the 20 states sampled was not significantly greater than the national average of 25%.

b. Rule for Rejecting H_0 in terms of t-statistic

$\mu_0 = 25\%$

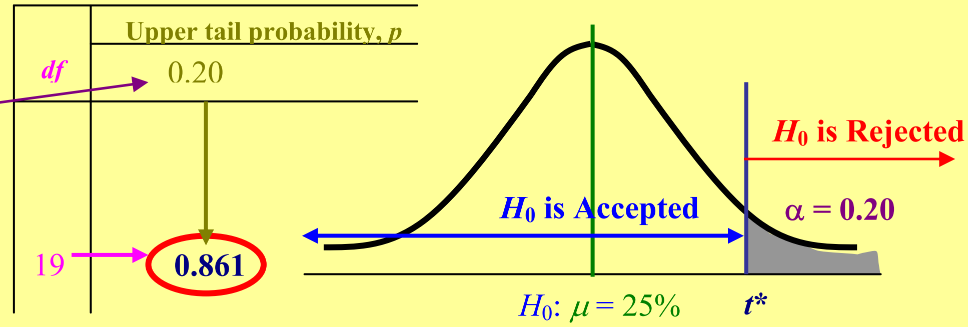
$s_{\bar{x}} = 7.087\%$

$n = 20$

$\alpha = 0.20$

1. Find critical value (t^*) using α as p and $df = 19$

$t^* = 0.861$



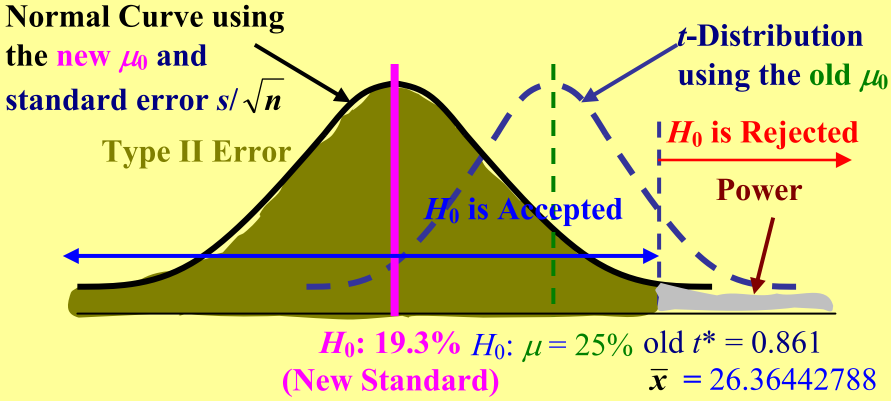
2. State the t-test statistic that we will reject H_0

Reject H_0 when $t \geq t^*$: Since $t = \dfrac{\bar{x} - \mu_0}{\left(s_{\bar{x}}/\sqrt{n}\right)} = \dfrac{(\bar{x} - 25)}{\left(7.087/\sqrt{20}\right)}$, **we reject H_0 when** $\dfrac{(\bar{x} - 25)}{\left(7.087/\sqrt{20}\right)} \geq$ **0.861**

c. Type I Error: When we believe that H_a is true (t-statistic is above t^*, $t \geq 0.861$) but the sample actually has $t < 0.861$ (H_0 is true).

P(Type I Error) = α = 0.20

d. Type II Error: When we believe that H_0 is true ($\bar{x} \leq 26.36442788$ with $\mu = 19.3\%$) but the sample actually has $\bar{x} > 26.36442788$ (H_a is true).



Raw Score of t^*:

$$\frac{(\bar{x} - 25)}{\left(7.087/\sqrt{20}\right)} = 0.861$$

$$\bar{x} = (0.861)\left(7.087/\sqrt{20}\right) + 25$$

$$\bar{x} = 26.36442788$$

P(Type II Error) = normalcdf(0, 26.36442788, 19.3, $7.087/\sqrt{20}$)

P(Type II Error) = β = 0.9999958579

Power = $1 - \beta$ = 1 − 0.9999958579

Power = 4.142×10^{-6}

Since Power is very small, the test is very sensitive for H_a.

11.1B Assignment

pg. 608–609 #11.15 and 11.17;

pg. 611–612 #11.19;

pg. 613–616 #11.21, 11.23, 11.27 and 11.29

11.2: Comparing Two Means

Two-Sample Problems: - problems that deal with comparative experiment (involving giving each group of subjects different treatments – tested drug versus placebo).
- this is different than matched pairs designs where subjects of the same group receive two different treatments – before and after tests).
- there may be two different sample sizes, two different sampling means, and two different sampling standard deviations.

Comparing Two Means: - there are two assumptions when comparing two means:

a. **Both Samples are Independent of each other and they are drawn from Random Samples.**
b. **Both Populations are somewhat Normally Distributed.**

Note: Similar to any z-Test/Interval or t-Test/Interval, *it is important to do a* **back to back stem plot** or **double box plots** to assess any outliers in both samples.

Difference of Two Sample Means

Difference of Two Population Means = $(\mu_1 - \mu_2)$

Difference of Two Sample Means = $(\bar{x}_1 - \bar{x}_2)$

Difference in Population Standard Deviations: $(\sigma_1 - \sigma_2) = \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

(Special Case when $\sigma_1 = \sigma_2$) $(\sigma_1 - \sigma_2) = \sigma\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$

Difference in Sample Standard Deviations: $(s_1 - s_2) = \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$

(Special Case when $s_1 = s_2$) $(s_1 - s_2) = s\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$

***Note*: Standard Deviations cannot be subtracted directly. They must be change to variance first before the difference can be found.**

z-Statistic and t-Statistic for the Difference of Two Sample Means

z-Statistics: $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$ **t-Statistics:** $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

Two-Sample *t*-Procedures: - there are two methods to find two-sample *t*-confidence interval or use *t*-test.

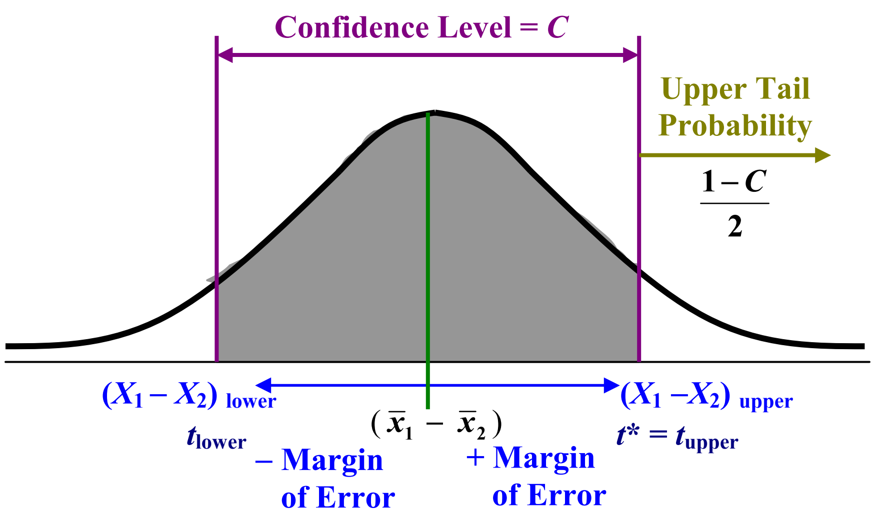
1. **Use the combined degree of freedom formula because $n_1 \neq n_2$.** $df_{combined} = \dfrac{\left(\dfrac{s_1^2}{n_1} - \dfrac{s_2^2}{n_2}\right)}{\dfrac{1}{n_1 - 1}\left(\dfrac{s_1^2}{n_1}\right)^2 + \dfrac{1}{n_2 - 1}\left(\dfrac{s_2^2}{n_2}\right)}$

   The formula is fairly accurate when both n_1 and $n_2 \geq 5$. The TI-83 Plus graphing calculator as well as other Statistics Computer Software uses this formula to generate the **approximate *t*-statistics** and ***p*-value.**

2. **Use the same *t*-procedure as before but refer to the table using the SMALLER Degree of Freedom from $df_1 = (n_1 - 1)$ and $df_2 = (n_2 - 1)$.**

Two-Sample *t*-Confidence Intervals [$(\bar{x}_1 - \bar{x}_2) \pm$ Margin of Error] at Level *C*

Margin of Error (*m*) = Critical Value (*t) × Standard Error of Statistic [$(s_1 - s_2) = \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$]**



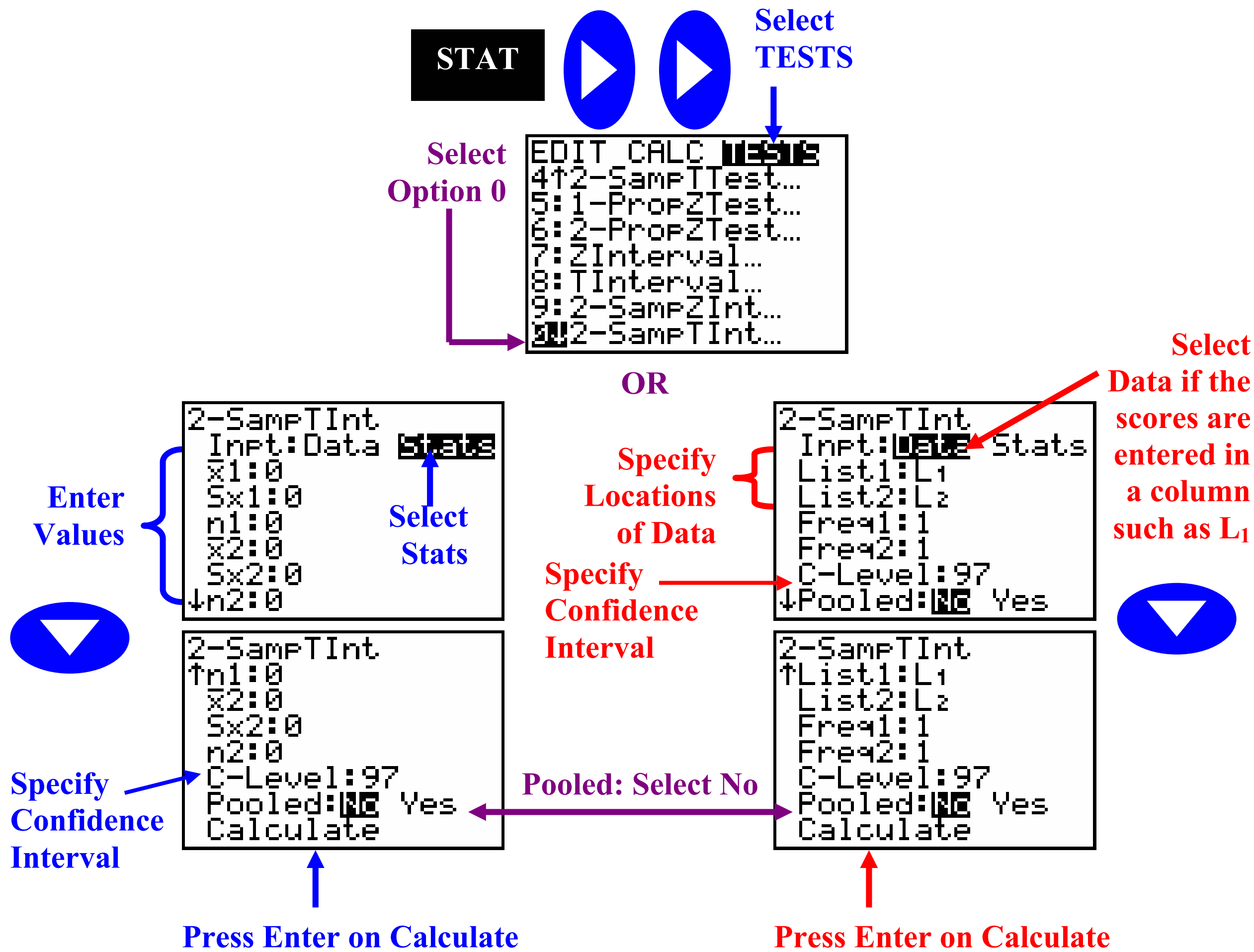
Confidence Interval for a Population Mean

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \quad \text{where } m = t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Note*: DO NOT use invNorm$\left(\dfrac{1-C}{2} + C\right)$ to find *t

(Must use the *t*-Distribution Table to find *t from Level *C* or Upper Tail Probability and the Smaller of df_1 and df_2)**

Two-Sample Confidence Interval (2-Samp *TInt* function) on TI-83 Plus



Pooled Data means when two sample variances are equal. Hence $df_{combined} = (n_1 - 1) + (n_2 - 1)$. In most cases, the two samples do not have the same variance. Therefore, we usually select No Pooled on the TI-83 Plus.

Example 1: Researchers set out to compare the amounts of money spent at Christmas at two competing department stores, X and Y. Fourteen shoppers at store X are chosen at random and sixteen shoppers at store Y say they spent the following amounts of money.

Department Store X			Department Store Y		
$89.81	$77.50	$87.13	$467.00	$5.00	$68.94
$89.00	$15.00	$148.00	$31.56	$2.00	$74.69
$29.00	$78.13	$5.50	$3.19	$13.50	$30.02
$19.56	$493.00	$89.79	$96.25	$10.13	$21.13
$71.75	$249.00		$10.25	$20.19	
			$11.56	$10.00	

Determine the 90% confidence interval for the difference in mean spending at the two department stores.

We have to enter the individual samples in L_1 and L_2 of the Stats Editor

STAT **ENTER**

```
L1      L2      L3     2
87.13   20.19
148     10
5.5     68.94
89.79   74.69
------  30.02
        21.13
L2(17) =
```

Then, we run 1-Var Stats on each L_1 and L_2 separately to obtain the Sample Means and Standard Deviations, ($\bar{x}_1$ and $\bar{x}_2$ with s_1 and s_2)

STAT ▶ **ENTER**

```
1-Var Stats
 x̄=110.155
 Σx=1542.17
 Σx²=377332.17
 Sx=126.3251098
 σx=121.7299217
↓n=14
```

```
1-Var Stats
 x̄=54.713125
 Σx=875.41
 Σx²=241098.468
 Sx=113.4906902
 σx=109.8868883
↓n=16
```

$\bar{x}_1 = 110.155$
$s_1 = 126.3251098$
$n_1 = 14$

$\bar{x}_2 = 54.713125$
$s_2 = 113.4906902$
$n_2 = 16$

$(\bar{x}_1 - \bar{x}_2) = 110.155 - 54.713125$

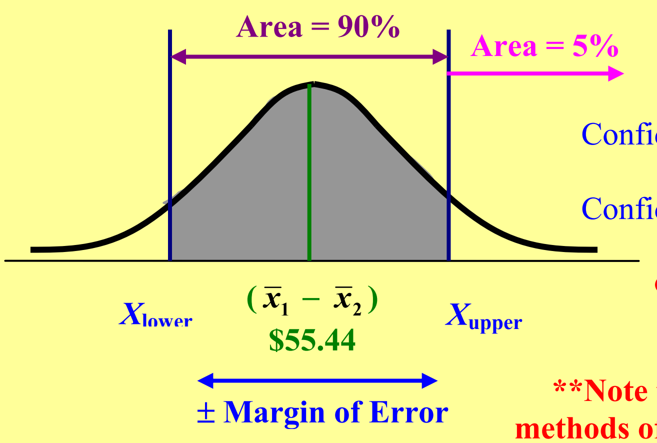
$(\bar{x}_1 - \bar{x}_2) = 55.441875$

$$(s_1 - s_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{(126.3251098)^2}{14} + \frac{(113.490602)^2}{16}}$$

$(s_1 - s_2) = 44.10064424$

$df_{combined} = n_1 - 1 = 13$ **(use smaller of the two sample size)**

	Upper tail probability, p
df	0.05
13	→ 1.771
	90%
	Confidence level C



$$\text{Confidence Interval} = (\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Confidence Interval = 55.441875 ± 1.771 × 44.10064424

Confidence Interval = \$55.44 ± \$78.10

****Note the difference between the two methods of calculating 2 –Sample *t*-Interval**

or Using the 2-SampTInt

STAT ▶ ▶ **Select Option 0**

Select TESTS

```
EDIT CALC TESTS
4↑2-SampTTest…
5:1-PropZTest…
6:2-PropZTest…
7:ZInterval…
8:TInterval…
9:2-SampZInt…
0:2-SampTInt…
```

Select Data

```
2-SampTInt
 Inpt:Data Stats
 List1:L1
 List2:L2
 Freq1:1
 Freq2:1
 C-Level:.9
↓Pooled:No Yes
```

```
2-SampTInt
↑List1:L1
 List2:L2
 Freq1:1
 Freq2:1
 C-Level:.9
 Pooled:No Yes
 Calculate
```

Select No Pooled **Press Enter to Calculate**

```
2-SampTInt
 (-19.73,130.62)
 df=26.42393542
 x̄1=110.155
 x̄2=54.713125
 Sx1=126.32511
↓Sx2=113.49069
```

```
2-SampTInt
 (-19.73,130.62)
↑x̄2=54.713125
 Sx1=126.32511
 Sx2=113.49069
 n1=14
 n2=16
```

$m = \pm (130.62 - 55.441875) = \pm$ **75.178125**

Confidence Interval =\$55.44 ± \$75.18

We can say with 90% confidence that the difference of spending by shoppers of the two stores is \$55.44 ± \$75.18 with a combined degree of freedom of 26.424.

Two-Sample *t*-Test: - a significance test comparing the two-sample means after one of them has converted to a *t*-score with the other sample mean as $t = 0$.

Four Steps of Two-Sample *t*-Test for a Population Mean:

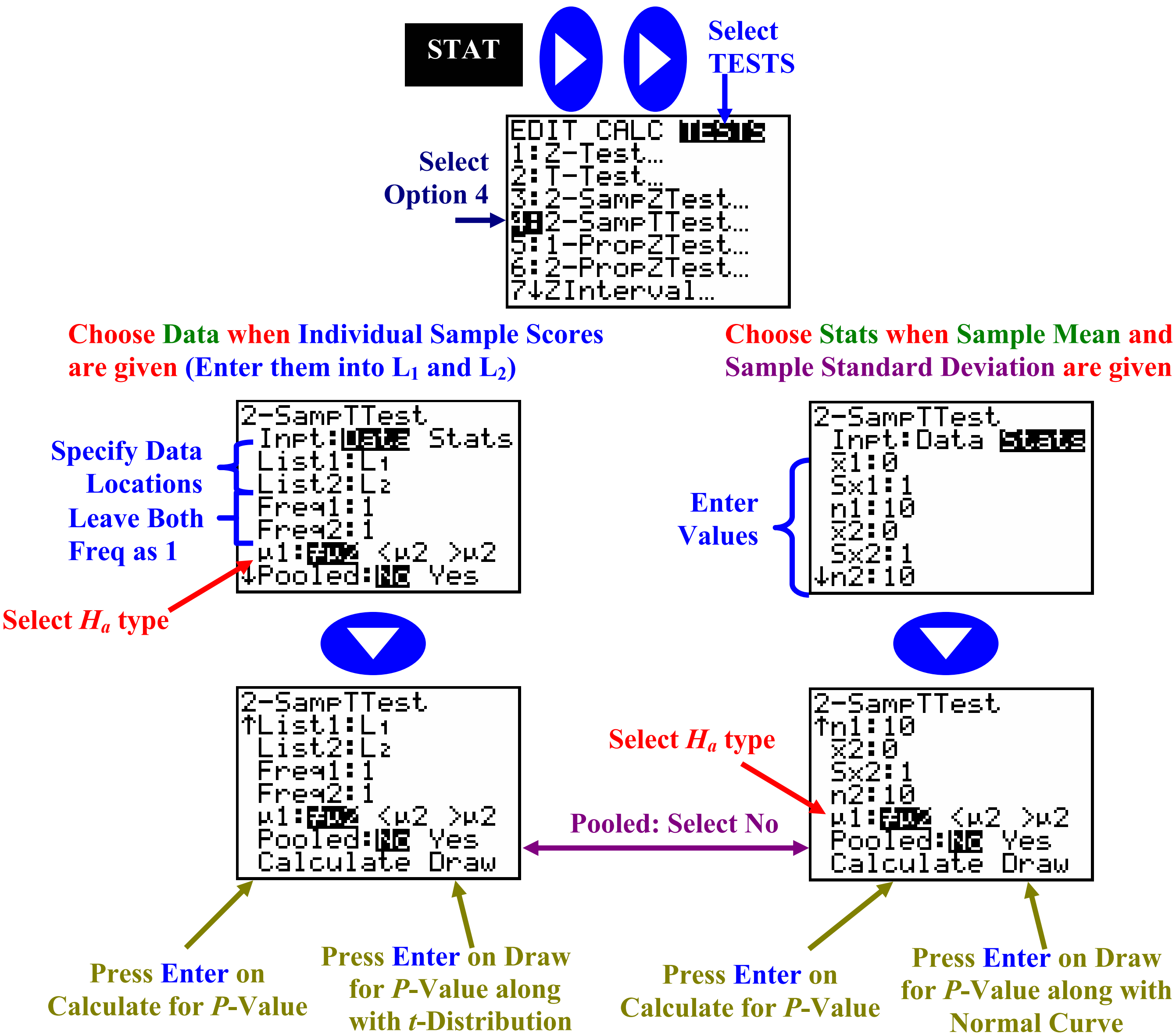
1. **State the Hypothesis** **(H_0: $\mu_1 = \mu_2$ and H_a: $\mu_1 \neq \mu_2$ or $\mu_1 > \mu_2$ or $\mu_1 < \mu_2$).**
2. **Calculate the Two-Sample *t*-test statistics.**
3. **Decide on the method to obtain df_{combined} (Pooled or Not Pooled)**
4. **Determine the *P*-Value.**

Two-Sample *t*-Test Statistics

$$t_{\bar{\mu}_1-\bar{\mu}_2} = \frac{(\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \text{ with}$$

combined degrees of freedom (df_{combined})

2-SampT-Test function on TI-83 Plus



Pooled Data means when two sample variances are equal. Hence $df_{\text{combined}} = (n_1 - 1) + (n_2 - 1)$. In most cases, the two samples do not have the same variance. Therefore, we usually select No Pooled on the TI-83 Plus.

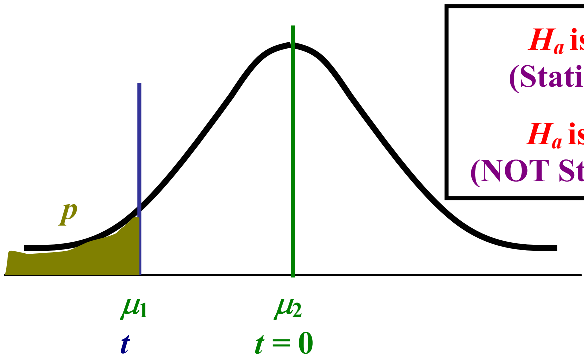
Fixed Significance Level Two-Sample *t*-Tests for a Population Mean

Given a fixed significance level, α, the CONCLUSION can be drawn after *t*-test is performed.

1. **If $p \leq \alpha$, then H_a is True (Statistically Significant)**
2. **If $p > \alpha$, then H_0 is True (NOT Statistically Significant)**

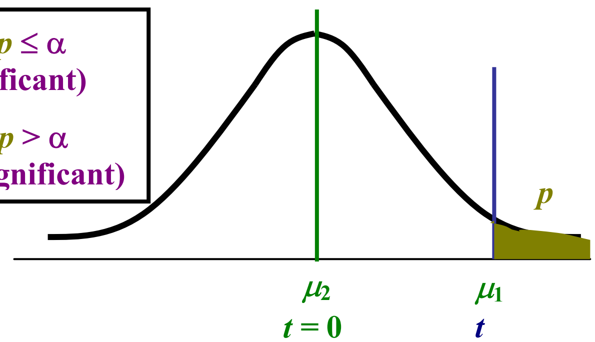
Two Sample One-Sided *t*-Tests

A Sample can be tested for Significance Level as α = tail probability. The CONCLUSION can then be drawn. (H_a: $\mu_1 < \mu_2$, or H_a: $\mu_1 > \mu_2$)



H_a is True when $p \leq \alpha$
(Statistically Significant)

H_a is False when $p > \alpha$
(NOT Statistically Significant)

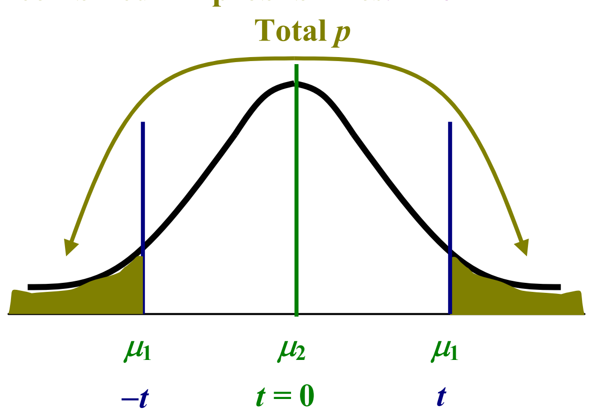


Two Sample Two-Sided *t*-Tests

A Sample can be tested for Significance Level as α = combined tail probabilities. The CONCLUSION can then be drawn. (H_a: $\mu_1 \neq \mu_2$)

H_a is True when Total $p \leq \alpha$
(Statistically Significant)

H_a is False when Total $p > \alpha$
(NOT Statistically Significant)



Example 2: Using Example 1 on pg. 153, determine if the average spending for shoppers of department store X is significantly higher than the average spending for shoppers of department store Y at a 5% level. State the hypothesis and perform a 2-sample *t*-test.

From the last example, we know that,

$\bar{x}_1 = \mu_1 = 110.155$ $\bar{x}_2 = \mu_2 = 54.713125$

$s_1 = 126.3251098$ $s_2 = 113.4906902$

$n_1 = 14$ $n_2 = 16$

1. **Stating Hypothesis**

H_0: $\mu_1 = \mu_2$

H_a: $\mu_1 > \mu_2$

2. Calculate ***t*-test statistics**

$$t_{\bar{\mu}_1 - \bar{\mu}_2} = \frac{(\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(110.155 - 54.713125)}{\sqrt{\frac{(126.3251098)^2}{14} + \frac{(113.4906902)^2}{16}}}$$

$t = 1.257166604$

3. $df_{\text{combined}} = n_1 - 1 = 13$
   (use smaller of the two sample size)

4. Fine *P*-Value from *t*-Distribution Critical Value Table using $df_{\text{combined}} = 13$

   $p \approx 0.13$

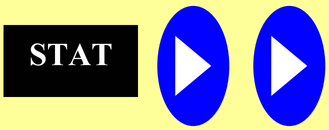
   (*t* is slightly lower than the average of the two *p* values. Therefore, it is closer to *p* as 0.15)

	Upper tail probability, *p*	
df	0.15	0.10
13 →	1.079	1.350

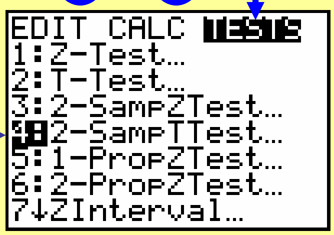
$t = 1.2572$ is between 1.079 and 1.350

Since p (0.13) > α (0.05), H_0 is true and H_a is false. Hence, the average spending of Department Store X shoppers sampled was not significantly higher than the average spending of Department Store Y shoppers sampled.

Or Using *T*-Test of the TI-83 Plus Calculator,



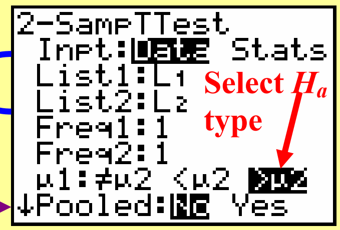
Select TESTS

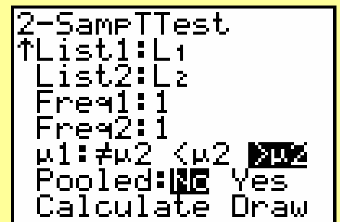


Select Option 4

Choose Data when Individual Sample Scores are given (Enter them into L_1 and L_2)

Enter Values





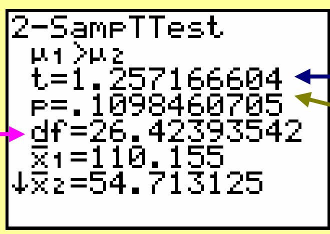
Select H_a type

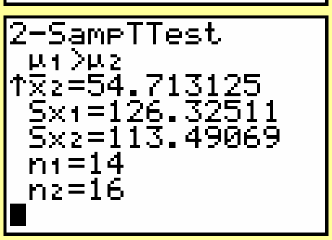
Select No Pooled

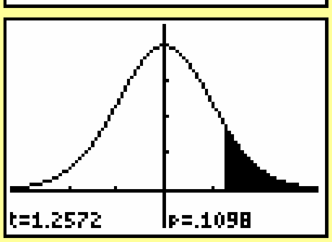
Press Enter on Calculate for *P*-Value

Press Enter on Draw for *P*-Value along with *t*-Curve

$df_{\text{combined}} = 26.42393542$

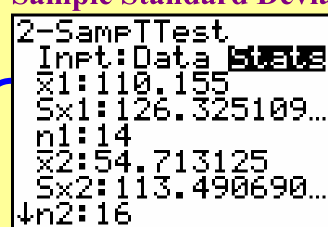




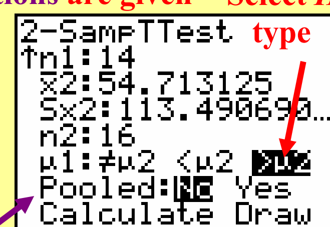


Choose Stats when Sample Means and Sample Standard Deviations are given

Enter Values



Select H_a type



Select No Pooled

Press Enter on Calculate for *P*-Value

Press Enter on Draw for *P*-Value along with *t*-Curve

$$t_{\bar{\mu}_1 - \bar{\mu}_2} = \frac{(\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(110.155 - 54.713125)}{\sqrt{\frac{(126.3251098)^2}{14} + \frac{(113.4906902)^2}{16}}}$$

$t = 1.257166604$ (same as calculation)

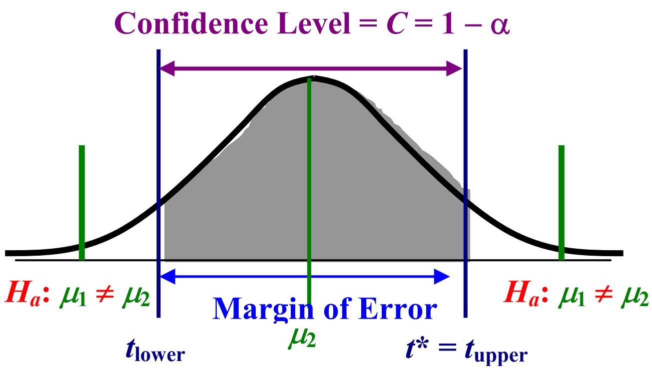
$p = 0.1098460705$ (different than before because df_{combined} is not the same)

Again, p (0.1098) > α (0.05), H_0 is true and H_a is false. Therefore, store X shoppers do not spend significantly more on average than store Y shoppers.

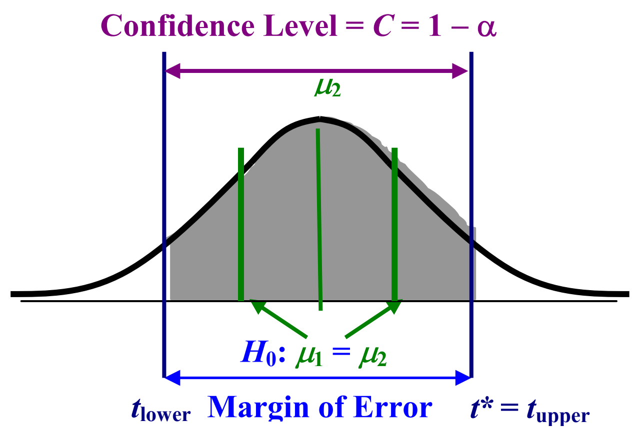
Two Sample *t*-Confidence Intervals with Two Sample Two-Sided *t*-Tests

A Confidence Interval of One of the Two Sample can be tested for Significance (Level $C = 1 - \alpha$) against the Other Sample Mean. The CONCLUSION can then be drawn.

1. **If $\mu_1 < (\mu_2 - m)$ or $\mu_1 > (\mu_2 + m)$, then H_a is True (Statistically Significant) as μ_1 falls OUTSIDE Level C of the sample mean.**



2. **If $(\mu_2 - m) < \mu_1 < (\mu_2 + m)$, then H_0 is True (NOT Statistically Significant) as μ_1 falls WITHIN Level C of the other sample mean.**



Robustness of Two-Sample *t*-Procedure:

1. **Generally, Two-Sample *t*-Procedure is more robust than One-Sample *t*-Procedure. This is especially true when the distributions are not symmetrical.**
2. **Even using the approximation of the lower *df* being the $df_{combined}$, the *t*-distribution table is fairly accurate down to $n_1 = n_2 = 5$.**
3. **For non-normal cases, an approximation can be use for $df_{combined} = n_1 + n_2$.**

11.2 Assignment

pg. 618 #11.31; pg. 628 #11.33 and 11.35;
pg. 631–632 #11.37 and 11.39;
pg. 637 #11.41; pg. 640 –642 #11.45, 11.47 and 11.49

Chapter 11 Review

pg. 647–652 #11.57, 11.59, 11.61, 11.63, 11,65 and 11.67

Chapter 12: Inference for Proportions

12.1A: Inference for a Population Proportion

Sample Proportion ($\hat{p}$): - the **experimental probability** (the ratio of favourable outcome and the total number of possible outcomes) of a sample.

Standard Error of Sample Proportion: - the standard deviation of a statistics as **estimated** from the sample proportion.
- instead of $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$, we use standard deviation of sample binomial probabilities $\sigma = \sqrt{\hat{p}(1-\hat{p})}$ and divide by $\sqrt{n}$

z-Statistics of Sample Proportion: - using the **z-distribution (normal curve)**, the **sample proportion ($\hat{p}$)** is compared to the **population proportion (or theoretical probability p)** as well as the its **standard error**.

Sample Proportion	Standard Error of a Sample Proportion
$\hat{p} = \frac{\text{\# of Favorable Outcomes in a Sample}}{\text{Total Number of Outcomes in a Sample}}$	$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

One-Sample z-Statistics of Sample Proportion

$$z = \frac{\hat{p} - p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}$$

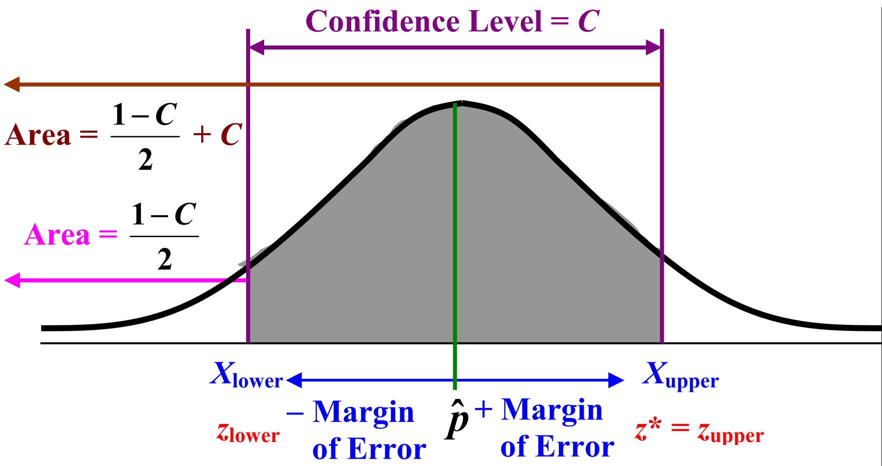
$\hat{p}$ = Sampling Proportion p = Population Proportion n = sample size

Assumptions for Inference about a Proportion:

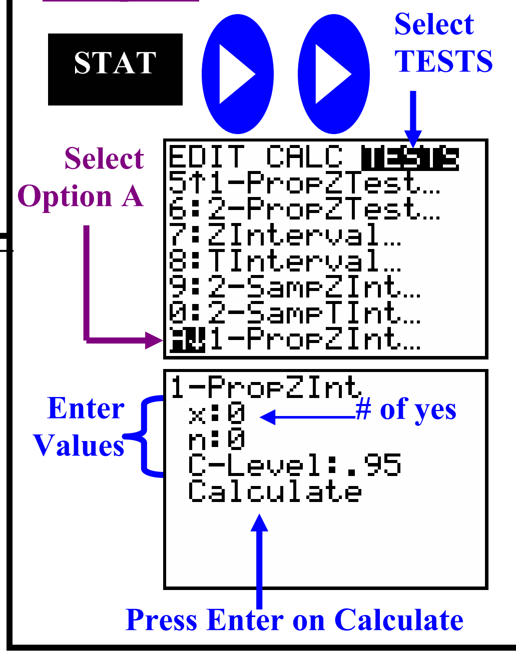
1. The data from our survey is from a **Simple Random Sample of size n from a Normal Population**.
2. Like the Normal Approximation to a Binomial Distribution, the **population must be 10 times the sample size (n).**
3. **For a Confidence Interval: $n\hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$**
4. **For a Significance Test of H_0: $p = p_0$: $np_0 \geq 10$ and $n(1-p_0) \geq 10$**

Confidence Intervals ($\hat{p}$ ± Margin of Error) at Level C

Margin of Error (m) = Critical Value (z^*) × Standard Error of Sampling Proportion $\left(\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$



Confidence Interval with 1-PropZInt function on TI-83 Plus



Confidence Interval for a Population Mean

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where $z^* = \text{invNorm}\left(\frac{1-C}{2} + C\right)$ and $m = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Example 1: From a random survey of 1000 people, 852 of them believe that the government should regulate the electricity industry. Calculate the 95% confidence intervals and the margin of error in percent. Report your final answer in complete sentences.

1. **We have to first evaluate $n\hat{p}$ and $n(1-\hat{p})$**

$n = 1000$ $\hat{p} = \frac{852}{1000} = 0.852$ $n\hat{p} = (1000)(0.852) = 852 \geq 10$

$n(1-\hat{p}) = (1000)(1 - 0.852) = 148 \geq 10$

(We can approximate using One-Sample Proportion z-Statistics)

2. Find z^* and Calculate **Margin of Error**

$z^* = \text{invNorm}\ (0.975)$ $\left(\frac{1-0.95}{2} + 0.95\right) = 0.975$

$z^* = \mathbf{1.959963986}$

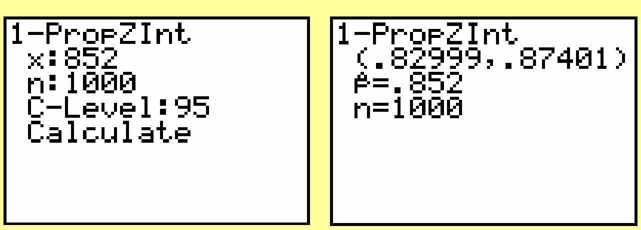
$m = \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = (1.959963986)\sqrt{\frac{(0.852)(1-0.852)}{1000}}$

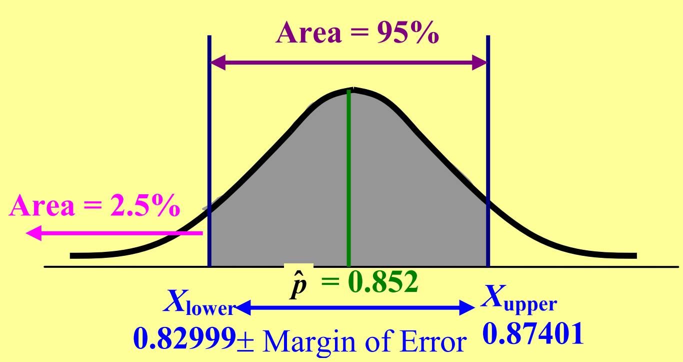
$m = \pm 0.022$

3. **State Confidence Interval**

We are 95% confident that the sample has a proportion of 0.852 ± 0.022. This means that 85.2% of the people sampled agree that government should regulate electricity. This is accurate within ±2.2%, 19 times out of 20.

Or Using TI-83 Plus **(0.82999, 0.87401)**
= 0.852 ± 0.02201





One-Sample z-Test for a Population Proportion: - a significance test comparing the **sample proportion** ($\hat{p}$) from a large population after it has converted to a z-score with the **population proportion (p_0)** as $z = 0$.

One-Sample z-test statistics for a Population Proportion ($z_{\hat{p}}$): - when the sample mean from a single variable is converted to a z-score for the comparison with the population mean using the sample standard deviation.

One-Sample z-Test Statistics for a Population Proportion (p_0)

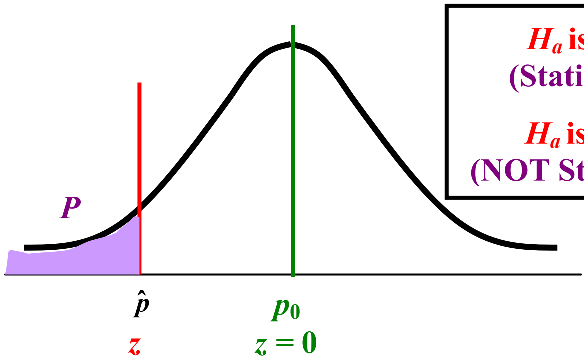
$$z_{\hat{p}} = \frac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1 - p_0)}{n}}}$$

Three Steps of One-Sample z-Test for a Population Proportion:

1. **State the Hypothesis (H_0: $p = p_0$ and H_a).**
2. **Calculate the One-Sample z-test statistics for a Population Proportion.**
3. **Determine the P-Value.**

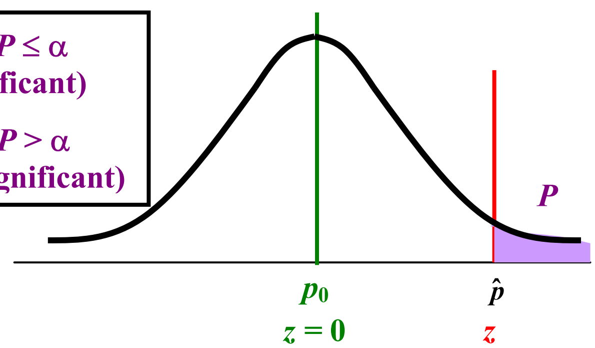
One Sample One-Sided z-Tests for a Population Proportion

A Sample can be tested for Significance Level as α = tail probability. The CONCLUSION can then be drawn. (H_a: $p < p_0$, or H_a: $p > p_0$)



H_a is True when $P \leq \alpha$
(Statistically Significant)

H_a is False when $P > \alpha$
(NOT Statistically Significant)

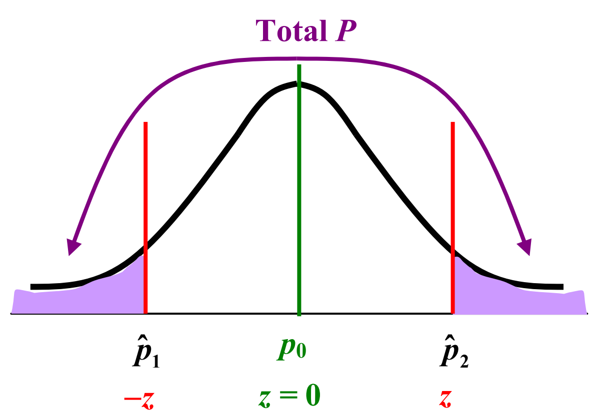


One Sample Two-Sided t-Tests for a Population Proportion

A Sample can be tested for Significance Level as α = combined tail probabilities. The CONCLUSION can then be drawn. (H_a: $p \neq p_0$)

H_a is True when Total $P \leq \alpha$
(Statistically Significant)

H_a is False when Total $P > \alpha$
(NOT Statistically Significant)

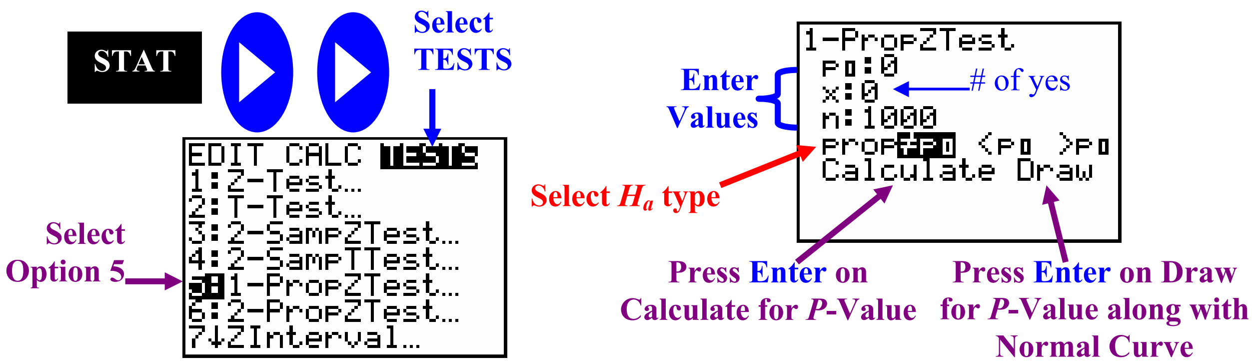


Fixed Significance Level One-Sample *z*-Tests for a Population Mean

Given a fixed significance level, α, the CONCLUSION can be drawn after *z*-test is performed.

1. **If $P \leq \alpha$, then H_a is True (Statistically Significant)**
2. **If $P > \alpha$, then H_0 is True (NOT Statistically Significant)**

1-PropZ-Test function on TI-83 Plus



Example 2: In 2003, the U.S. Department of Transportation, National Highway Traffic Safety Administration reported that 67% of all fatally injured automobile drivers were under the influence. A random sample of 200 records of automobile driver fatalities in San Mateo County, California, in 2003 showed that 111 involved a driver under the influence. Do these data indicate that the sample proportion of driver fatalities caused by driving under the influence in San Mateo County is significantly different compared to the national proportion? State your hypotheses and use the significance level of 0.01 to evaluate.

$p_0 = 67\% = 0.67$
$\hat{p} = 111/200 = 0.555$
$n = 200 \qquad \alpha = 0.01$

Evaluate $n\hat{p}$ and $n(1 - \hat{p})$

$n\hat{p} = (200)(0.555) = 111 \geq 10$

$n(1 - \hat{p}) = (200)(1 - 0.555) = 89 \geq 10$

(We can approximate using One-Sample Proportion *z*-Statistics)

1. **Stating Hypothesis**

H_0: $p = 0.67$
H_a: $p \neq 0.67$

2. Calculate **z-test statistics**

$$z_{\hat{p}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{(0.555 - 0.67)}{\left(\sqrt{\frac{(0.67)(1-0.67)}{200}}\right)}$$

$$z_{\hat{p}} = \pm 3.458744383$$

(± because this is a two-sided alternative – H_a: $p \neq p_0$)

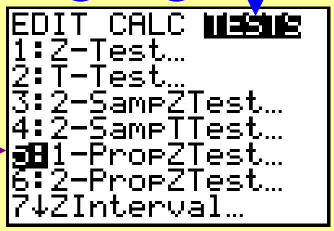
3. Calculate **P-Value**

$P = 1 - \text{normalcdf}(-3.458744383, 3.458744383)$ $\quad$ $P = 5.4279 \times 10^{-4}$

Since P (5.4279×10^{-4}) < α (0.01), H_a is true and H_0 is false. Therefore, the sample proportion for driver fatalities under the influence from San Mateo County is significantly different than the national proportion.

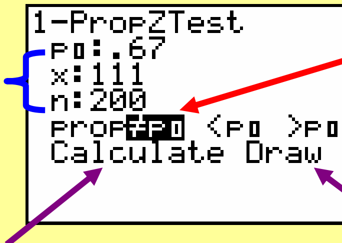
Or Using 1-Prop*Z*-Test of the TI-83 Plus Calculator,

STAT ▶ ▶ **Select TESTS**



Select Option 5

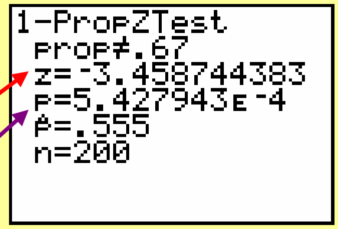
Enter Values

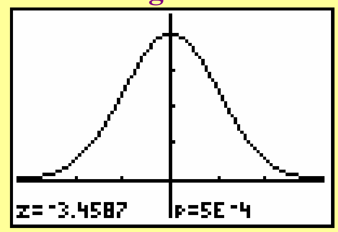


Select H_a type

Press Enter on Calculate for *P*-Value

Press Enter on Draw for *P*-Value along with *t*-Curve





$$z_{\hat{p}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{(0.555 - 0.67)}{\left(\sqrt{\frac{(0.67)(1-0.67)}{200}}\right)}$$

$$z_{\hat{p}} = \pm 3.458744383$$

$$P = 5.4279 \times 10^{-4}$$

Again, $P\ (5.4279 \times 10^{-4}) < \alpha\ (0.01)$, H_a is true and H_0 is false.
Therefore, it is significantly different compare to national population proportion.

Example 3: 150 bipolar patients are given a new drug that may improve the 45% effective treatment rate for the medication currently available.

a. State the hypotheses that need to be tested.
b. Determine the decision rule for which α will be 0.05.

a. $p_0 = 45\% = 0.45$ $n = 150$ **State Hypothesis**

H_0: $p = 0.45$ H_a: $p > 0.45$

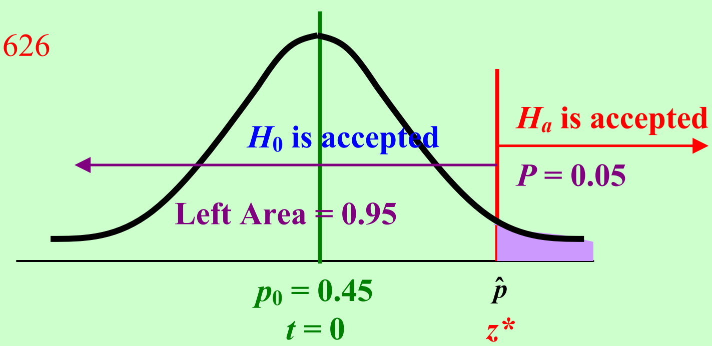
b. Decision Rule means finding $\hat{p}$ such that $P\ (z = z^*) = \alpha = 0.05$

$z^* = \text{invNorm}\ (0.95) = 1.644853626$

$$z^* = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$$\hat{p} = z^*\left(\sqrt{\frac{p_0(1-p_0)}{n}}\right) + p_0$$

$$\hat{p} = (1.644853626)\left(\sqrt{\frac{(0.45)(1-0.45)}{150}}\right) + 0.45$$



$\hat{p} = 0.5168$

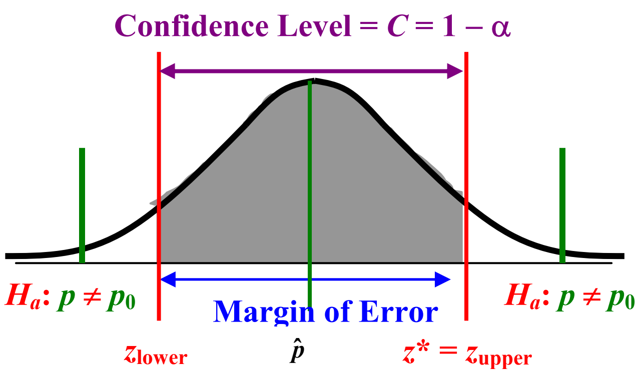
In terms of number of patients: $X = n\hat{p} = (150)(0.5168) = 77.52$ **$X = 78$ patients**

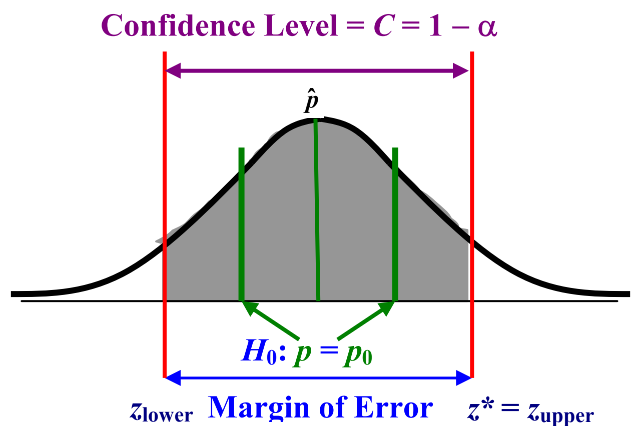
When the new drug has an effective rate **$\hat{p} \geq 0.5168$ or 51.68% or (78 or more patients)**, then it will be **significantly more effective** than the old drug. If the new drug has an effective rate **of $\hat{p} <$ 0.5168 or 51.68% or less than 78 patients**, then it is **not significantly different** than the old drug.

z-Confidence Intervals with One Sample Two-Sided z-Tests for a Population Proportion

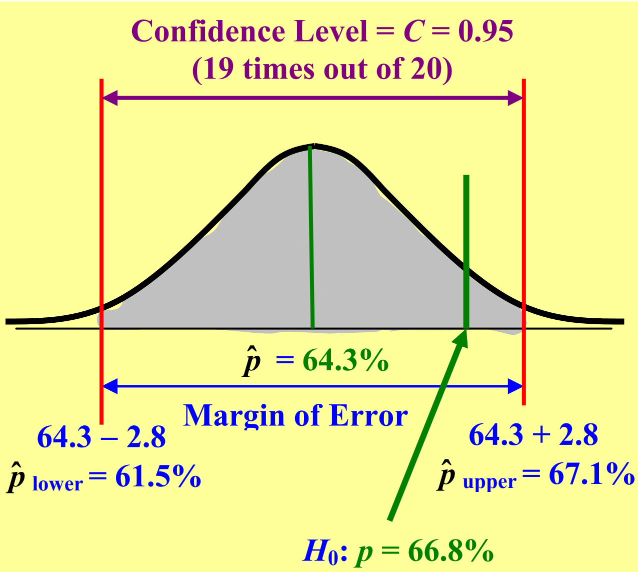
A Confidence Interval of a Sample can be tested for Significance (Level $C = 1 - \alpha$). The CONCLUSION can then be drawn.

1. **If $p_0 < (\hat{p} - m)$ or $p_0 > (\hat{p} + m)$, then H_a is True (Statistically Significant) as p_0 falls OUTSIDE Level C of the sample.**
2. **If $(\hat{p} - m) < p_0 < (\hat{p} + m)$, then H_0 is True (NOT Statistically Significant) as p_0 falls WITHIN Level C of the sample.**





Example 4: A survey of local parents' attitude towards school uniform in public schools was conducted with a sample size of 500 people. The result of this local survey indicated that 64.3% of the parents believes school uniforms with improve students' behaviours in and out of class. The local survey further stated that it was accurate within ±2.8%, 19 times out of 20. If the national survey shows that 66.8% of the parents' favours the implementation of school uniforms in public schools, is the local survey significantly than its national counterpart?



Since $p_0 = 66.8\%$, it is within the confidence interval of the sample proportion (64.3% ± 2.8%). Therefore, the local survey is NOT significantly different than the national survey regarding parents' attitude towards uniform in public schools.

12.1A Assignment

pg. 660 #12.1 and 12.3; pg.664 #12.5; pg. 668–669 #12.6 to 12.9

12.1B: Sample Size for Desired Margin of Error

Sample Size for Desired Margin of Error: - to calculate the **sample size (n) needed to yield a specific margin of error (m)**, **we have to know the standard error of the population proportion** $\left(\sqrt{\dfrac{p^*(1-p^*)}{n}}\right)$ and the **confidence interval** (so we can find z^*).

> **Sample Size for Desired Margin of Error**
>
> $$z^*\sqrt{\frac{p^*(1-p^*)}{n}} \le m \quad \text{or} \quad n \ge \left(\frac{z^*}{m}\right)^2 p^*(1-p^*)$$
>
> **p^* = "guessed" population proportion (because we really don't know the ideal population proportion unless a census is taken)**
>
> **The minimum n of the inequality can be obtained when $p^* = 0.5$. This is due to $y = x(1-x)$ reaches its maximum when $x = 0.5$**

Example 1: Determine the minimum n needed for the survey where the margin of error is no greater than 1% at a 97% confidence interval.

$m = 0.01$

$p^* = 0.5$ (since we do not know the ideal population proportion – this will give us the worst case minimum n needed)

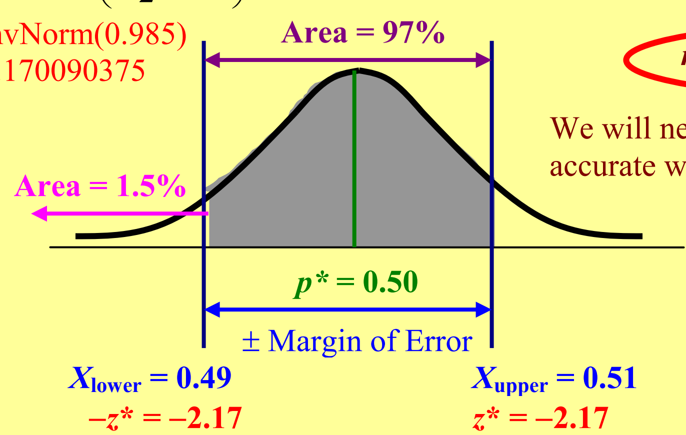
$C = 0.97$

$$z^* = \text{invNorm}\left(\frac{1-C}{2} + C\right)$$

$z^* = \text{invNorm}(0.985)$

$z^* = 2.170090375$

$n = ?$



$$z^*\sqrt{\frac{p^*(1-p^*)}{n}} \le m$$

$$n \ge \left(\frac{z^*}{m}\right)^2 p^*(1-p^*)$$

$$n \ge \left(\frac{2.170090375}{0.01}\right)^2 (0.5)(1-0.5)$$

$$n \ge 11773.23059$$

$n \ge 11774$ (always ROUND UP!)

We will need 11774 people in order to make a survey accurate within 1% at a 97% confidence interval.

> **12.1B Assignment**
>
> **pg. 672–677 #12.10 and 12.11; pg. 675 #12.13, 12.15, 12.17 and 12.19**

12.2: Comparing Two Proportions

Two-Sample Problems: - problems that deal with comparative experiment (involving surveying opinions of each population groups).
- this is different than matched pairs designs where subjects of the same group receive two surveys – before and after events).
- there may be two different sample sizes, two different sampling proportion means, and two different sampling proportion standard deviations.

Comparing Two Means: - there are several assumptions when comparing two means:

a. **Both Samples are Independent of each other and they are drawn from Random Samples.**

b. **Both Populations are Large, Binomial in Nature, and Normally Distributed.**

c. Like the Normal Approximation to a Binomial Distribution, the **population must be 10 times the sample size (n).**

d. **For a Confidence Interval: $n_1\hat{p}_1 \geq 5$; $n_1(1-\hat{p}_1) \geq 5$; $n_2\hat{p}_2 \geq 5$ and $n_2(1-\hat{p}_2) \geq 5$**

e. **For a Significance Test of (H_0: $\hat{p}_1 = \hat{p}_2$): $n_1\hat{p}_1 \geq 5$; $n_1(1-\hat{p}_1) \geq 5$; $n_2\hat{p}_2 \geq 5$ and $n_2(1-\hat{p}_2) \geq 5$**

Difference of Two Sample Proportion Means

Difference of Two Population Proportion Means = $(p_1 - p_2)$

Difference of Two Sample Proportion Means = $(\hat{p}_1 - \hat{p}_2)$

Difference in Population Proportion Standard Deviations = $\sqrt{\frac{p_1(1-p_1)}{n_1}+\frac{p_2(1-p_2)}{n_2}}$

(Special Case when $p_1 = p_2$) = $\sqrt{(p(1-p))\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$

Difference in Sample Proportion Standard Deviations = $\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$

(Special Case when $\hat{p}_1 = \hat{p}_2$) = $\sqrt{(\hat{p}(1-\hat{p}))\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$

***Note*: Standard Deviations cannot be subtracted directly. They must be change to variance first before the difference can be found.**

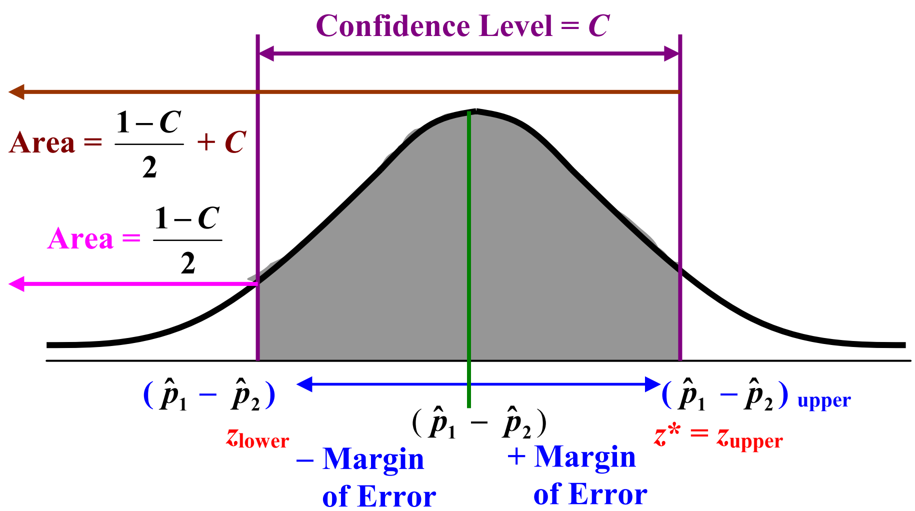
z-Statistic for the Difference of Two Sample Proportion Means

z-Statistics: $$z = \frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$$

Two-Sample Sampling Proportions z-Confidence Intervals [$(\hat{p}_1 - \hat{p}_2) \pm$ Margin of Error] at Level C

Margin of Error (m) = Critical Value (z^*) × Standard Error of Statistic

$$\left[SE = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}\right]$$

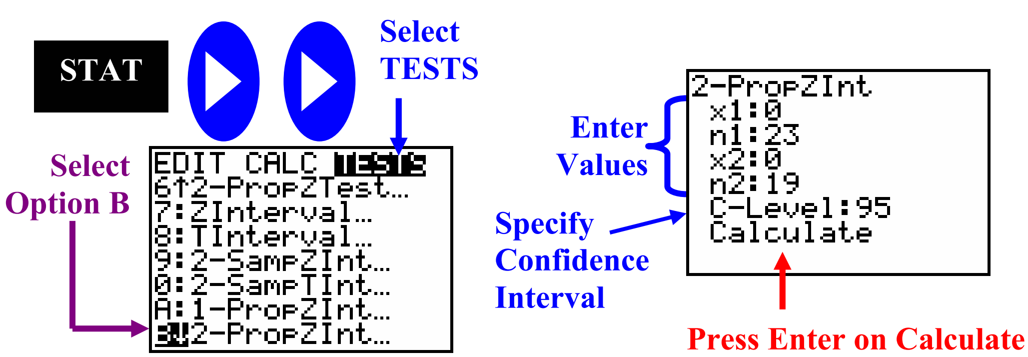


Confidence Interval for a Difference in Sampling Proportion Mean

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \quad \text{where } m = z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

***Note*: Use invNorm$\left(\frac{1-C}{2} + C\right)$ to find z^***

Two-Sample Proportion Confidence Interval (2-Prop ZInt function) on TI-83



Example 1: At a community hospital, the radiotherapy department is experimenting with a new cancer treatment for early detection of stomach cancer. A random group of one sample of 316 patients with stomach cancer found that 259 went into remission after the treatment. Another random sample of 419 patients with stomach cancer underwent the traditional surgery to remove the cancer part of the stomach. For this group, it was found that 94 went into remission after the surgical procedure. Let $\hat{p}_2$ be the sample proportion of all patients with stomach cancer that underwent surgeries and $\hat{p}_1$ be the sample proportion of all patients with stomach cancer that received the new radiotherapy treatments. Find the 95% confidence interval for the difference between the two samples.

$n_2 = 419$

$x_2 = 94$

$\hat{p}_2 = \frac{94}{419}$

$\hat{p}_2 = 0.2243436754$

$n_1 = 316$

$x_1 = 259$

$\hat{p}_1 = \frac{259}{316}$

$\hat{p}_1 = 0.8196202532$

C-Level = 0.95

$z^* = \text{invNorm}(0.975) = 1.959963986$

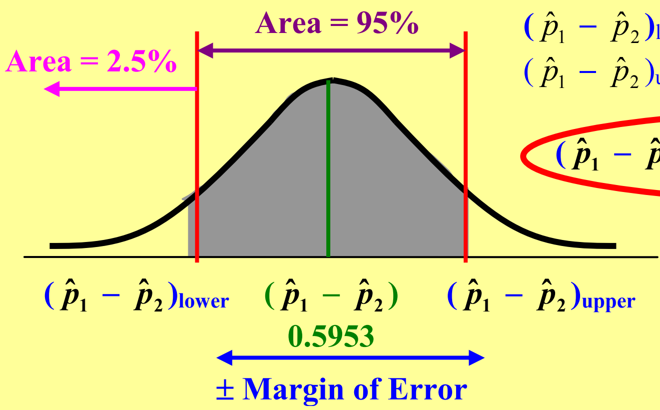
$(\hat{p}_1 - \hat{p}_2) = 0.8196202532 - 0.2243436754$

$(\hat{p}_1 - \hat{p}_2) = \mathbf{0.5952765778}$

$$\text{SE} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$\text{SE} = \sqrt{\frac{(0.8196202532)(1-0.8196202532)}{(316)} + \frac{(0.2243436754)(1-0.2243436754)}{(419)}}$$

SE = 0.029718078



$(\hat{p}_1 - \hat{p}_2)_{lower} = \text{invNorm}(0.025, 0.5952765778, 0.029718078)$

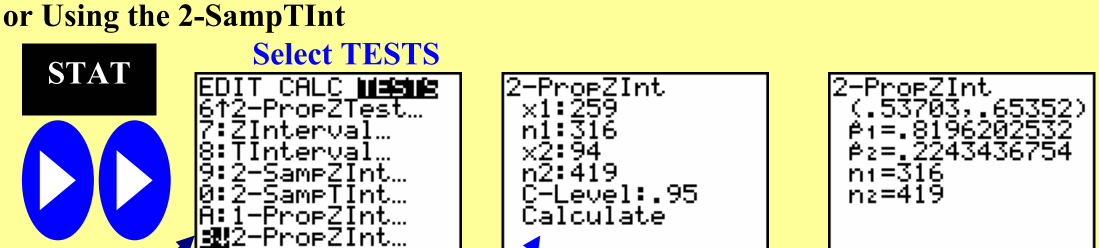
$(\hat{p}_1 - \hat{p}_2)_{upper} = \text{invNorm}(0.975, 0.5952765778, 0.029718078)$

$(\hat{p}_1 - \hat{p}_2)_{lower} = \mathbf{0.5370}$ $(\hat{p}_1 - \hat{p}_2)_{upper} = \mathbf{0.6535}$

$m = \pm z^*(\text{SE}) = \pm 1.959963986 \times 0.029718078$

Margin of Error = ± 0.0582

or Using the 2-SampTInt



Select Option B

Press Enter to Calculate

$$(\hat{p}_1 - \hat{p}_2) = \frac{(0.65352 - 0.53703)}{2}$$

$(\hat{p}_1 - \hat{p}_2) = \mathbf{0.595275}$

$m = \pm(0.65352 - 0.595275) = \pm\, \mathbf{0.058245}$

Confidence Interval = 0.5953 ± 0.0582

We can say with 95% confidence that the difference between the two procedures (radiotherapy compared to surgeries) in terms of the cancer remission rate is 0.5953 ± 0.0582

Two-Sample *z*-Test: - a significance test comparing the two independent sample proportion means after one of them has converted to a *z*-score with the other sample proportion means as $z = 0$.

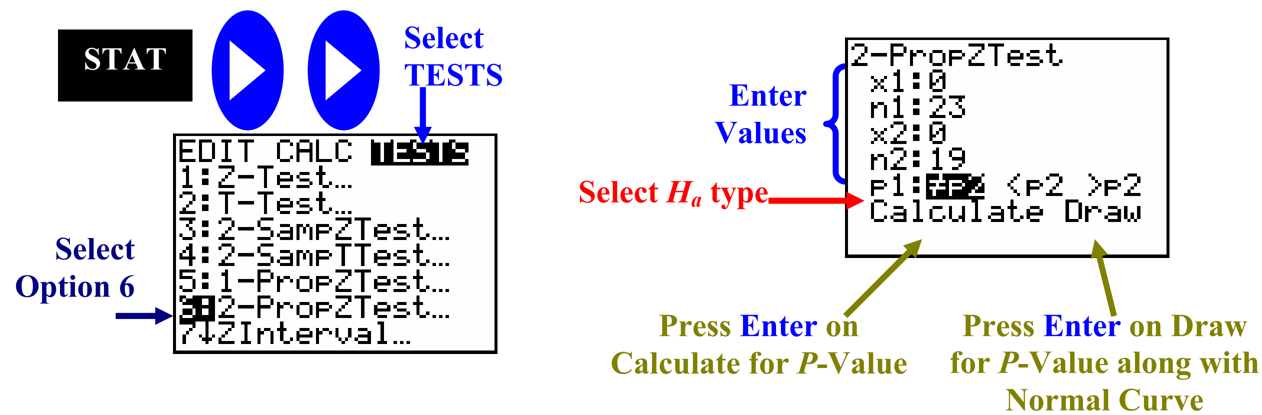
Steps for Two-Sample *z*-Test for a Proportion Mean:

1. **State the Hypothesis** (H_0: $\hat{p}_1 = p_2$ **and** H_a: $\hat{p}_1 \neq p_2$ **or** $\hat{p}_1 > p_2$ **or** $\hat{p}_1 < p_2$).
2. **Calculate the Two-Sample *z*-test statistics.**
3. **Determine the *P*-Value.**

Two-Sample *z*-Test Proportion Statistics

$$z_{\hat{p}_1 - \hat{p}_2} = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\dfrac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$$

2-PropZ-Test function on TI-83 Plus



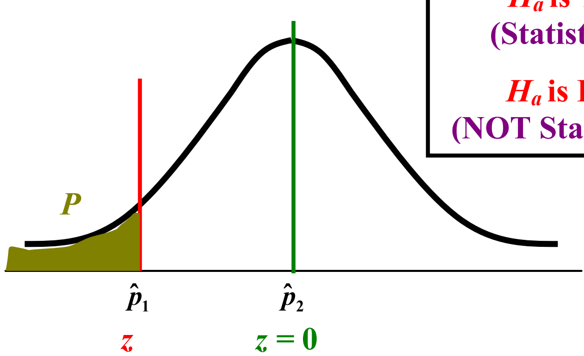
Fixed Significance Level Two-Sample *z*-Tests for Sample Proportion Means

Given a fixed significance level, α, the CONCLUSION can be drawn after *z*-test is performed.

1. **If $P \leq \alpha$, then H_a is True (Statistically Significant)**
2. **If $P > \alpha$, then H_0 is True (NOT Statistically Significant)**

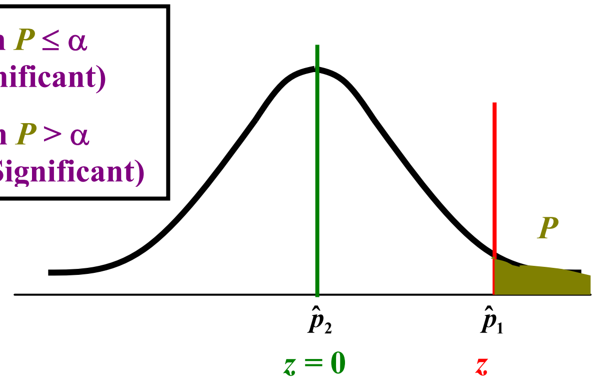
Two Sample Proportions One-Sided *z*-Tests

A Sample can be tested for Significance Level as α = tail probability. The CONCLUSION can then be drawn. (H_a: $\hat{p}_1 < \hat{p}_2$, or H_a: $\hat{p}_1 > \hat{p}_2$)



H_a is True when $P \leq \alpha$ (Statistically Significant)

H_a is False when $P > \alpha$ (NOT Statistically Significant)

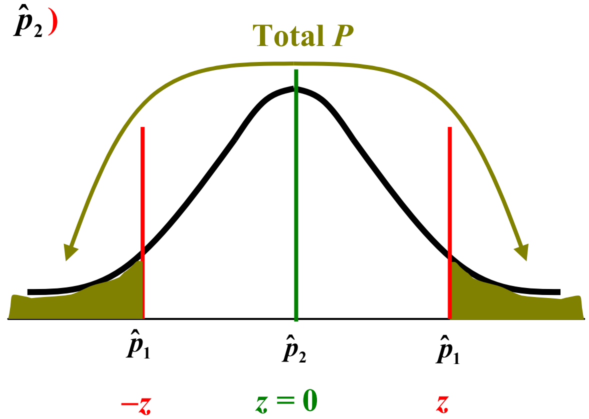


Two Sample Proportions Two-Sided *t*-Tests

A Sample can be tested for Significance Level as α = combined tail probabilities. The CONCLUSION can then be drawn. (H_a: $\hat{p}_1 \neq \hat{p}_2$)

H_a is True when Total $P \leq \alpha$
(Statistically Significant)

H_a is False when Total $P > \alpha$
(NOT Statistically Significant)



Example 2: The Santa Clara County Clerk wishes to improve voter registration. One method under consideration is to send reminders in the mail to all citizens in the county who are eligible to register. As a pilot study to determine if this method will actually improve voter registration, a random sample of 1500 potential voters was taken. Then this sample was randomly divided into two groups. One group consists of 800 people. No reminders to register were sent to them. The number of potential voters from this group who registered was 378. Another group contains 700 people. Reminders were sent in the mail to each member in this group, and the number who registered to vote was 392. The county clerk claims that the proportion of people to register was significantly greater with the group that received the reminders. On the basis of this claim, the clerk recommends that the project be funded for the entire population of Santa Clara County. Use a 5% significance to test the claim that the proportion of potential voters who registered in the group that received reminders was significantly greater. Find the *P*-value of the sample test statistics.

$x_1 = 392$ $\quad x_2 = 378$

$n_1 = 700$ $\quad n_2 = 800$

$\hat{p}_1 = \dfrac{392}{700}$ $\quad \hat{p}_2 = \dfrac{378}{800}$

$\hat{p}_1 = 0.56$ $\quad \hat{p}_2 = 0.4725$

1. Stating Hypothesis

H_0: $\hat{p}_1 = \hat{p}_2$

H_a: $\hat{p}_1 > \hat{p}_2$

2. Calculate **Standard Error**

$$SE = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$SE = \sqrt{\frac{(0.56)(1-0.56)}{700} + \frac{(0.4725)(1-0.4725)}{800}}$$

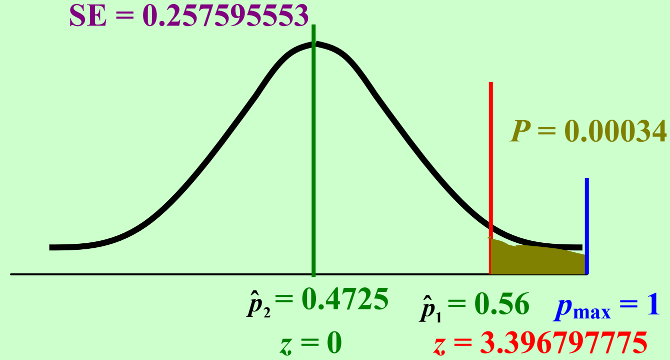
SE = 0.257595553

3. Calculate ***z*-test statistics**

$$z_{\hat{p}_1-\hat{p}_2} = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$$

$$z_{\hat{p}_1-\hat{p}_2} = \frac{(0.56-0.4725)}{\sqrt{\frac{(0.56)(1-0.56)}{700} + \frac{(0.4725)(1-0.4725)}{800}}}$$

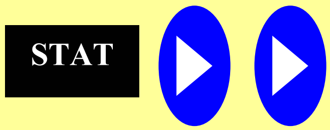
$$z_{\hat{p}_1-\hat{p}_2} = 3.396797775$$



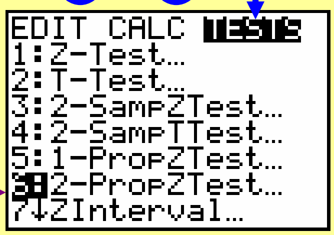
4. *P*-Value = normalcdf(0.56, 1, 0.4725, 0.0257595553) = 3.4095×10^{-4} **P-Value = 0.00034**

Since P (0.00034) < α (0.05), H_a is true and H_0 is false. Hence, the proportion from the group that received reminder in the mail that actually registered to vote was significantly higher than the proportion of registered voters from the other group that did not get the mail reminder to vote. Therefore, the county clerk is correct to make the claim that mail reminders will significantly increase voter registrations.

Using 2-Prop*Z*Test of the TI-83 Plus Calculator,

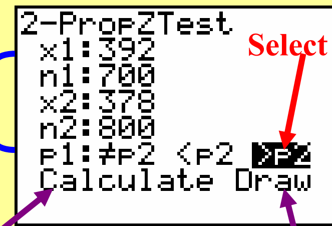


Select TESTS



Select Option 6

Enter Values



Select H_a type

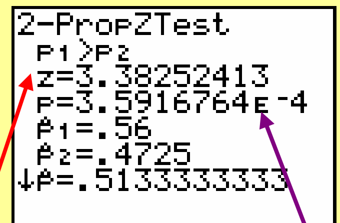
Press Enter on Calculate for P-Value

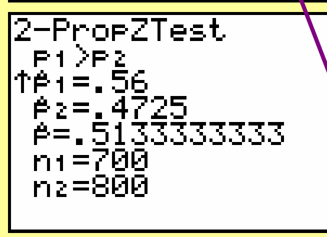
Press Enter on Draw for P-Value along with Normal Curve

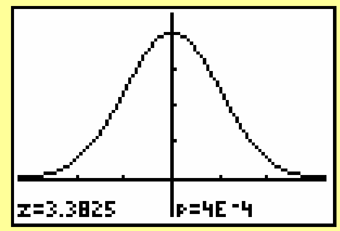
$$z_{\hat{p}_1-\hat{p}_2} = \frac{(\hat{p}_1-\hat{p}_2)}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$$

$$z_{\hat{p}_1-\hat{p}_2} = \frac{(0.56-0.4725)}{\sqrt{\frac{(0.56)(1-0.56)}{700}+\frac{(0.4725)(1-0.4725)}{800}}}$$

$z_{\hat{p}_1-\hat{p}_2}$ **= 3.396797775 (very close)**







P-Value = normalcdf(0.56, 1, 0.4725, 0.0257595553)

***P*-Value = 3.4095 × 10^{-4}**

12.2 Assignment

pg. 682 #12.21; pg. 668 #12.23;
pg. 690–693 # 12.25, 12.27, 12.29, 12.31 and 12.33

Chapter 12 Review

pg. 695–696 #12.35 to 12.40

Chapter 13: Inference for Tables: Chi-Square Procedure

13.1: Test for Goodness of Fit

Chi-Square (χ^2) Test for Goodness of Fit: - an overall test to **compare ALL Categories** of a Sample Distribution against a Population Distribution with the SAME Categories in order to judge **whether they are significantly different.**
- it is very **useful** when dealing with **Distributions with Categorical Variables** (Bar Graphs and Pie Charts) as well as **Distributions with Quantitative Variables** (Dot plots, Histograms, and Stem plots).
- involves calculating the **chi-square (χ^2) statistics** and the **degree of freedom (*df*)**.

Chi-Square (χ^2) Statistics (X^2): - the **sum of all comparisons** between the **observed count (*O*) from each category of the sample distribution** with the **expected count (*E*) from the same category of the population distribution**.
- similar to the variance quantities, it measures the overall amount of deviation between the categories of sample and the population distributions.

Degree of Freedom (***df***): - measures the number of ways to compare with the sample distribution based on the number of categories in the sample.
- equals to **(n – 1) where n = number of categories in the sample**.

Chi-Square (χ^2) Statistics

$$X^2 = \sum \frac{(O-E)^2}{E} \quad \text{with } (n-1) \text{ degrees of freedom } (df)$$

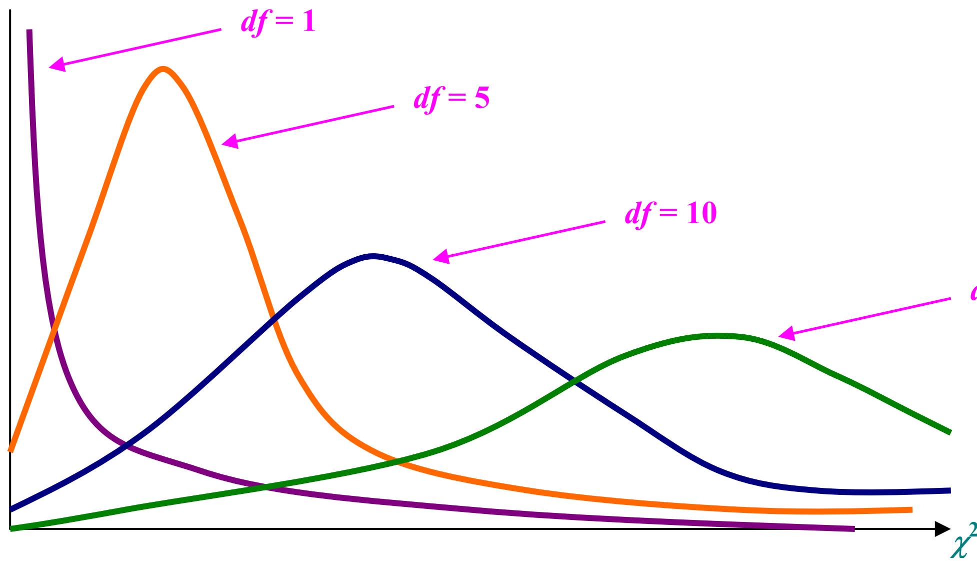
***O* = Observed Count of Each Category in Sample Distribution**
***E* = Expected Count of Each Category in Population Distribution**
***n* = number of categories in the distribution**

Counts: - number of subjects that fit into a particular category in the distribution.
- also refer to as frequency.

Characteristics of χ^2-Distribution:

1. It **takes positive values because X^2 is the sum of all average variance** between two distributions. Therefore, the **distribution starts at 0 on the horizontal axis.**
2. The **total area of the distribution curve is 1**.
3. The **curve is skewed to the right**.
4. As the **degree of freedom increases, the shape of the χ^2-distribution approaches the symmetrical normal curve**. This is because of the law of large numbers and the central limit theorem. **In essence, the bigger X^2 value is required to achieve the same upper tail probability**. This makes sense since the more number of categories that we have in a distribution; the probability that two distributions are significantly different requires a larger overall deviation between them.

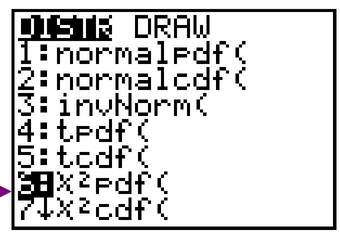
Families of χ^2 Distributions



Drawing χ^2 Distributions on TI-83 Plus

χ^2pdf (X, *df*) to be drawn in **Y=** **screen**

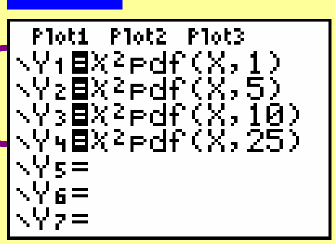
X,T,θ,n → **2nd** **DISTR VARS** → **Select Option 6**

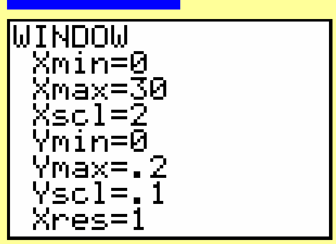


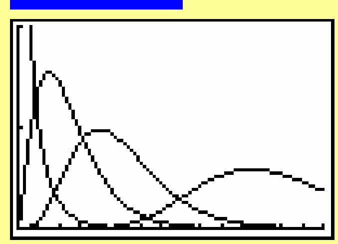
***Note*: Using χ^2pdf to draw χ^2 Distribution does not calculate *P*-value. <u>To calculate *P*-value, one must use the χ^2-Distribution Table or χ^2cdf.</u>**

Example: Draw the χ^2 distributions for degree of freedom of 1, 5, 10, and 25. Use Window Settings x: [0, 30, 2] and y: [0, 0.2, 0.1].

Enter χ^2pdf Functions — **Y=** — **WINDOW** — **GRAPH**







Goodness of Fit Test: - is required to **compare the sample distribution (actual population) categories** expressed in percentages **are significantly different than the population distribution (hypothesized) categories** expressed in percentages.

Four Steps of Goodness of Fit Test:

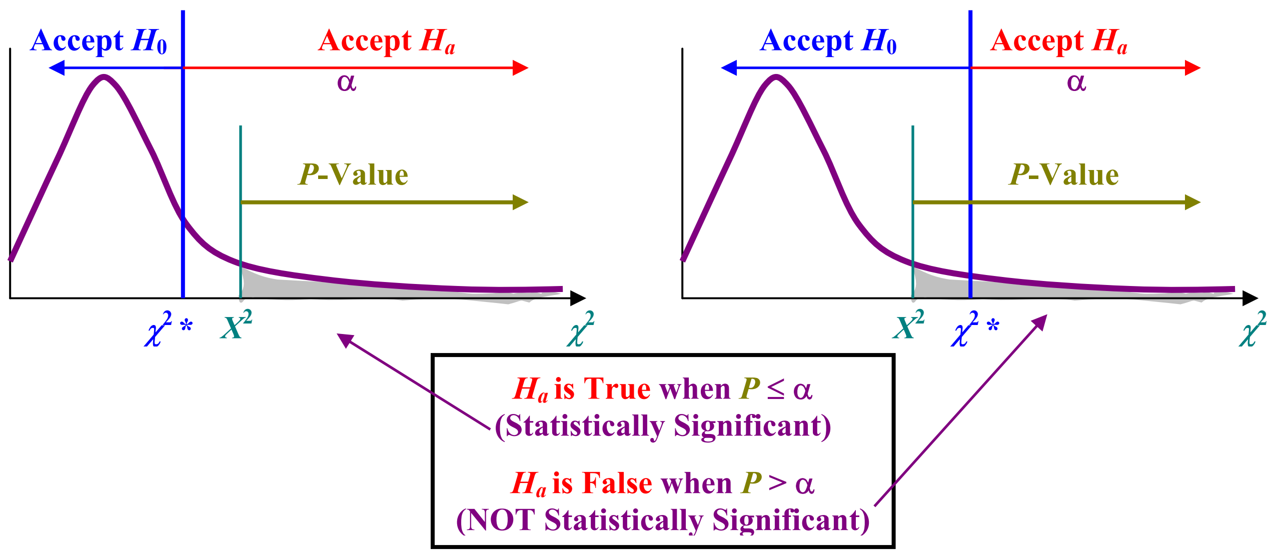
1. **State the Hypothesis**
   (*H_0*: *Actual Population Percentages* = *Sample Distribution Percentages* and
   ***H_a*: *Actual Population Percentages* ≠ *Sample Distribution Percentage*)**
2. **Calculate the Chi-Square (χ^2) Statistics**: $X^2 = \sum \frac{(O - E)^2}{E}$.
3. **Determine $df = (n - 1)$** where **n = number of categories**.
4. **Find the *P*-Value** either using the **χ^2- Distribution Table** or **χ^2cdf** Function of the TI-83 Plus Calculator.

Assumptions Using the Goodness of Fit Test:

a. **All individual count must be at least 1. (No categories with 0% or 0 count.)**

b. **There should be no more than 20% of all expected counts are 5 or less. (That is, if there are 10 categories, only 2 categories can have counts that are 5 or less.)**

Critical Chi-Square (χ^2 *): - the **chi-square value corresponding to the α-level** of the goodness of fit test.



χ^2cdf Function on TI-83 Plus

χ^2cdf (X^2_{low}, X^2_{high}, df) = Upper Tail Probability

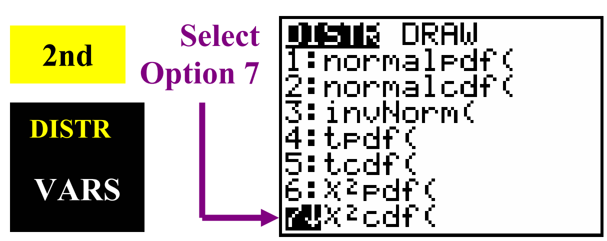


Table entry for p is the point (χ^2) with probability p lying above it.

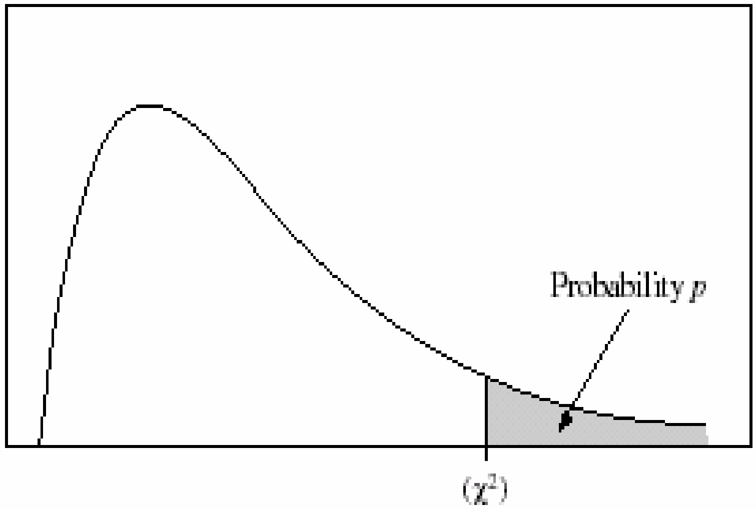


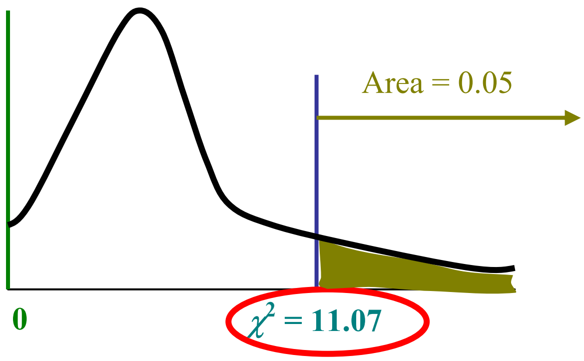
Table C χ^2 critical values

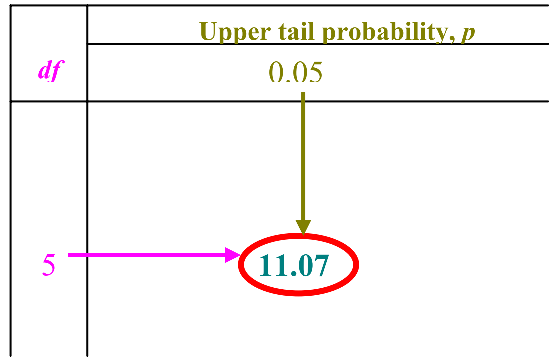
	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

Using the χ^2-Distribution Critical Values Table:

1. **Converting Upper Tail Probability (Area RIGHT of the χ^2-score boundary) to χ^2-score**
   - **a.** Look up the ***df* (degree of freedom) from the row heading**.
   - **b.** Look up the **upper tail probability, *p*, from the column heading**.
   - **c.** Follow that row and column to find the **χ^2-score**.

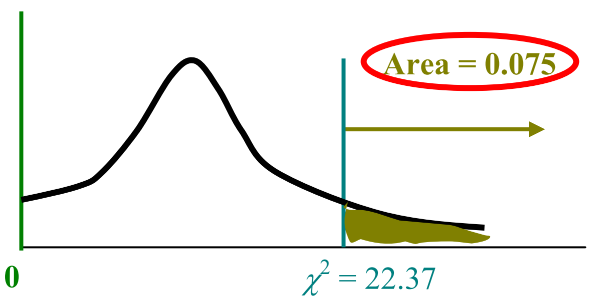
Example: Find χ^2 when $p = 0.05$ and $n = 6$ ($df = 5$).

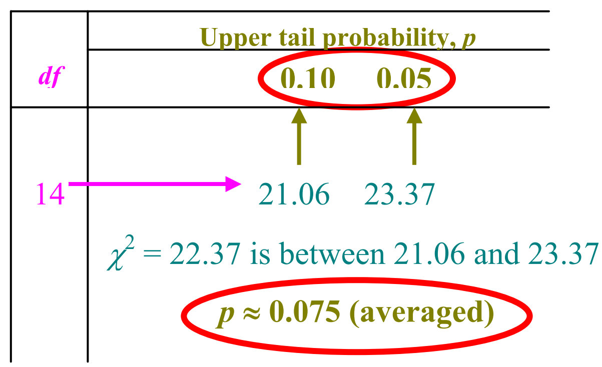




2. **Converting χ^2-score back to Upper Tail Probability (Area Right of the χ^2-score boundary)**
   - **a.** Along the row of *df* (degree of freedom), look up the **closest χ^2-score(s)** from **INSIDE** the table.
   - **b.** Follow that column(s) back UP to the heading and locate the corresponding **upper tail probability**.
   - **c.** May have to average the probabilities or guessed if the *t*-score used on the table is not exact.

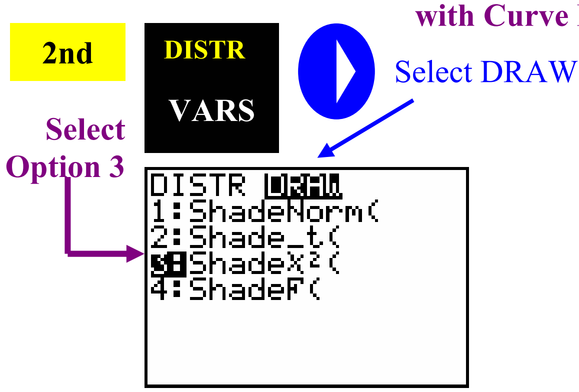
Example: Find $P(\chi^2 \geq 22.37)$ when $n = 15$ ($df = 14$)





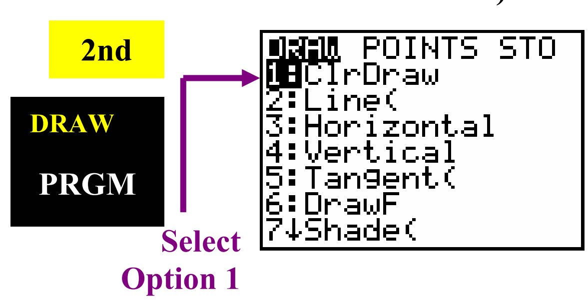
Shadeχ^2 Function on TI-83 Plus

Shadeχ^2 (X^2_{low}, X^2_{high}, *df*) = Upper Tail Probability with Curve Drawn



ClrDraw Function on TI-83 Plus

(To be used to CLEAR any Previous Draw or Shade Functions)



Example 1: The final marks of all the Algebra II classes at Suburbia High School are distributed against all the Algebra II classes in the County. Using the significance level of 0.05, evaluate whether the school performance in its Algebra II classes is significantly different than those in the county.

Results of 273 Algebra II Students in Suburbia High School	
Final Marks	**Percents of Population**
90% – 100%	5.56%
80% – 89%	18.33%
70% – 79%	21.11%
60% – 69%	15.33%
50% – 59%	18.00%
below 50%	21.67%

Results of all Algebra II Students in the County	
Final Marks	**Percents of Population**
90% – 100%	8.56%
80% – 89%	21.33%
70% – 79%	25.67%
60% – 69%	12.11%
50% – 59%	20.89%
below 50%	11.44%

1. Calculate the Counts in Each Distribution using sample size of 273.

	Results of 273 Algebra II Students in Suburbia High School (Actual Population)		**Results of all Algebra II Students in the County (Hypothesized Population using a Total of 273 Students)**	
Final Marks	**% Population**	**Observed Count (O)**	**% Population**	**Expected Count (E)**
90% – 100%	5.56%	**273 (0.056) = 15.288**	8.56%	**273 (0.0856) = 23.3688**
80% – 89%	18.33%	**273 (0.1833) = 50.0409**	21.33%	**273 (0.2133) = 58.2309**
70% – 79%	21.11%	**273 (0.2111) = 57.6303**	24.67%	**273 (0.2467) = 67.3491**
60% – 69%	15.33%	**273 (0.1533) = 41.8509**	12.11%	**273 (0.1211) = 33.0603**
50% – 59%	18.00%	**273 (0.1800) = 49.14**	15.11%	**273 (0.1511) = 41.2503**
below 50%	21.67%	**273 (0.2167) = 59.1591**	15.33%	**273 (0.1533) = 41.8509**

2. Check Conditions for Goodness of Fit Test.

a. **No categories have a count of zero in both distributions.**

b. **More than 20% of the counts (2 out of 6 categories) have counts more than 5 in both distributions.**

3. State Hypothesis

H_0: ***Actual Population Percentages = Hypothesized Distribution Percentages***

H_a: ***Actual Population Percentages ≠ Hypothesized Distribution Percentages***

4. Calculate χ^2 Statistics

Final Marks	$\frac{(O-E)^2}{E}$
90% – 100%	$\frac{(15.288-23.3688)^2}{23.3688} = \mathbf{2.794295327}$
80% – 89%	$\frac{(50.0409-58.2309)^2}{58.2309} = \mathbf{1.151898734}$
70% – 79%	$\frac{(57.6303-67.3491)^2}{67.3491} = \mathbf{1.40246972}$
60% – 69%	$\frac{(41.8509-33.0603)^2}{33.0603} = \mathbf{2.337384971}$
50% – 59%	$\frac{(49.14-41.2503)^2}{41.2503} = \mathbf{1.509016082}$
Below 50%	$\frac{(59.1591-41.8509)^2}{41.8509} = \mathbf{7.158120548}$

$$X^2 = \sum \frac{(O-E)^2}{E} \qquad X^2 = 16.35318538$$

5. Find P-Value from χ^2-Distribution Table

(n = 6 categories; $df = (n - 1) = 5$)

df	**Upper tail probability, p**	
	0.01	**0.005**
5	15.09	16.75

χ^2 = 16.35 is between 15.09 and 16.35

$P \approx 0.0057$ (X^2 closer to 16.75)

Since the P (0.0057) < α (0.05), the distribution from the Algebra II classes in Suburbia High School is <u>significantly different</u> than the distribution of all the Algebra II classes in the county. This is most likely due to the number of students in the below 50% category (largest deviation with Suburbia school having a larger percentage compared to that of the county).

Or Using the TI-83 Plus Calculator.

1. Enter Data in L_1 and L_2

L_1 = Actual (Sample) Population Proportions (Suburbia High)
L_2 = Ideal (Hypothesized) Population Proportions (County)

STAT ENTER

```
L1      L2      L3      2
.0556   .0856   ------
.1833   .2133
.2111   .2467
.1533   .1211
.18     .1511
.2167   .1533
------  ------
L2(7) =
```

2. Calculate $\frac{(O-E)^2}{E}$ in L_3

For L_3, type:
"$(273 \times L_1 - 273 \times L_2)^2/(273 \times L_2)$"

ALPHA MEM " +

```
L1      L2      L3      3
.0556   .0856   2.8703
.1833   .2133   1.1519
.2111   .2467   1.4025
.1533   .1211   2.3374
.18     .1511   1.509
.2167   .1533   7.1581
------  ------  ------
L3 =…²/(273*L2)"
```

Use Quotation Mark for Formula

3. Sum (L_3) for X^2.

2nd LIST STAT ▶ ▶

```
NAMES OPS MATH
1:min(
2:max(
3:mean(
4:median(
5:sum(
6:prod(
7↓stdDev(
```

Select MATH

Select Option 5

```
sum(L3)
          16.42921716
```

X^2

4. To Find P-Value of X^2, use either χ^2cdf or Shadeχ^2 functions
χ^2cdf (16.42921716, 1×10^{99}, 5) or Shadeχ^2(16.42921716, 1×10^{99}, 5)

2nd DISTR VARS

```
DISTR DRAW
1:normalpdf(
2:normalcdf(
3:invNorm(
4:tpdf(
5:tcdf(
6:X²pdf(
7↓X²cdf(
```

Select Option 7

```
sum(L3)
          16.42921716
X²cdf(Ans,1E99,5
)
          .0057197215
```

***P*-Value = 0.00572**

***Note*: May have to do <u>ClrDraw</u> and <u>erase all functions</u> from Y= screen before using Shadeχ^2.**

a. Set Window WINDOW

```
WINDOW
 Xmin=0
 Xmax=20
 Xscl=1
 Ymin=-.05
 Ymax=.2
 Yscl=.05
 Xres=1
```

b. Get Shadeχ^2 Function

2nd DISTR VARS ▶ Select DRAW

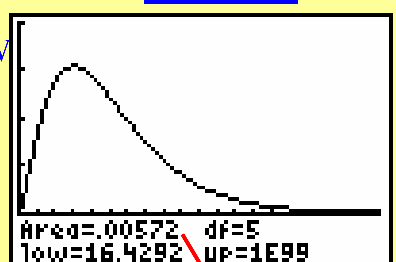
Select Option 3

```
DISTR DRAW
1:ShadeNorm(
2:Shade_t(
3:ShadeX²(
4:ShadeF(
```

c. Enter Parameters

```
ShadeX²(16.42921
716,1E99,5)
```

d. Press ENTER



***P*-Value = 0.00572**

Since the P (0.00572) < α (0.05), the distribution from the Algebra II classes in Suburbia High School is <u>significantly different</u> than the distribution of all the Algebra II classes in the county. This is most likely due to the number of students in the below 50% category (largest deviation with Suburbia school having a larger percentage compared to that of the county).

Example 2: A six-sided dice is rolled 100 times and the following results were recorded. Determine whether the dice is significantly different than expected at a 0.05 level.

Results	Frequency
1	18
2	12
3	24
4	20
5	10
6	16

1. Calculate the Expected Counts using Theoretical Probability. $P(\text{a specific number}) = \frac{1}{6}$

Results	Observed Count	Expected Count	$\frac{(O-E)^2}{E}$
1	18	$100 \times \frac{1}{6} = 16.667$	$\frac{(18-16.667)^2}{16.667} = \mathbf{0.10667}$
2	12	$100 \times \frac{1}{6} = 16.667$	$\frac{(12-16.667)^2}{16.667} = \mathbf{1.3067}$
3	24	$100 \times \frac{1}{6} = 16.667$	$\frac{(24-16.667)^2}{16.667} = \mathbf{3.2267}$
4	20	$100 \times \frac{1}{6} = 16.667$	$\frac{(20-16.667)^2}{16.667} = \mathbf{0.66667}$
5	10	$100 \times \frac{1}{6} = 16.667$	$\frac{(10-16.667)^2}{16.667} = \mathbf{2.6667}$
6	16	$100 \times \frac{1}{6} = 16.667$	$\frac{(16-16.667)^2}{16.667} = \mathbf{0.02667}$

$$X^2 = \sum \frac{(O-E)^2}{E} \qquad X^2 = 8$$

2. Check Conditions for Goodness of Fit Test.

a. No categories have a count of zero in both distributions.

b. More than 20% of the counts (2 out of 6 categories) have counts more than 5 in both distributions.

3. State Hypothesis

H_0*: *Theoretical Count = Sample Experimental Count

H_a*: *Theoretical Count ≠ Sample Experimental Count

4. Calculate χ^2 Statistics (see above)

$X^2 = 8$

5. Find *P*-Value from χ^2-Distribution Table

(n = 6 categories; $df = (n - 1) = 5$)

df	Upper tail probability, *p*	
	0.20	0.15
5	7.29	8.12

$X^2 = 8$ is between 7.29 and 8.12

$P \approx 0.16$ (X^2 closer to 8.12)

Since the P (0.16) > α (0.05), the experimental frequency distribution is NOT significantly different than the theoretical distribution of rolling a six-sided dice 100 times.

Or Using the TI-83 Plus Calculator.

1. Enter Data in L_1 and L_2

L_1 = Sample (Experimental) Count
L_2 = Theoretical (Hypothesized) Count

STAT → ENTER

```
L1      L2      L3     2
18      16.667  ------
12      16.667
24      16.667
20      16.667
10      16.667
16      16.667
------  ------
L2(7) =
```

2. Calculate $\frac{(O-E)^2}{E}$ in L_3

For L_3, type: "$(L_1 - L_2)^2/L_2$"

ALPHA → MEM " → +

Use Quotation Mark for Formula

```
L1      L2      L3     3
18      16.667  .10667
12      16.667  1.3067
24      16.667  3.2267
20      16.667  .66667
10      16.667  2.6667
16      16.667  .02667
------  ------  ------
L3 ="(L1-L2)²/L2"
```

3. Sum (L_3) for X^2.

2nd → LIST STAT → ▶ → ▶

```
NAMES OPS MATH
1:min(
2:max(
3:mean(
4:median(
5:sum(
6:prod(
7↓stdDev(
```

Select MATH

Select Option 5

```
sum(L3)
                8
```

X^2

5. To Find *P*-Value of X^2, use either χ^2cdf or Shadeχ^2 functions

χ^2cdf $(8, 1 \times 10^{99}, 5)$ or Shade$\chi^2(8, 1 \times 10^{99}, 5)$

2nd → DISTR VARS

```
DISTR DRAW
1:normalpdf(
2:normalcdf(
3:invNorm(
4:tpdf(
5:tcdf(
6:X²pdf(
7:X²cdf(
```

Select Option 7

```
sum(L3)
                8
X²cdf(8,1E99,5)
       .1562356273
```

P-Value = 0.1562

Note: May have to do ClrDraw and erase all functions from Y= screen before using Shadeχ^2.

a. Set Window WINDOW

```
WINDOW
 Xmin=0
 Xmax=20
 Xscl=1
 Ymin=-.05
 Ymax=.2
 Yscl=.05
 Xres=1
```

b. Get Shadeχ^2 Function

2nd → DISTR VARS → ▶ (Select DRAW)

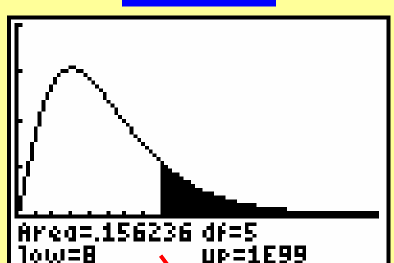
Select Option 3

```
DISTR DRAW
1:ShadeNorm(
2:Shade_t(
3:ShadeX²(
4:ShadeF(
```

c. Enter Parameters

```
ShadeX²(8,1E99,5
)
```

d. Press ENTER



P-Value = 0.1562

Since the *P* (0.1562) > α (0.05), the experimental frequency distribution is NOT significantly different than the theoretical distribution of rolling a six-sided dice 100 times.

13.1 Assignment

pg. 710–711 #13.1, 13.3 and 13.4; pg. 716 #13.9, 13.10 and 13.12

13.2: Inference for Two-Way Tables

Multiple Comparison: - a chi-square comparison of a two-way table (*row* × *column*) – (horizontal by vertical) between the sample distribution and a theoretical distribution.

Example: A Two-way table of with 4 rows by 2 columns (4 × 2)

Highest Education Level	Never or Only Applied for Social Assistance Once within the last 10 years	Applied for Social Assistance More than Once within the last 10 years
Did not finish High School	38%	62%
High School Diploma Only	63%	37%
Attended College Courses Only	79%	21%
Attained a Post-Secondary Diploma or Degree	89%	11%

Cell: - an element from a specific row and column within a two-way table.

Example: The cell from row 3 and column 2 of the above table indicates that 21% of the people surveyed who only took some college courses applied for social assistance more than once within the last 10 years.

Expected Counts of a Two-Way Table: - because each cell of the two-way table is two-dimensional (row and column), its expected count has to account for the row total and column total of that cell.

Expected Counts of a Two-Way Table

The expected count (*E*) of any cell of a two-way table can be calculated as follows:

$$\text{Expected Count} = \frac{\text{Row Total} \times \text{Column Total}}{\text{Table Total}} \qquad E = \frac{r_{total} \times c_{total}}{T_{total}}$$

Example 1: A survey was conducted recently from a local social security office to understand the number of applications for social assistance based on the applicants' highest education levels. 100 samples of each of the four highest levels of education (did not finish high school, finished high school only, some college courses only, and college diploma or certificate) were randomly taken, and the number of times they applied for social assistance in the last ten years were recorded as shown below. Calculate the expected count of the cell that describes the number of people who took some college courses and have applied for social assistance more than once within the last 10 years.

Highest Education Level	Never or Only Applied for Social Assistance Once within the last 10 years	Applied for Social Assistance More than Once within the last 10 years	Total
Did not finish High School	38	62	**100**
High School Diploma Only	63	37	**100**
Some College Courses Only	79	21	**100**
Attained a Post-Secondary Diploma or Degree	89	11	**100**
Total	**269**	**131**	**400**

For the cell that describes the number of people who took some college courses and have applied for social assistance more than once within the last 10 years (row 3 and column 2), $r_{\text{total}} = 100$, $c_{\text{total}} = 131$, and $T_{\text{total}} = 400$

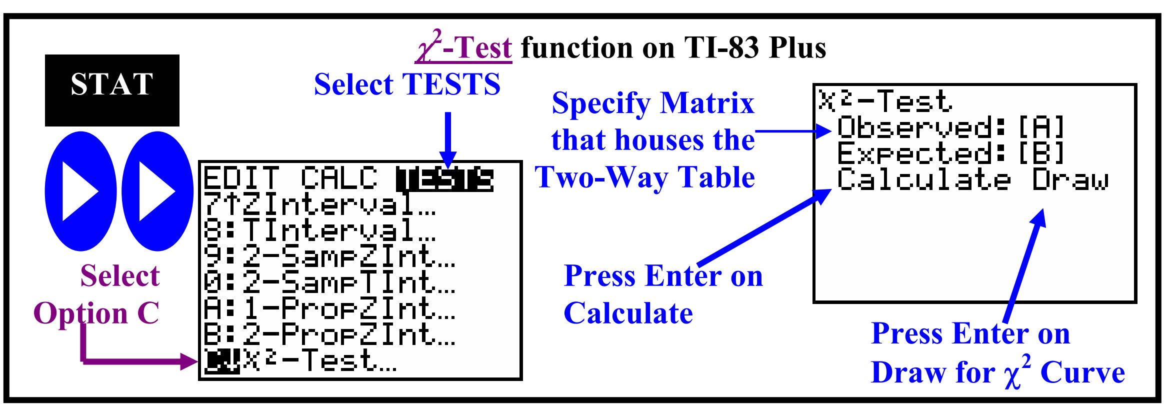
$$E = \frac{r_{total} \times c_{total}}{T_{total}} = \frac{100 \times 131}{400}$$

$E = 32.75$ (for row 3 and column 2)

Chi-Square Test of a Two-Way Table: - a significance test using the χ^2-distribution by comparing the observed count of a sample distribution with the expected count of a theoretical distribution from a two-way table.
- the null hypothesis is all proportions from one category are equalled to all others.
- the degree of freedom is also two-dimensional $(r - 1) \times (c - 1)$.

Five Steps of Chi-Square Test of a Two-Way Table:

1. **State the Hypothesis (p_1, p_2, p_3, … = sample proportion in each category)**
   (H_0: $p_1 = p_2 = p_3 = \ldots$ and H_a: *not all of* p_1, p_2, and p_3 are equal)
2. **Calculate the Chi-Square (χ^2) Statistics**: $X^2 = \sum \frac{(O - E)^2}{E}$ **from all the cells.**
3. **Determine $df = (r - 1) \times (c - 1)$** where **$r$ = number of rows and c = number of columns.**
4. **Find the *P*-Value** either using the **χ^2- Distribution Table** or **χ^2-Test** Function of the TI-83 Plus Calculator.
5. **Follow-up Analysis**: - by examining the numbers in the two-way table, provide a reason for the conclusion as demonstrated with the *P*-value and α-level.

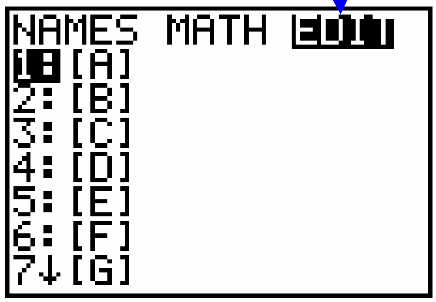


Using a Graphing Calculator to Operate with Matrices

A. To Enter a Matrix:

1. Press **2nd** **MATRX** x^{-1}

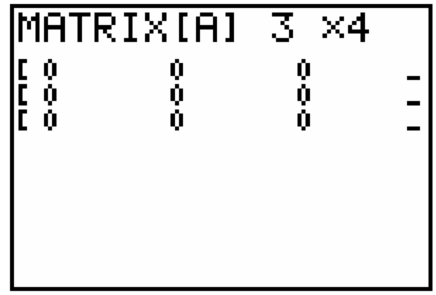
2. Use ▶ to access EDIT



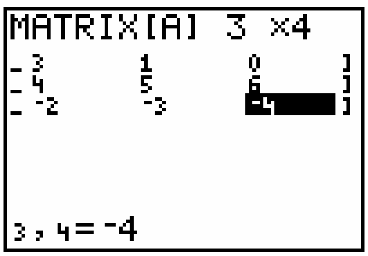
3. Select Option 1 if the desired name of the Matrix is [A]. Otherwise select other options for other names.

4. Press **ENTER**

5. Enter the dimensions of the matrix.



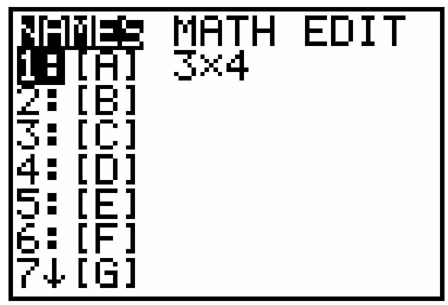
6. Enter the elements of the matrix (along each row).



7. Press **2nd** **QUIT** **MODE** when finished.

B. To Recall a Matrix from the Home Screen:

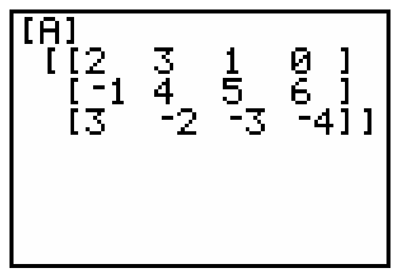
1. Press **2nd** **MATRX** x^{-1}



2. Select Option 1 if the desired matrix to be recalled is [A]. Otherwise select other options for other matrices.

3. Press **ENTER**

4. Press **ENTER** again to see the entire matrix on the home screen. (Highly recommended for matrices bigger than 3 × 3 to verify if there are any mistakes while entering elements.)



C. To Delete a Matrix:

1. Press **2nd** **MEM** **+**

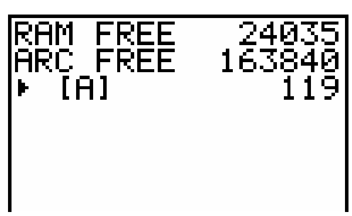
2. Select Option 2.

3. Press **ENTER**

4. Select Option 5.

5. Press **ENTER**

6. Press **INS** **DEL** next to the matrix that needs to be deleted.



Example 1: Using the table on page 182, evaluate if the evidence shows that the proportions of people who never or applying for social assistance once in the past 10 years are significantly different based of their levels of education.

1. State the Hypothesis: H_0: $p_1 = p_2 = p_3 = p_4$ **and** H_a: *not all of* p_1, p_2, p_3, and p_4 are equal

2. **Calculate χ^2-Statistics**

   a. Determine **Expected Count** for all Cells in the Two-Way Table ($E = \frac{r_{total} \times c_{total}}{T_{total}}$)

Highest Education Level	Never or Only Applied for Social Assistance Once within the last 10 years	Applied for Social Assistance More than Once within the last 10 years	Total
Did not finish High School	$E = \frac{100 \times 269}{400} = 67.25$	$E = \frac{100 \times 131}{400} = 32.75$	**100**
High School Diploma Only	$E = \frac{100 \times 269}{400} = 67.25$	$E = \frac{100 \times 131}{400} = 32.75$	**100**
Some College Courses Only	$E = \frac{100 \times 269}{400} = 67.25$	$E = \frac{100 \times 131}{400} = 32.75$	**100**
Attained a Post-Secondary Diploma or Degree	$E = \frac{100 \times 269}{400} = 67.25$	$E = \frac{100 \times 131}{400} = 32.75$	**100**
Total	**269**	**131**	**400**

   b. **Calculate $\frac{(O-E)^2}{E}$ and χ^2-Statistics of each cell.**

Highest Education Level	Never or Only Applied for Social Assistance Once within the last 10 years	Applied for Social Assistance More than Once within the last 10 years	Total
Did not finish High School	$\frac{(38-67.25)^2}{67.25} = 12.722119$	$\frac{(62-32.75)^2}{32.75} = 26.1240458$	**100**
High School Diploma Only	$\frac{(63-67.25)^2}{67.25} = 0.2685874$	$\frac{(37-32.75)^2}{32.75} = 0.55152672$	**100**
Some College Courses Only	$\frac{(79-67.25)^2}{67.25} = 2.0529740$	$\frac{(21-32.75)^2}{32.75} = 4.21564886$	**100**
Attained a Post-Secondary Diploma or Degree	$\frac{(89-67.25)^2}{67.25} = 7.0343866$	$\frac{(11-32.75)^2}{32.75} = 14.44465649$	**100**
Total	**269**	**131**	**400**

$$X^2 = \sum \frac{(O-E)^2}{E} = 12.722119 + 0.2685874 + 2.0529740 + 7.0343866 + 26.1240458 + 0.55152672 + 4.21564886 + 14.44465649$$

$$X^2 = 67.4139$$

1. **Determine *df*.**

$df = (r-1) \times (c-1) = (4-1) \times (2-1)$ $df = 3$

2. **Find *P*-Value from χ^2-Distribution Table ($df = 3$).**

df	Upper tail probability, *p*: 0.0005
3	17.73

$X^2 = 67.4139$ is beyond 17.73

P << 0.0005 (X^2 is way smaller than the last reading for the row)

3. **Follow-up Analysis**:

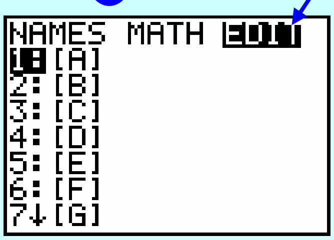
Since the P << 0.0005 << α (0.05), the proportions of people who never or applying for social assistance once in the past 10 years are <u>significantly different</u> based on their levels of education. We can see it most clearly with the group who has never finished high school as compared to the group who has finished college. The high school dropouts have a higher than expected proportion to apply for social assistance more than once within the last 10 years. This is in contrast to college grads that have a lower than expected proportion to apply for social assistance more than once within the last 10 years.

Or Using TI-83 Plus Calculator

1. Enter Two-Way Table in Matrix A.

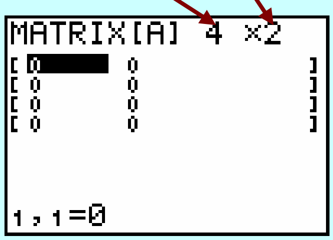
1. Press **2nd** **MATRX x^{-1}**

2. Use ▶ to access EDIT



3. Press **ENTER**

4. Enter the dimensions of the matrix.
(4 rows × 2 columns)



5. Enter the elements of the matrix (along each row).

6. Press **2nd** **QUIT MODE** **when finished.**

2. **Perform χ^2-Test**

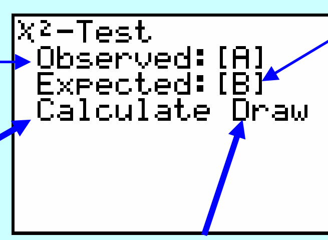
STAT ▶ ▶

Select TESTS

Select Option C

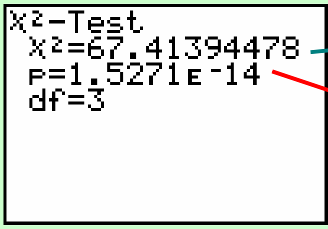
Specify Matrix that houses the Two-Way Table

Expected Counts Automatically Calculated



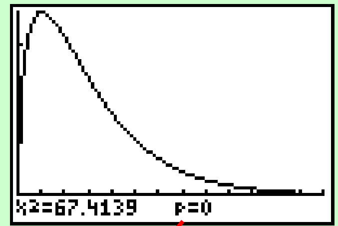
Press Enter on Calculate

Press Enter on Draw for χ^2 Curve



X^2 = 67.4139

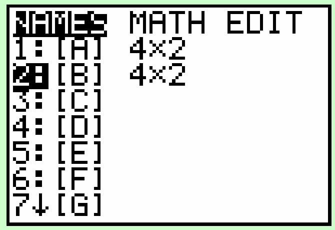
P-Value = $1.5271 \times 10^{-14} \approx 0$



4. Press ENTER again

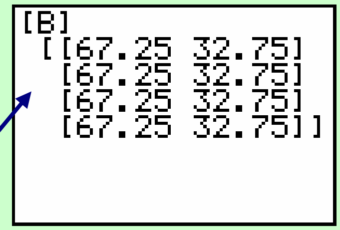
3. Double Check Expected Count Table

1. Press 2nd MATRX x^{-1}



2. Select Option 2 to recall Matrix [B].

3. Press ENTER



Same as calculated before

Since the *P* (1.5271×10^{-14}) << α (0.05), the proportions of people who never or applying for social assistance once in the past 10 years are <u>significantly different</u> based on their levels of education. We can see it most clearly with the group who has never finished high school as compared to the group who has finished college. The high school dropouts have a higher than expected proportion to apply for social assistance more than once within the last 10 years. This is in contrast to college grads that have a lower than expected proportion to apply for social assistance more than once within the last 10 years.

<u>Other uses of Chi-Square Test</u>: - instead of testing the H_0 being no difference between different proportions, there are other situations where we test for no relationship between two categorical variables.

<u>Situations where H_0: *No Relationship between Two Categorical Variables*</u>

1. Individuals that are from two independent SRSs using several populations, and are classified using one categorical variable with the other variable specifying which populations they came from.

Example: Two independent random surveys were conducted on the attitude of young people towards various political issues. Their results were compared to evaluate their reliability.

2. Individuals from a single SRS are classified into two categorical variables.

Example: A random survey was conducted on the attitude of young people towards various political issues based on their education levels. The results are tested to see if there is any relationship between the two variables.

3. Individuals form an entire population are classified into two categorical variables.

Example: All students in a particular a high school was asked to participate on a survey regarding their attitude towards various political issues based on their grade levels. The results are tested to see of there is any relationship between the two variables.

Example 2: All students in a particular a high school was asked to participate on a survey on how often they follow current events. The following results are categorized based on their grade level. Using the result collected, is there any relationship between grade level and how often these high school students follow current events. Evaluate with a significance level of 0.03.

	Number of Times Following Current Events in a Week				
Grade	**Never**	**Once or Twice**	**Three or Four Times**	**Five Times or More**	**Total**
9	123	114	87	30	354
10	91	107	99	45	342
11	74	98	109	56	337
12	58	85	119	99	361
Total	346	404	414	230	1394

1. State Hypothesis:

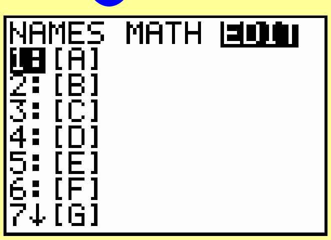
***H_0*: No Relationship between Grade Level and Number of Times Students Following Current Events.**
***H_a*: There is a Relationship between Grade Level and Number of Times Students Following Current Events.**

2. Using TI-83 Plus Calculator, calculate χ^2-Statistic and *P*-Value.

a. Enter Two-Way Table in Matrix A.

1. Press **2nd** **MATRX x^{-1}**

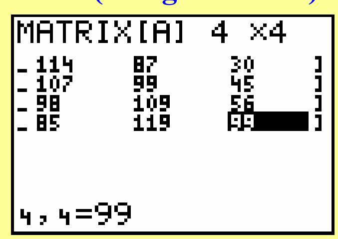
2. Use ▶ to access EDIT



3. Press **ENTER**

4. Enter the dimensions of the matrix.
(4 rows × 4 columns)

5. Enter the elements of the matrix (along each row).



6. Press **2nd** **QUIT MODE** **when finished.**

b. Perform the χ^2-Test

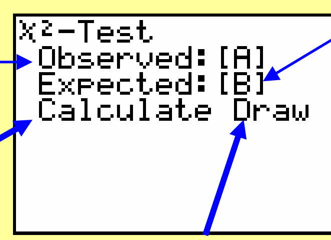
STAT ▶ ▶

Select TESTS

Select Option C

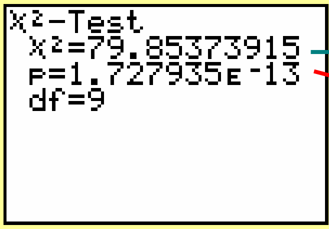
Specify Matrix that houses the Two-Way Table

Expected Counts Automatically Calculated



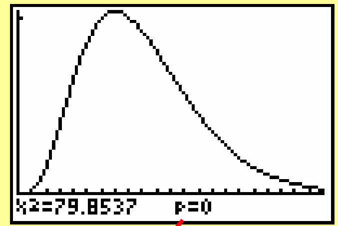
Press Enter on Calculate

Press Enter on Draw for χ^2 Curve



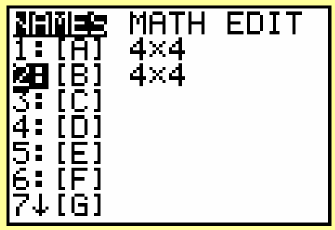
$X^2 = 79.8537$

P-Value $= 1.7279 \times 10^{-13} \approx 0$



3. Double Check Expected Count Table

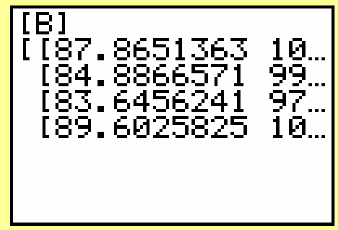
1. Press **2nd** **MATRX** x^{-1}



2. Select Option 2 to recall Matrix [B].

3. Press **ENTER**

4. Press **ENTER** again



Since the P $(1.7279 \times 10^{-14}) \ll \alpha$ (0.03), there is a relationship between grade level and the number of times a high school student follows current events in a week. As the grade level increases, there is a higher proportion of students following current events more frequently (three to four times and even more than five times a week). We CANNOT ascertain a "cause and effect" to this relationship, but we can hypothesized that the higher grade level a student become, the more current event knowledge one requires for social studies courses.

13.2 Assignment

pg. 722–723 #13.13 and 13.14; pg. 728 to 729 #13.15 to 13.17
pg. 735–736 #13.19 and 13.21; pg. 738 to 740 #13.23, 13.25 and 13.27

Chapter 13 Review

pg. 742–747 #13.29 to 13.31, 13.33 and 13.37

Chapter 14: Inference for Regression

14.1: Inference About a Model

Linear Regression Model: - ($\hat{y} = a + bx$) or ($\hat{y} = b_0 + b_1 x$) outlines parameters of **slope (b or b_1)** and **y-intercept (a or b_0)**.
- because linear regression came from statistical points, the slope and y-intercept calculated can be subjected to confidence interval as well as significance test.

Assumptions for Regression Inference

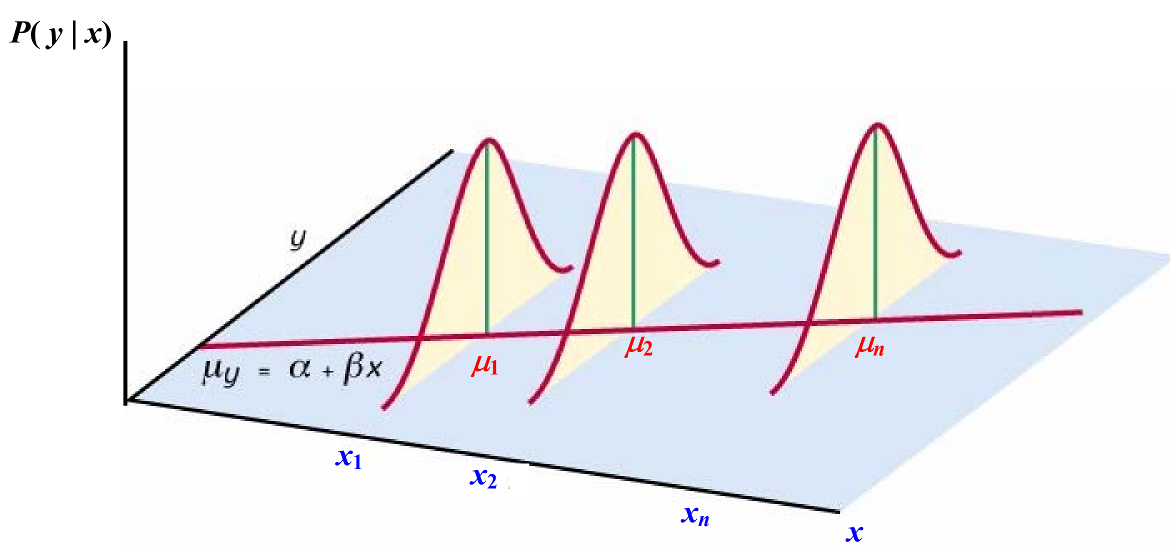
1. **Each value of the y-variable is independent from each other**. That is, one response taken is unrelated from another.
2. **Each value of the y-variable can be normally distributed and have the same unknown standard deviation**.
3. The mean response has a **linear relationship** with the explanatory variable.

True Regression Line: - the average linear relationship that measures the mean response (the mean of each y-value) against x.
- **the slope, b, becomes the unbiased estimator of the true slope (β).**
- **the constant (y-intercept), a, becomes the unbiased estimator of the true intercept, (α).**

True Regression Line

$$\mu_y = \alpha + \beta x$$

μ_y = mean y-value **α = true intercept** **β = true slope**



Standard Error About a Line: - the standard deviation of a line as **estimated** from the sum of all sample residuals of the response variable [**Residual = $y - \hat{y}$ (Observed – Predicted)**].

- the **degree of freedom** is **(n – 2)** due to the two dimensional nature of a linear relationship.

Standard Error and Variance About a Line

$$\text{Standard Error About a Line} = s = \sqrt{\frac{1}{n-2}\sum(\text{Residual})^2} = \sqrt{\frac{1}{n-2}\sum(y-\hat{y})^2}$$

$$\text{Variance About a Line} = s^2 = \frac{1}{n-2}\sum(\text{Residual})^2 = \frac{1}{n-2}\sum(y-\hat{y})^2$$

Example 1: Using the data below,

a. Determine the equation of linear regression model.
b. Calculate the variance and standard error about the line.

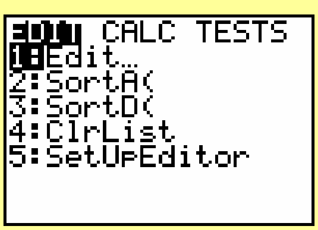
Comparison between Number of Absences and Final Score for an Algebra I Class in Suburbia Public High School

Student ID Number	Number of Absences	Final Marks (%)	Student ID Number	Number of Absences	Final Marks (%)
1	3	75	11	2	52
2	6	67	12	3	65
3	8	51	13	3	88
4	1	88	14	4	67
5	2	80	15	8	72
6	4	78	16	1	91
7	10	42	17	0	83
8	7	55	18	2	67
9	3	70	19	3	63
10	5	65	20	4	85

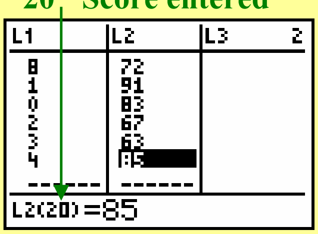
a. Determine the equation of linear regression model.

Entering Data using TI-83 Plus Calculator:

STAT ENTER Enter Values



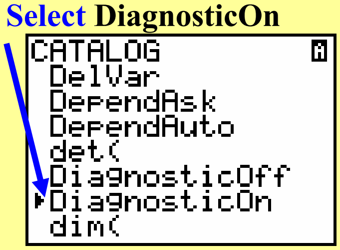
20th Score entered

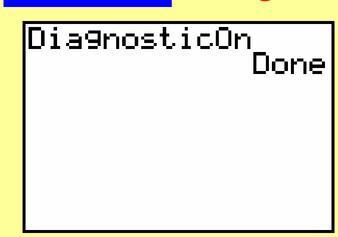


1. Turn Diagnostic On

2nd CATALOG 0 ▼ ENTER ENTER Again

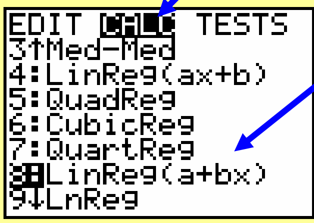
Select DiagnosticOn





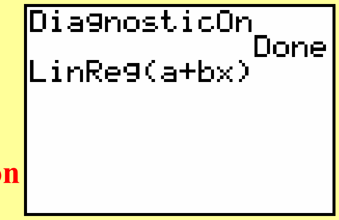
2. **Obtain LinReg ($a + bx$) and copy Equation to** **Y=** **Screen**

STAT Select **CALC**, use ► ▼

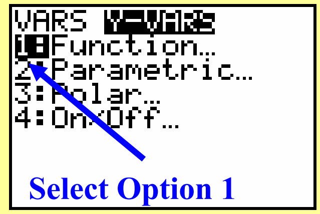


Choose Option 8 Linear Regression ($a + bx$)

ENTER



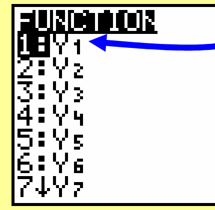
VARS ►



Select Option 1

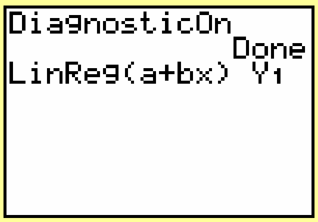
To access Y_1, press **ENTER**

Select Option 1 for Y_1

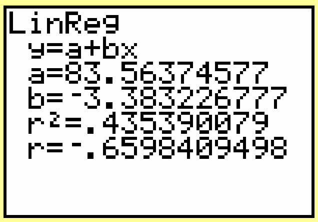


LinReg ($a + bx$) Y_1 calculates the equation and copied into the **Y=** **Screen.**

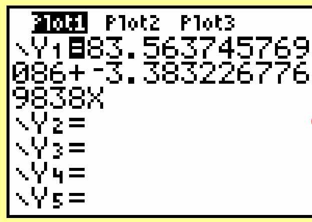
ENTER



ENTER



Y=



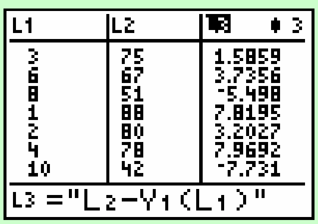
Least Square Line Equation

$\hat{y} = 83.56375 - 3.38323x$

$r = -0.6598$

b. Calculate the variance and standard error about the line

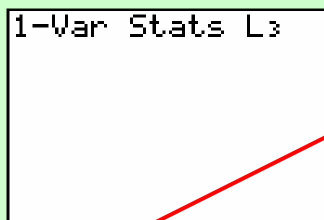
1. **Use L_3 = "(L_2 – Y_1(L_1))" for $(y - \hat{y})$**

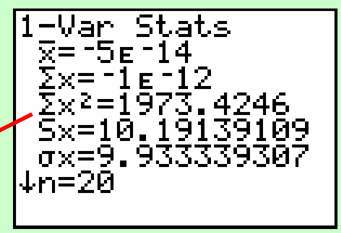


Y_1(L_1) means each y value for each of the individual x values in L_1

***Note:* LinReg ($a + bx$) Y_1 must be ran prior to entering formula for L_3.**

2. **For $\Sigma(y - \hat{y})^2$, run 1–Var Stats on L_3**





$\Sigma(y - \hat{y})^2 = 1973.4246$

Variance: $s^2 = \dfrac{1}{n-2}\sum(y-\hat{y})^2 = \dfrac{1}{(20-2)}(1973.4246)$ $\quad s^2 = 109.6347$

Standard Error: $s = \sqrt{\dfrac{1}{n-2}\sum(y-\hat{y})^2} = \sqrt{s^2} = \sqrt{109.6347}$ $\quad s = 10.4707$

<u>Confidence Interval of Regression Slope</u>: - this is a confidence interval based on t-distribution (two-variable statistics usually deal with small number of data sets collected).

<u>Confidence Interval of Regression Slope</u>

$$b \pm t^{*}SE_b$$

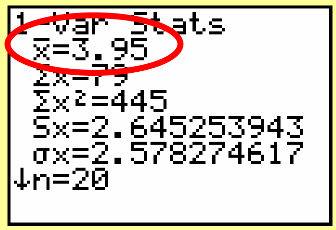
where SE_b = Standard Error of Slope $= \dfrac{s}{\sqrt{\sum(x-\bar{x})^2}}$

Example 2: Using the data from the previous example, determine the 95% confidence interval for the regression slope.

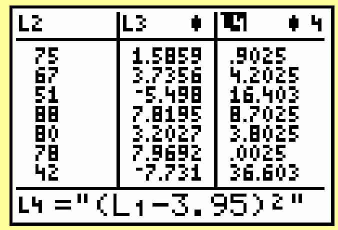
First, we have to calculate the standard error of the slope, SE_b.

$$SE_b = \frac{s}{\sqrt{\sum(x-\bar{x})^2}} \qquad s = 10.4707 \text{ and } b = -3.38323 \text{ (from last example)}$$

1. For $\bar{x}$, run 1–Var Stats on L_1



2. Use L_4 = "$(L_1 - 3.95)^2$" for $(x-\bar{x})^2$

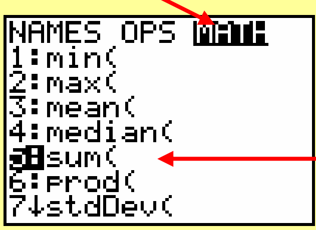


3. Sum L_4 for $\sum(x-\bar{x})^2$

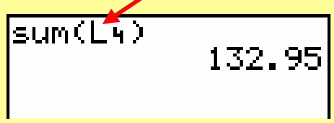
$$SE_b = \frac{s}{\sqrt{\sum(x-\bar{x})^2}} = \frac{10.4707}{\sqrt{132.95}}$$

SE_b = 0.908096

Select MATH on Menu



Select Option 5



	Upper tail probability, *p*
df	0.025
18	**2.145**
	95%
	Confidence level *C*

Finally, at $C = 0.95$ and $df = (n - 2) = 18$, $t^* = 2.145$

$b \pm t^* SE_b = -3.38323 \pm (2.145)(0.908096)$ **(-3.38323 ± 1.94786) % /day of absence**

We are confident that a student's final average will drop 1.43537% to 5.33109% per day of absence 19 times out of 20.

<u>Significance Test for Regression Slope</u>:

1. **To test whether the regression slope has a slope of 0 (no relationship as null hypothesis) or there is in fact a linear relationship (alternate hypothesis) when H_0: $\beta = 0$ and H_a: $\beta \neq 0$.**
2. **To test whether the regression slope has a slope of 0 (no relationship as null hypothesis) or there is in fact a POSITVE linear relationship (alternate hypothesis) when H_0: $\beta = 0$ and H_a: $\beta > 0$.**
3. **To test whether the regression slope has a slope of 0 (no relationship as null hypothesis) or there is in fact a NEGATIVE relationship (alternate hypothesis) when H_0: $\beta = 0$ and H_a: $\beta < 0$.**

***t*-Ratio**: - similar to *t*-statistic, it is the *t*-score (ratio of slope versus the standard error of slope) that we use to compare with the critical t^* of a pre-set significance level.

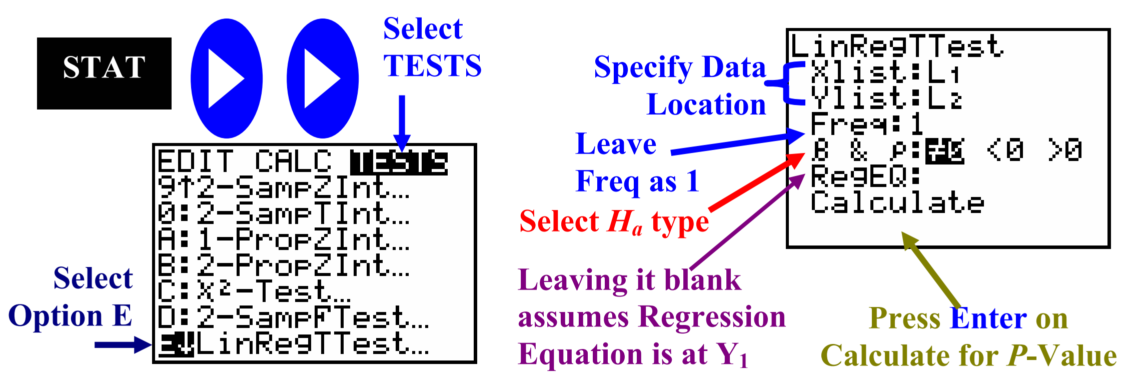
Four Steps of Linear Regression *t*-Test:

1. State the Hypothesis (H_0: $\beta = 0$ and H_a).
2. Calculate the *t*-ratio.
3. Determine the *P*-Value.
4. Compare with significance level and conclude the case.

***t*-ratio**

$$t = \frac{b}{SE_b}$$ with $(n - 2)$ degrees of freedom (*df*)

LinRegT-Test function on TI-83 Plus



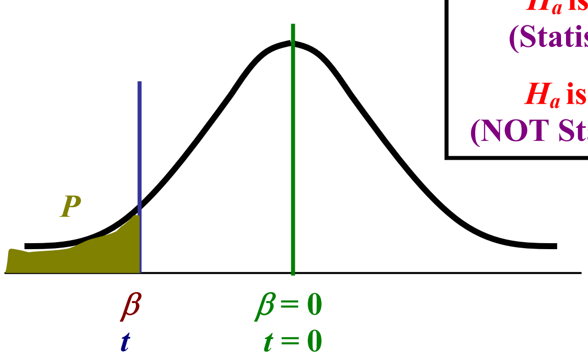
Fixed Significance Level Linear Regression *t*-Tests for Linear Relationship

Given a fixed significance level, α, the CONCLUSION can be drawn after *t*-test is performed.

1. If $P \le \alpha$, then H_a is True (Evidence strongly shown there is a Linear Relationship)
2. If $P > \alpha$, then H_0 is True (No Linear Relationship)

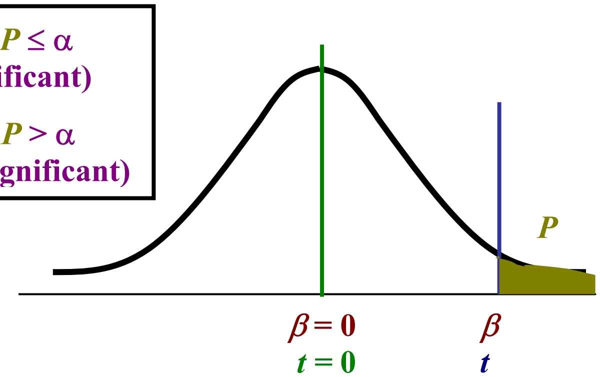
Two Variable Statistics One-Sided Linear Regression *t*-Tests

A Sample can be tested for Significance Level as α = tail probability. The CONCLUSION can then be drawn. (H_a: $\beta < 0$, or H_a: $\beta > 0$)



H_a is True when $P \le \alpha$
(Statistically Significant)

H_a is False when $P > \alpha$
(NOT Statistically Significant)

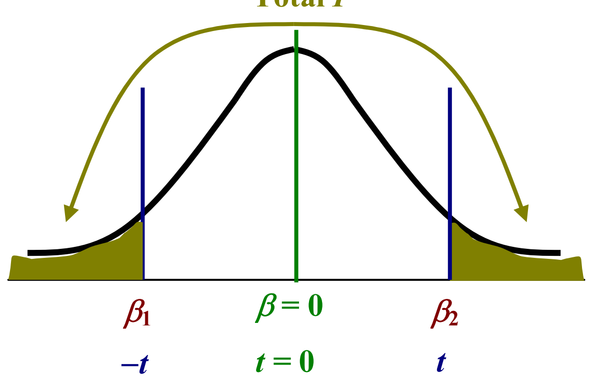


Two Variable Statistics Two-Sided Linear Regression *t*-Tests

A Sample can be tested for Significance Level as α = combined tail probabilities. The CONCLUSION can then be drawn. (H_a: $\beta \neq 0$)

H_a is True when Total $P \leq \alpha$
(Statistically Significant for a Linear Relationship)

H_a is False when Total $P > \alpha$
(NOT Statistically Significant for a Linear Relationship)



Example 3: Using the data from the Example 1 of this section and a significance level of 0.03, evaluate the evidence that there is a linear relationship between number of days absent and the final mark. Determine if there is indeed a negative relationship between the two variables to the same significance level.

$b = -3.38323$
$SE_b = 0.908096$
$n = 20$
$df = (n - 2) = 18$
$\alpha = 0.03$

1. Stating Hypothesis

H_0: $\beta = 0$
H_a: $\beta \neq 0$

2. Calculate ***t*-ratio**

$$t = \frac{b}{SE_b} = \frac{-3.38323}{0.908096}$$

$t = -3.7256$

3. Fine ***P*-Value from *t*-Distribution Critical Value Table using *df* = 18**

$P \approx 0.00075$

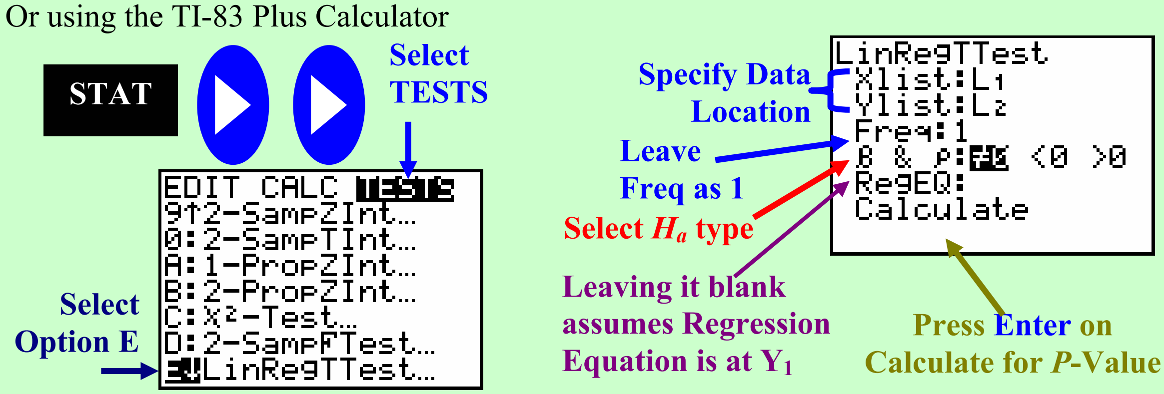
(*t* is in the middle of the two *p* values. Therefore, we can take *P* as 0.0015)

df	Upper tail probability, p	
	0.001	0.0005
18	3.611	3.922

$t =$ 3.7256 is between 3.611 and 3.922

Since P (0.00075) < α (0.03), H_0 is false and H_a is true. Hence, there is significantly enough evidence for a linear relationship between the number of absences and the final average.

Or using the TI-83 Plus Calculator

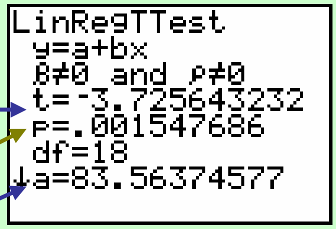


$$t = \frac{b}{SE_b} = \frac{-3.38323}{0.908096}$$

***t*-ratio = –3.7256**

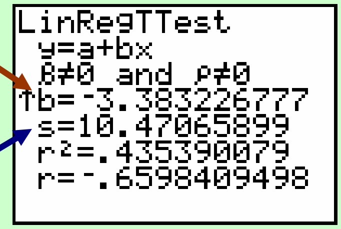
***P*-value = 0.001548**

true intercept α = 83.56



true slope
***β* = –3.38323**

standard error about a line
***s* = 10.470**



Since *p* (0.0015) < α (0.03), H_0 is false and H_a is true. Hence, there is a <u>significantly evidence for a linear relationship</u> between the number of absences and the final average.

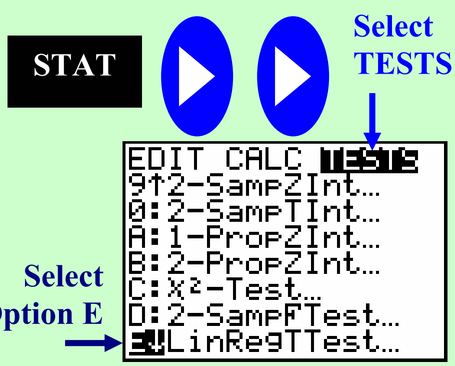
<u>Data from Computer Assisted Program Minitab on Example 3</u>

	Predictor	Coef (*a* and *b*)	Stdev (Standard Error)	t-ratio (*t*-ratio)	P (*P*-Value)
(*a* or α)	Constant	83.5637	0.0577	1448.24	0.0000
(*b* or *β*)	AbDays	−3.383227	0.908096	−3.73	0.0015
			(Standard Error of Slope)		(Only gives Two-Sided *P*-Value)

s = 10.4707 R-sq = 43.5% R-sq(adj) = 44.0%

(Standard Error About a Line)

Testing if there is a Negative Linear Relationship between the two variables:

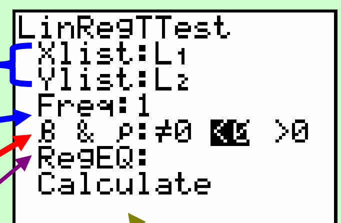


Specify Data Location

Leave Freq as 1

Select H_a type

Leaving it blank assumes Regression Equation is at Y_1

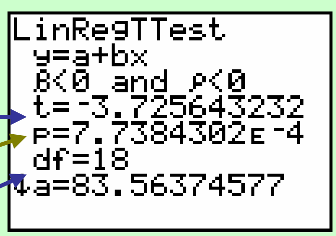


Press Enter on Calculate for *P*-Value

***t*-ratio = –3.7256**

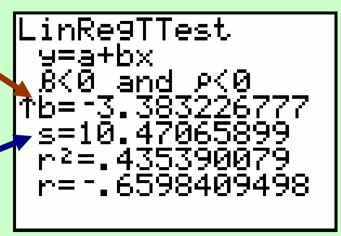
***P*-value = 7.7 × 10^{-4}**

true intercept α = 83.56



true slope
***β* = –3.38323**

standard error about a line
***s* = 10.470**



Since *p* (7.7 × 10^{-4}) << α (0.03), H_0 is false and H_a is true. Hence, there is a <u>significant evidence for a negative linear relationship</u> between the number of absences and the final average.

<u>14.1 Assignment</u>

pg. 760–761 #14.1 and 14.2; pg. 768–769 #14.5, 14.6, 14.8 and 14.9

14.2: Inference About Prediction

Prediction Interval: - a confidence interval of a prediction using the linear regression model with *a* and *b* as the estimators of the true constant and slope (α and β).
- when calculating prediction interval, we **use the symbol, x^***, instead x.

Prediction Interval
$\hat{y} \pm t^* \text{SE}$

Two kinds of Prediction Intervals

1. **Prediction Interval for the Mean Response**: - when the predicted response ($\hat{y}$) found is for **ALL Observations of x^***.
   - the prediction interval is **narrower than if the predicted response ($\hat{y}$) for a single observation of x^***.

2. **Prediction Interval for a Single Observation**: - when the predicted response ($\hat{y}$) found is for a **Single Observations of x^***.
   - the prediction interval is **wider than if the predicted response ($\hat{y}$) for a all observations of x^*.**

Prediction Interval for the Mean Response	Prediction Interval for a Single Observation
$\hat{y} \pm t^* \text{SE}_{\hat{\mu}}$	$\hat{y} \pm t^* \text{SE}_{\hat{y}}$
$\text{SE}_{\hat{\mu}} = s\sqrt{\dfrac{1}{n} + \dfrac{(x^* - \bar{x})^2}{\sum (x - \bar{x})^2}}$	$\text{SE}_{\hat{y}} = s\sqrt{1 + \dfrac{1}{n} + \dfrac{(x^* - \bar{x})^2}{\sum (x - \bar{x})^2}}$
Note: In both cases, look up t^* from the t-Distribution critical values table.	

Example 1: Using the *Number of Absences and Final Scores for an Algebra I Class in Suburbia Public High School* data from Section 14.1 and the linear regression equation, $\hat{y}$ = 83.56375 – 3.38323x, calculate the final average with a 95% prediction interval for the mean response and for a single observation when there are 4 absences.

First, for $x^* = 4$ absences, calculate $\hat{y}$ using the linear regression equation.

$\hat{y} = 83.56375 - 3.38323x^*$

$\hat{y} = 83.56375 - 3.38323(4)$

$\hat{y} = 70.03\%$

Next, we look at the t-distribution table for t^* at 95% confidence interval using $df = (n - 2) = (20 - 2) = 18$.

	Upper tail probability, *p*
df	0.025
18	2.145
	95%
	Confidence level *C*

For $SE_{\hat{\mu}}$ and $SE_{\hat{y}}$ we will need s, $\bar{x}$, and $\Sigma(x - \bar{x})^2$.

$s = 10.4707$ (Example 1, Section 14.1) $\bar{x} = 3.95$ and $\Sigma(x - \bar{x})^2 = 132.95$ (Example 2, Section 14.1)

To calculate **prediction interval for the mean response**, we need to determine $SE_{\hat{\mu}}$.

$$SE_{\hat{\mu}} = s\sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum(x - \bar{x})^2}} = (10.4707)\sqrt{\frac{1}{20} + \frac{(4 - 3.95)^2}{132.95}} \qquad \mathbf{SE_{\hat{\mu}} = 2.341759919}$$

$$\hat{y} \pm t^* SE_{\hat{\mu}} = 70.03\% \pm (2.145)(2.341759919) \qquad \mathbf{70.03\% \pm 5.02\%}$$

To calculate **prediction interval for a single observation**, we need to determine $SE_{\hat{y}}$.

$$SE_{\hat{y}} = s\sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum(x - \bar{x})^2}} = (10.4707)\sqrt{1 + \frac{1}{20} + \frac{(4 - 3.95)^2}{132.95}} \qquad \mathbf{SE_{\hat{y}} = 10.72937081}$$

$$\hat{y} \pm t^* SE_{\hat{y}} = 70.03\% \pm (2.145)(10.72937081) \qquad \mathbf{70.03\% \pm 23.01\%}$$

Data from Computer Assisted Program Minitab on Example 1

Fit	St dev. Fit	95% C.I.	95% P.I.
70.0308	2.3418	(65.0077, 75.0539)	(47.0163, 93.0453)
($\hat{y}$)	($SE_{\hat{\mu}}$)	**(Mean Response)**	**(Single Observation)**

(Only Calculate with 95% Confidence Interval)

14.3: Check the Regression Assumptions

14.2 Assignment
pg. 772–774 #14.10 to 14.12

1. **Linear Relationship Exists between the Two Variables.**
   a. **Look at scatterplot to see if a linear relationship is possible.**
   b. Obtain a **residual plot** and **check for any unusual pattern like a non-linear relationship** (curved pattern on the residual plot).

2. **Standard Deviation is the SAME for each value of the *y*-variable.**
   a. **Examine the scatterplot for uniform spread** of the data points about the linear regression line.
   b. The residual plot should have the same number points randomly above and below the base line (residual = 0). **Look for any outliers** (points that are way above or below of the residual plot's base line).

3. **The values of the *y*-variable are normally distributed about the Linear Regression Line.**
   a. **Check for skewness** by doing a stemplot of the residual values.
   b. **Throw out any influential points** (lone points that are on the far right of the residual plot).

14.3 Assignment
pg. 777–779 #14.13 and 14.14

Chapter 14 Review
pg. 781–784 #14.15, 14.17, 14.19