Algebra 2: Themes for the Big Final Exam

Final will cover the whole year, focusing on the big main ideas.

Graphing:

Overall: x and y intercepts, fct vs relation, fct vs inverse, x, y and origin symmetries, types of roots (single, double, complex) for polynomials

Transformations: $y=a(f(b(x+c)) + d$, what does each letter do to the graph

Specific types: linear, conics, polynomials, exponential, logarithmic, sin/cos, **other trig for honors**, **rational for honors**

Functions:

domain vs range, meaning of each in formula, notation, fct vs relation, fct vs inverse, interpreting meaning in word problems, notation with and without calculator

Solve:

Linear, quadratic, rational, radical, polynomial degree 3+, exponential, logarithmic, trig, linear ineq, poly ineq, absolute value equation, absolute value inequality, system of equations, system of inequalities

Rules/laws: radical, exponent, logarithmic, trig (SOH-CAH-TOA, unit circle, special angles)

Word Problems: making equation to match situation, using given equation, interpreting results

Key skills:

Simplify, graph, use formula, make formula, solve, calculator

Algebra 2
Emphasis in C1 to C3:

- Exponentials, Rational exponents
- Factoring: general, including $A^2 – B^2$ formula
- Quadratic equations: factoring and quad formula, including emphasis on substitution
- Complex numbers: add, subtract, multiply, divide
- Add, subtract, multiply, divide inequalities
- Graphing by plotting points
- Equation of a circle
- Symmetry along x, y origin axes
- Graphing by calculator
- Definition of function: domain, range, vertical line test
- Transformations of functions, reflection
- One to one functions and their inverses: including horizontal line test

Algebra 2: Year in Review

Solving:

The first step to solving an equation is to recognize the general structure of the equation. Is there anything in common that can be pulled out (reverse distribute)? Is it linear? Is it quadratic? A higher order polynomial? A key idea is to see an expression as the variable and not the just the *x*.

Examples: $e^{2x} - e^{x} - 6 = 0$ is quadratic in e^{x}. $4\sin x = 2$ is linear in $\sin x$. $4+3|x-6| = 22$ is linear in $|x-6|$.

Linear equations you solve by isolating the variable expression. Basic equations (see 1-1) also follow this approach. Quadratic equations you solve by factoring or using the quadratic equation (see 1-3 with factoring reviewed in P6 or the cookbook). Higher order polynomials (degree 3 or higher) rarely have a variable expression involved, just an *x*. We solve them by factoring by grouping (p. 46 or p. 340 Ex 1) or testing/ using rational roots with synthetic division (see 4-3 and 4-4).

There are also some special cases like fractions where you need to multiply through by the LCD to remove all the denominators (section 1-1) and square roots (section 1-5) where you square both sides. In both of these cases, you are multiplying both sides of the equation by a variable so you need to check your answers in the original to make sure they are not **extraneous solutions.** (p. 112-113) You never want to divide by a variable (this will lose solutions) but factor out what is common instead.

Once the variable expression is isolated, you can solve for *x* by either using the definition of the expression ($|x-2|=6$ means $x-2 = 6$ or -6 or $\sin x = \frac{1}{2}$ means $x = \frac{\pi}{3}$ or $\frac{2\pi}{3}$) or rewriting the expression using its inverse ($\log_2 x = 4$ becomes $2^4 = x$ or $\sqrt{x} = 6$ becomes $x = 6^2$).

We also solved two equations with 2 variables (you will not see 3 equations with 3 variables on the final). Your options for this are substitution, elimination or graphing.

If you can solve an equation, you can also solve the corresponding inequality. The answers to $f(x) = 0$ gives us the places where $f(x)$ potentially changes sign from positive to negative or vice versa. We can then check what type of region each piece (all points between two zeroes is a piece) is (positive or negative) by plugging in one point from this region. We can then keep the regions that we want. (ex 3 on p. 122) We saw this in section 1-6 with polynomials and also in 1-7 with absolute values. We can also use the fact that we know what the graph of many functions look like to figure out our solutions. For example, $(x-4)(x+5) > 0$ will have a solution outside the *x* intercepts because this is a parabola opening up. A common mistake is to treat an inequality polynomial problem exactly the same as a polynomial equation. Be able to use appropriate notation like interval notation, a number line or inequality format (section P-1, don't worry about union and intersection)

To graph an inequality with 2 variables (or a system of inequalities), you first graph the equality version and then shade on one side or the other of the graph. If it is a system, the final answer is the region that is shaded from all the graphs. You can figure out where to shade by testing points or thinking about the inequality.

Functions:

The idea of **functions** and working with functions was a major theme of this class. (chapter 3) A function is a "machine" that takes certain numbers (the domain) and turns them into another set of numbers (the range). If a machine has more than one output for any one input it is not a function but a **relation**. Functions can be given four ways (this is sometimes called the "rule of four"). A function can be given as an equation, a table of values, a graph or given through words. Working from an equation may be the most common but the other three show up at times as well. The notation (way of writing) of functions is important to understand. $f(2) = 8$ means that an input of 2 produces an output of 8 using the rule called *f*. More complicated expressions are used also at times. For example: $f(g(-2))=7$ or $(f+g)(x) = 4$. Understanding the connection between *f* and f^{-1} is important (section 3-7). The **inverse** (f^{-1}) of *f*, is just looking at the *f* rule in the opposite order. The input and outputs are switched. If $f(8) = -3$ then $f^{-1}(-3) = 8$. This switching of *x* and *y* (the input and output) causes the graphs to be reflection of each other over the line $y = x$. To find the formula for the inverse, you take the formula of *f*, switch the *x* and *y*, and then solve. Not all functions have an inverse. To have an inverse, a function must have each output coming from only one input. When looking at a graph of a function, we can check for it having an inverse by using the **horizontal line test**. In a similar way, we can check to see if the original is really a function by using the **vertical line test** on it.(p. 223). If you can understand that the horizontal line test is just the vertical line test with the domain and range of the original switched, you are doing great! (if not, you will survive)

The last function idea is the one of **transformations** (section 3-4). This idea can be applied to any type of function. The most general model is $y = a + bf(c(x-d))$ where *a*, *b*, *c* and *d* are constants (numbers) and *f* is the name of the function we are using ($f(x)$ could be a trig fct or exponential or log or …) The changes outside the *f* effect the *y* or vertical part of the graph. (*a* and *b*). The changes inside the function effect the *x* or the horizontal part of the graph. Numbers that multiply (*b* and *c*) stretch or compress. Numbers that add/subtract (*a* or *d*) shift. Finally, the numbers inside the function with the *x* (*c* and *d*) effect things opposite of what you think they should while numbers outside (*a* and *b*) effect things as you expect. For example $y = f(x-2)$ slides the *f* graph 2 to the right while $y = f(x) - 2$ slides the graph two down.

Graphing:

We graphed many different types of functions this year. Some general facts are finding the *x* and *y* intercepts, the general shape of each type and shifting the basic shape of the graphs using transformations. You may see problems asking to make the graph, give the equation of the graph, find out facts about the graph or give the equation/ graph matching certain information.

Linear- graph is a line, slope is an important characteristic (section 2-4), know standard, point-slope and slope-intercept forms

Quadratic – graph is a parabola, vertex and axis of symmetry are important, equation will look like $y = a(x-h)^2 + k$ or $y = ax^2 + bx + c$, be able to work with either one. Complete the square and $-b/2a$ are helpful tools (section 1-3)

Circles – find the center and radius and use them to graph (section 2-2)

Polynomial – degree 3 or higher, find roots and type (single/double), is leading term positive or negative, think about general shape (also called the end behavior) (section 4-1) and how complex roots effect things (section 4-4)

Exponential- general shape, passes through (0,1), D: $(-\infty,\infty)$ R: $(0,\infty)$ (section 5-1)

Logarithmic- general shape, passes through (1,0), D: $(0,\infty)$ R: $(-\infty,\infty)$ (section 5-2)

Sine/ Cosine – general shape, amplitude, average value, period, phase shift, connection between the "k" and the period (section 7-3)

Absolute Value – graph looks like a “v”, it mimics $y = x^2$, except it is straight and not curved

General things to know:

Rules of exponents/ radicals- See P3/ P4 or Night of Chapter 7 HW Worksheet (online)

Factoring- See P6 or factoring cookbook (online)

Complex numbers – key fact is “*i*” terms are only like terms with “*i*” terms, $i^2 = \sqrt{-1}$ so replace negatives under a square root with an i out front $i^{35} = (i^2)^{17} i = (-1)^{17} i = -i$, see 1-4 for more info

Log Facts –they work as exponentials in reverse (they are the inverse of exponentials) , see 5-3 and 5-4 for rules/properties, formula will be given in word problems, you will not have to make it

Trigonometry – understand radian locations on the unit circle (first quadrant will be given), know $x^2 + y^2 = 1$, know all 6 trig definitions in terms of *x* and *y* and each other (chapter 7)

Graphing calculator skills – use as tool to graph (input Y= correctly, set appropriate window), use to solve equations (using intercept or zero), use to solve inequalities (see where one is larger than the other on the graph), connect what you are seeing on the graph to what you are being asked for in a word problem

Word Problems – The two contexts in which you will have to make and solve a word problem completely (an equation is not given) are D=RT and mixture problems. A good guide to doing word problems is understand (read,read, read, write down key facts, make pictures), translate (choose a variable, define it and get everything possible in terms of it), equation (make and solve the equation), reflect (is the answer reasonable, units, did you answer the question)

Honors ideas:

Some honors problems will just be harder versions of the above concepts. However there were a few ideas that were exclusively honors.
Graphing rational functions- vertical asymptotes, horizontal asymptotes, general shape, zeroes
Graphing Tan/Cot/Sec/Csc – how to adapt sine/cosine to get these
Partial fractions- the idea from 10-8

No Ch 12 or Ch 13 on the test. Test will be two-thirds second semester (Ch 4, 5, 6/7, 8, 10) and one-third the first semester.

Algebra 2 Practice Final Exam (2005 – 2006 2nd Semester)

1. Graph the system, $\begin{cases} x - y \ge 0 \\ 3x + y \le 4 \\ y > -4 \end{cases}$, and find its vertices:

2. Solve $\begin{cases} 2x + 3y = 4 \\ 5x + 4y = 3 \end{cases}$

3. If $f(x) = x^2 + 2$ and $g(x) = 1 - x$, find:

a. $f(3)$ **b.** $g(4x - 1)$ **c.** $f(g(-2))$ **d.** $f^{-1}(x)$

4. Simplify $\left(\frac{16x^4y^7}{9y^4}\right)\left(\frac{81y^2}{4x^5}\right)$ using positive exponents only.

5. Simplify the expression $\frac{4x^2 - 144}{2x^2 - 24x + 72} \times \frac{5x - 30}{10x^2 + 80x + 120}$

6. Simplify the following expressions:

a. $(2\sqrt{3} + 3\sqrt{27})^2$ **b.** $\sqrt{3} - \frac{1}{2\sqrt{3}}$ **c.** $\frac{2 + 3i}{4 - 5i}$

7. Solve for x for the following equations.

a. $3x - 5\sqrt{x} = 2$ **b.** $4x^2 = 81$ **c.** $(x - 5)^2 - 25 = 11$

8. Find all solutions to the equation, $x^4 - 3x^3 - 7x^2 + 27x - 18 = 0$.

9. Find the equation of the circle whose center is (4, 4) and contains the origin on its path.

10. Solve the system of equations, $\begin{cases} 16x^2 + 4y^2 = 16 \\ 4x - y = -4 \end{cases}$

11. For the points (−7, 5) and (−9, −21), find the midpoint and distance between the two.

12. Solve for x. Give exact and approximate answers (if applicable):

a. $\log x = 0.8531$ **b.** $25^{(x+2)} = 5^{(3x-3)}$ **c.** $10^{5x} = 2$

13. Over the past twenty years, the city of San Jose has grown by an average of 1.4% per year, compounded yearly.

a. If the population in 1995 was 980,000, what is the estimated population in 2010?
b. Estimate the year the population was 500,000.

14. Coach Bookin can line the soccer field by himself faster than Coach D can. In fact, he can complete the job by himself ten minutes faster than Coach D. If they work together, they can finish the job in 20 minutes. If Coach Bookin breaks something and can't help, how long would it take Coach D to do the job himself?

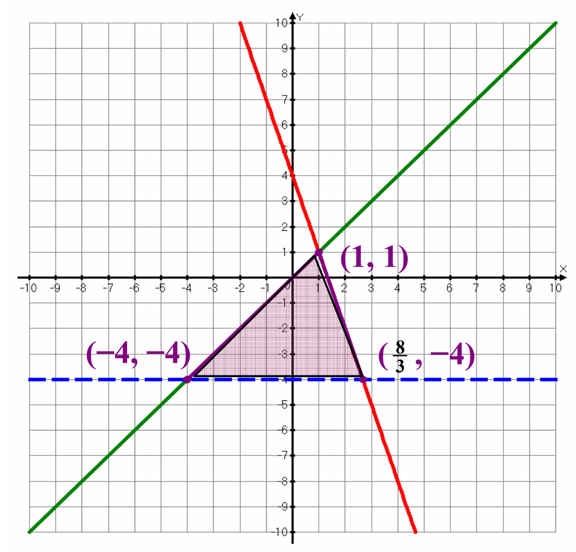
15. A plane whose air speed is 200 mph flew from San Francisco to San Diego in 2 hours with a tail wind. On the return trip, against the same wind, the plane was still 50 miles away from San Francisco after 2 hours of flying. Find the wind speed and the distance from San Francisco to San Diego.

16. Sketch $y = -3\sin(2x)$ in the range $-\pi < x < \pi$.

17. Sketch $y = -5\cos[2(x - \pi)] - 4$ over two periods.

<u>Answers to Algebra 2 Practice Final Exam (2005 – 2006 2nd Semester)</u>:

1.



2. $(-1, 2)$

3. a. 11 **b.** $-4x + 2$ **c.** 11 **d.** $\sqrt{x-2}$

4. $\dfrac{36y^5}{x}$ **5.** $\dfrac{1}{(x+2)}$

6. a. 363 **b.** $\dfrac{5\sqrt{3}}{6}$ **c.** $\dfrac{-7+22i}{41}$

7. a. 4 (only answer that verifies)

b. $\pm\dfrac{9}{2}$ **c.** 11 and -1

8. Using synthetic division: $x = 1, 2, 3$ and -3

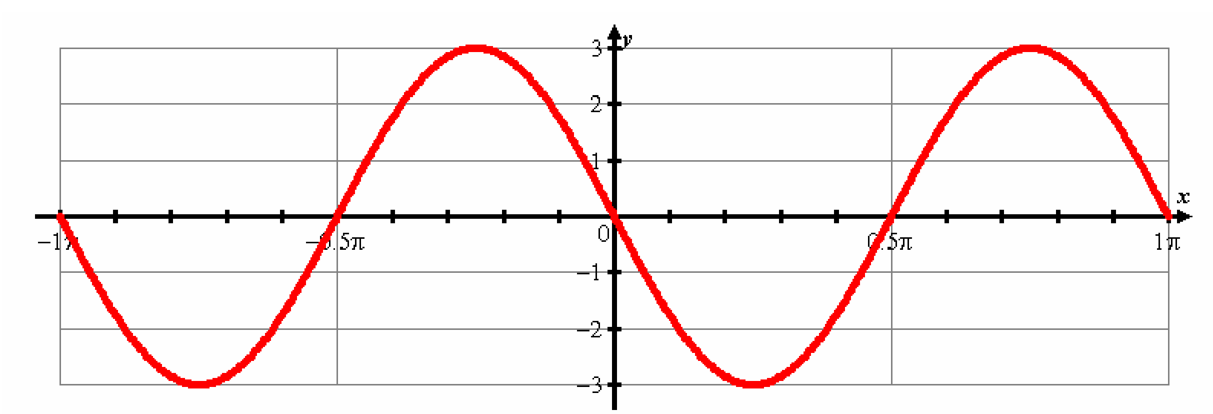
9. $(x-4)^2 + (y-4)^2 = 32$ **10.** $(-1, 0)$ and $\left(-\dfrac{3}{5}, \dfrac{8}{5}\right)$ **11.** $M = (-8, -8)$; $d = 2\sqrt{170}$

12. a. 7.13 **b.** 7 **c.** 0.0602 **13. a.** 1,207,244 **b.** 1947

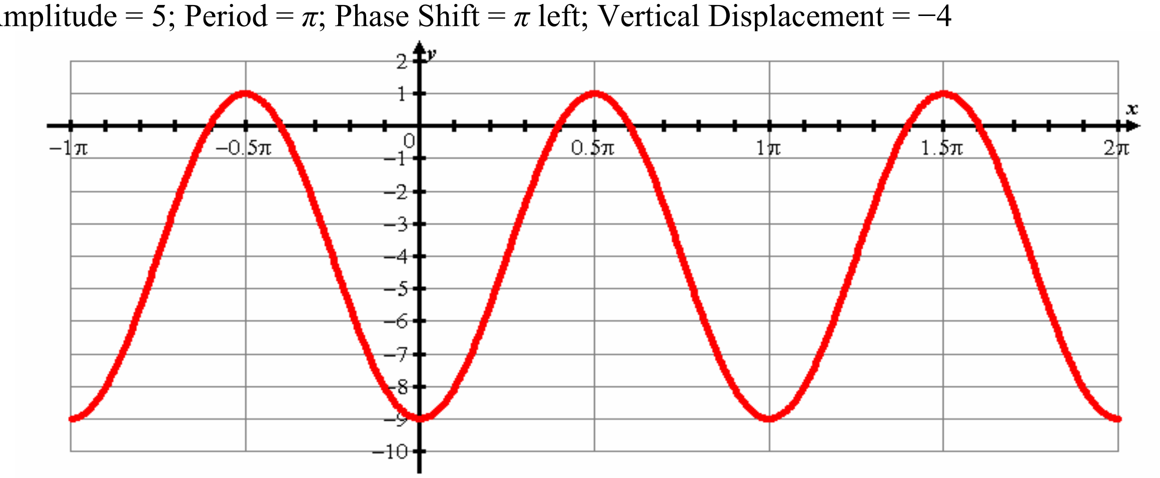
14. 45.62 min **15.** wind speed =12.5 miles/hr; $d = 400$ miles

(1, 1)

16. Amplitude = 3; Period = π; No Phase Shift; No Vertical Displacement



17. Amplitude = 5; Period = π; Phase Shift = π left; Vertical Displacement = -4



1. Simplify the expression.

$$\left(\frac{a^2b^{5/3}}{a^{1/3}b^{2/3}}\right)^6$$

2. Find all real solutions of the equation.

$$\frac{x+2}{x-2}=\frac{3x}{3x-6}$$

3. Two points P and Q are given.

$$P(0,-8),\quad Q(-11,-8)$$

(a) Find the distance from P to Q.
(b) Find the midpoint of the line segment PQ.

4. Find an equation for the line that passes through the point $(5,1)$ and is perpendicular to the line $x-3y+16=0$.

5. In a certain city, the property tax collected for a home is varies directly to the valuation of the property. The tax collected on a \$105,000 home is \$2,846 per year. What is the value of a home if the tax collected is \$1,735 ?

6. Find the domain of the function.

$$f(x)=0.5x-\frac{2}{\sqrt{x+1}}$$

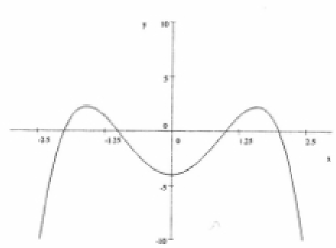
7. Find the maximum or minimum value of the function $f(x)=x^2-4x+2$.

8. Evaluate $f(-2)$, $f(-1)$, $f(0)$, $f(1)$, and $f(5)$ for the piecewise-defined function.

$$f(x)=\begin{cases}3x^2 & \text{if } x<0\\ 2x+1 & \text{if } x\ge 0\end{cases}$$

9. Use $f(x)=3x-2$ and $g(x)=3+2x^2$ to evaluate the expression $(f\circ g)(2)$.

10. Determine whether the function in the figure is even, odd, or neither.



11. Find the inverse of the function.

$f(x)=\sqrt{25-x^2}$, $0 \le x \le 5$

find its real zeroes

12. Use a graphing device to graph the polynomial $y=2x^5+3x^2-2x+1$ and ~~describe its end behavior~~.

13. Find the quotient and remainder.

$$\frac{x^3+3x-2}{x-1}$$

14. Use the Factor Theorem to show that $(x-4)$ is a factor of the polynomial.

$P(x)=x^5+x^4-36x^3-16x^2+320x$

15. Find all rational, irrational, and complex zeros (and state their multiplicities).

$P(x)=x^4+10x^2-2x^3-18x+9$

16. Graph the rational function. Show clearly all x- and y-intercepts and asymptotes.

$y=\dfrac{2x+4}{x-2}$ HONORS ONLY

17. Sketch the graph of the function. State the domain, range, and asymptote.

$f(x)=2+\log_2(x+4)$

18. Expand the logarithmic expression.

$$\log\left(\frac{4x^3}{y(x-2)^6}\right)$$

19. Find the solution of the equation $3^{x+2}=5^{2x}$, correct to four decimal places.

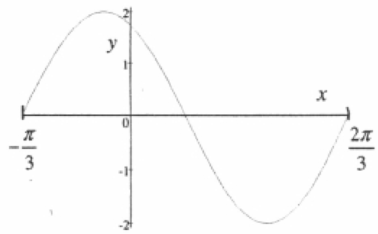
20. A culture contains 680 bacteria. After just 30 minutes the count is 3250. Find the number of bacteria after 3.5 hours. After how many minutes will the number of bacteria triple?

21. Suppose that the terminal point determined by t is the point $\left(\frac{5}{13},\frac{12}{13}\right)$ on the unit circle. Find the terminal point determined by $\pi+t$.

22. If the terminal point determined by t is $\left(\frac{12}{13},-\frac{5}{13}\right)$, find $\sin t$, $\cos t$, and $\tan t$.

23. If $\cos t=\frac{3}{5}$ and the terminal point for t is in quadrant IV, find $\tan t+\sec t$.

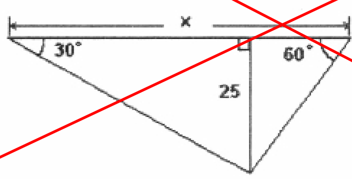
24. The graph shown is one period of a function of the form $y = a\sin k(x-b)$. Determine the function.



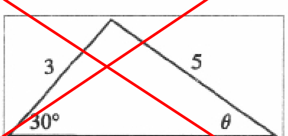
25. Find the period of the function $y = \frac{1}{2}\tan\left(x - \frac{\pi}{3}\right)$ and sketch its graph.

26. Find the measure of the central angle θ in a circle of radius 6 ft if the angle is subtended by an arc of length 7 ft.

27. Solve for x correct to one decimal place.

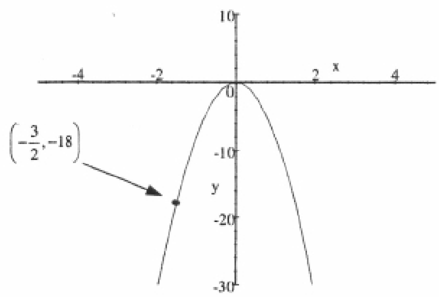


28. Find the indicated angle θ.



41. Find the partial fraction decomposition of $\dfrac{x+15}{x^2+3x}$. HONORS

42. Find an equation for the conic whose graph is shown.



43. Sketch the graph of the ellipse.

$$\frac{(x-3)^2}{9}+\frac{y^2}{36}=1$$

Cumulative Review Exercises

1. Subtract: $\dfrac{4x^2}{x^2+x-2} - \dfrac{3x-2}{x+2}$

2. Factor: $2x^6 + 16$

3. Multiply: $\sqrt{2y}\left(\sqrt{8xy} - \sqrt{y}\right)$

4. Simplify: $\left(\dfrac{x^{-\frac{3}{4}} \cdot x^{\frac{3}{2}}}{x^{-\frac{5}{2}}}\right)^{-8}$

5. Solve: $5 - \sqrt{x} = \sqrt{x+5}$

6. Solve: $2x^2 - x + 7 = 0$

7. Solve by the addition method: $3x - 3y = 2$
$6x - 4y = 5$

8. Solve: $2x - 1 > 3$ or $1 - 3x > 7$

9. Evaluate the determinant: $\begin{vmatrix} -3 & 1 \\ 4 & 2 \end{vmatrix}$ **−10**

10. Write $\log_5 \sqrt{\dfrac{x}{y}}$ in expanded form.

11. Solve for x: $4^x = 8^{x-1}$

12. Write the 5th and 6th terms of the sequence whose nth term is given by the formula $a_n = n(n-1)$. $a_5 = 20, a_6 = 30$

13. Evaluate the series $\sum_{n=1}^{7} (-1)^{n-1}(n+2)$. **6**

14. Solve by the addition method:
$x + 2y + z = 3$ **(2, 0, 1)**
$2x - y + 2z = 6$
$3x + y - z = 5$

15. Solve for x: $\log_6 x = 3$

16. Divide: $(4x^3 - 3x + 5) \div (2x + 1)$

17. For $g(x) = -3x + 4$, find $g(1 + h)$.

18. Find the range of $f(a) = \dfrac{a^3 - 1}{2a + 1}$ if the domain is $\{0, 1, 2\}$.

19. Graph: $3x - 2y = -4$

20. Graph the solution set of $2x - 3y < 9$.

y

21. A new computer can complete a payroll in 16 min less time than it takes an older computer to complete the same payroll. Working together, both computers can complete the payroll in 15 min. How long would it take each computer, working alone, to complete the payroll?

22. A boat traveling with the current went 15 mi in 2 h. Against the current, it took 3 h to travel the same distance. Find the rate of the boat in calm water and the rate of the current.

23. An 80-milligram sample of a radioactive material decays to 55 mg in 30 days. Use the exponential decay equation $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{k}}$, where A is the amount of radioactive material present after time t, k is the half-life, and A_0 is the original amount of radioactive material, to find the half-life of the 80-milligram sample. Round to the nearest whole number.

24. A "theater in the round" has 62 seats in the first row, 74 seats in the second row, 86 seats in the third row, and so on in an arithmetic sequence. Find the total number of seats in the theater if there are 12 rows of seats.

1536 seats

25. To test the "bounce" of a ball, the ball is dropped from a height of 10 ft. The ball bounces to 80% of its previous height with each bounce. How high does the ball bounce on the fifth bounce? Round to the nearest tenth.

Final Exam

1. Simplify: $12 - 8[3 - (-2)]^2 \div 5 - 3$

2. Evaluate $\dfrac{a^2 - b^2}{a - b}$ when $a = 3$ and $b = -4$.

3. Simplify: $5 - 2[3x - 7(2 - x) - 5x]$

4. Solve: $\dfrac{3}{4}x - 2 = 4$

5. Solve: $\dfrac{2 - 4x}{3} - \dfrac{x - 6}{12} = \dfrac{5x - 2}{6}$

6. Solve: $8 - |5 - 3x| = 1$

7. Solve: $|2x + 5| < 3$

8. Solve: $2 - 3x < 6$ and $2x + 1 > 4$

9. Find the equation of the line that contains the point $(-2, 1)$ and is perpendicular to the line $3x - 2y = 6$.

10. Simplify: $2a[5 - a(2 - 3a) - 2a] + 3a^2$

11. Simplify: $\dfrac{3}{2 + i}$

12. Write a quadratic equation that has integer coefficients and has solutions $-\frac{1}{2}$ and 2.

13. Factor: $8 - x^3y^3$

14. Factor: $x - y - x^3 + x^2y$

15. Divide: $(2x^3 - 7x^2 + 4) \div (2x - 3)$

16. Divide: $\dfrac{x^2 - 3x}{2x^2 - 3x - 5} \div \dfrac{4x - 12}{4x^2 - 4}$

17. Subtract: $\dfrac{x - 2}{x + 2} - \dfrac{x + 3}{x - 3}$

18. Simplify: $\dfrac{\frac{3}{x} + \frac{1}{x + 4}}{\frac{1}{x} + \frac{3}{x + 4}}$

19. Solve: $\dfrac{5}{x - 2} - \dfrac{5}{x^2 - 4} = \dfrac{1}{x + 2}$

20. Solve $a_n = a_1 + (n - 1)d$ for d. $d = \dfrac{a_n - a_1}{n - 1}$

21. Simplify: $\left(\dfrac{4x^2y^{-1}}{3x^{-1}y}\right)^{-2}\left(\dfrac{2x^{-1}y^2}{9x^{-2}y^2}\right)^3$

22. Simplify: $\left(\dfrac{3x^{\frac{2}{3}}y^{\frac{1}{2}}}{6x^2y^{\frac{4}{3}}}\right)^6$

23. Subtract: $x\sqrt{18x^2y^3} - y\sqrt{50x^4y}$

24. Simplify: $\dfrac{\sqrt{16x^5y^4}}{\sqrt{32xy^7}}$

25. Solve by using the quadratic formula: $2x^2 - 3x - 1 = 0$

26. Solve: $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 6 = 0$

27. Find the equation of the line containing the points $(3, -2)$ and $(1, 4)$.

28. Solve: $\frac{2}{x} - \frac{2}{2x + 3} = 1$

29. Solve by the addition method: $3x - 2y = 1$
$5x - 3y = 3$

30. Evaluate the determinant: $\begin{vmatrix} 3 & 4 \\ -1 & 2 \end{vmatrix}$ **10**

31. Solve for x: $\log_3 x - \log_3(x - 3) = \log_3 2$

32. Write $\sum_{i=1}^{5} 2y^i$ in expanded form.
$\mathbf{2y + 2y^2 + 2y^3 + 2y^4 + 2y^5}$

33. Find an equivalent fraction for $0.5\overline{1}$. $\mathbf{\frac{23}{45}}$

34. Find the 3rd term in the expansion of $(x - 2y)^9$.
$\mathbf{144x^7y^2}$

35. Solve: $x^2 - y^2 = 4$
$x + y = 1$

36. Find the inverse function of $f(x) = \frac{2}{3}x - 4$.

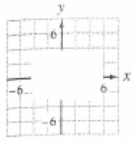
37. Write $2(\log_2 a - \log_2 b)$ as a single logarithm with a coefficient of 1.

38. Graph $2x - 3y = 9$ by using the x- and y-intercepts.

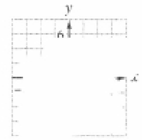
39. Graph the solution set of $3x + 2y > 6$.

40. Graph: $f(x) = -x^2 + 4$

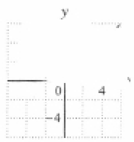
41. Graph: $\dfrac{x^2}{16} + \dfrac{y^2}{4} = 1$



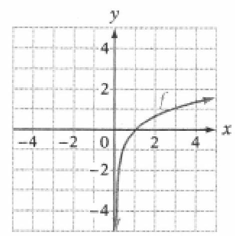
42. Graph: $f(x) = \log_2(x + 1)$

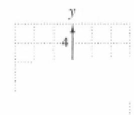


43. Graph $f(x) = x + 2^{-x}$ and approximate to the nearest tenth the values of x for which $f(x) = 2$.



44. Given the graph of $f(x) = \ln x$ shown below, graph $g(x) = \ln(x + 3)$.



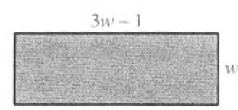


45. An average score of 70–79 in a history class receives a C grade. A student has grades of 64, 58, 82, and 77 on four history tests. Find the range of scores on the fifth test that will give the student a C grade for the course. **69 or better**

46. A jogger and a cyclist set out at 8 A.M. from the same point headed in the same direction. The average speed of the cyclist is two and a half times the average speed of the jogger. In 2 h, the cyclist is 24 mi ahead of the jogger. How far did the cyclist ride? **40 mi**

47. You have a total of $12,000 invested in two simple interest accounts. On one account, a money market fund, the annual simple interest rate is 8.5%. On the other account, a tax-free bond fund, the annual simple interest rate is 6.4%. The total annual interest earned by the two accounts is $936. How much do you have invested in each account?

48. The length of a rectangle is 1 ft less than three times the width. The area of the rectangle is 140 ft^2. Find the length and width of the rectangle.



49. Three hundred shares of a utility stock earn a yearly dividend of $486. How many additional shares of the utility stock would give a total dividend income of $810? **200 additional shares**

50. An account executive traveled 45 mi by car and then an additional 1050 mi by plane. The rate of the plane was seven times the rate of the other car. The total time of the trip was $3\frac{1}{4}$ h. Find the rate of the plane.

51. An object is dropped from the top of a building. Find the distance the object has fallen when the speed reaches 75 ft/s. Use the equation $v = \sqrt{64d}$, where v is the speed of the object and d is the distance. Round to the nearest whole number. **88 ft**

Answers to Final Exam Form A

1. $a^{10}b^6$ 2. no solution 3. a) 11 b) (-11/2, -8) 4. y = 16-3x 5. $64,010
6. $(-1,\infty)$ 7. -2 8. 12, 3, 1, 3, 11 9. 31 10. even
11. $g(x)=\sqrt{x^2-25}, 0\le x\le 5$ 12. 13. $x^2+x+4, R2$ 14. yes, p(4)=0 15. 1, 1, $\pm 3i$
16. 17. 18. 2log2+3logx-logy-6log(x-2) 19. 1.0363 20. 21 minutes
21. (-5/13, -12/13) 22. -5/13, 12/13, -5/12 23. 1/3 24. 2sin2(x+π/3)
25. 26. 7/6 rad 27. $25\sqrt{3}+25/3\sqrt{3}$ 28. 17.5 34. $\theta=\tan^{-1}(2/x)$
41. 5/x – 4/(x+3) 42. $y=-8x^2$ 43.

Answers to Cumulative Review Exercises

1. $\dfrac{x^2+5x-2}{(x+2)(x-1)}$ 2. $2(x^2+2)(x^4-2x^2+4)$ 3. $4y\sqrt{x}-y\sqrt{2}$ 4. $\dfrac{1}{x^{26}}$

5. 4 6. $\dfrac{1\pm i\sqrt{55}}{4}$ 7. (7/6,1/2) 8. x<-2 or x>2 10. $\dfrac{1}{2}(\log_5 x-\log_5 y)$
11. 3 15. 216 16. $2x^2-x-1\ R6$ 17. -3h+1 18. -1, 0, 7/5
19. 20. 21. 24 min new, 40 min old 22. boat 6.25 mph, current 1.25 mph
23. 55 days 25. 3.3 ft

Answers to Final exam p. 723

1. -31 2. -1 3. 33-10x 4. 8 5. 2/3 6. 4, -2/3 7. -4<x<-1 8. x>3/2
9. y=-2/3x-1/3 10. $6a^3-5a^2+10a$ 11. 6/5-3/5i 12. $2x^2-3x-2=0$
13. $(2-xy)(4+2xy+x^2y^2)$ 14. (x-y)(1+x)(1-x) 15. $x^2-3x-3\ R-5$
16. $\dfrac{x(x-1)}{2x-5}$ 17. $-\dfrac{10x}{(x+2)(x-3)}$ 18. $\dfrac{x+3}{x+1}$ 19. -7/4 21. $\dfrac{y^4}{162x^3}$
22. $\dfrac{1}{64x^8y^5}$ 23. $-2x^2y\sqrt{2y}$ 24. $\dfrac{x^2\sqrt{2y}}{2y^2}$ 25. $\dfrac{3\pm\sqrt{17}}{4}$ 26. 27,-8 27. y=-3x+7
28. 3/2, -2 29. (3,4) 31. 6 35. (5/2, -3/2) 36. $f^{-1}(x)=3/2x+6$
37. $\log_2\dfrac{a^2}{b^2}$ 38. (9/2, 0), (0,-3) 43. -2, 1.7 47. $8000, $4000 48. 20 by 7
50. 420mph

2006-2007 Algebra II Second Semester Final **Name: ______________________**

Basic Calculator only on this part

1. Make the equation of the line (in any form) that passes through (−2, 6) and is parallel to $y = 4x - 5$ (2 pts)

2. Use the line, $8x - 3y = 9$, to:
a) Give the x-intercept. (1 pt) **b)** Give another point on the line. (1 pt)

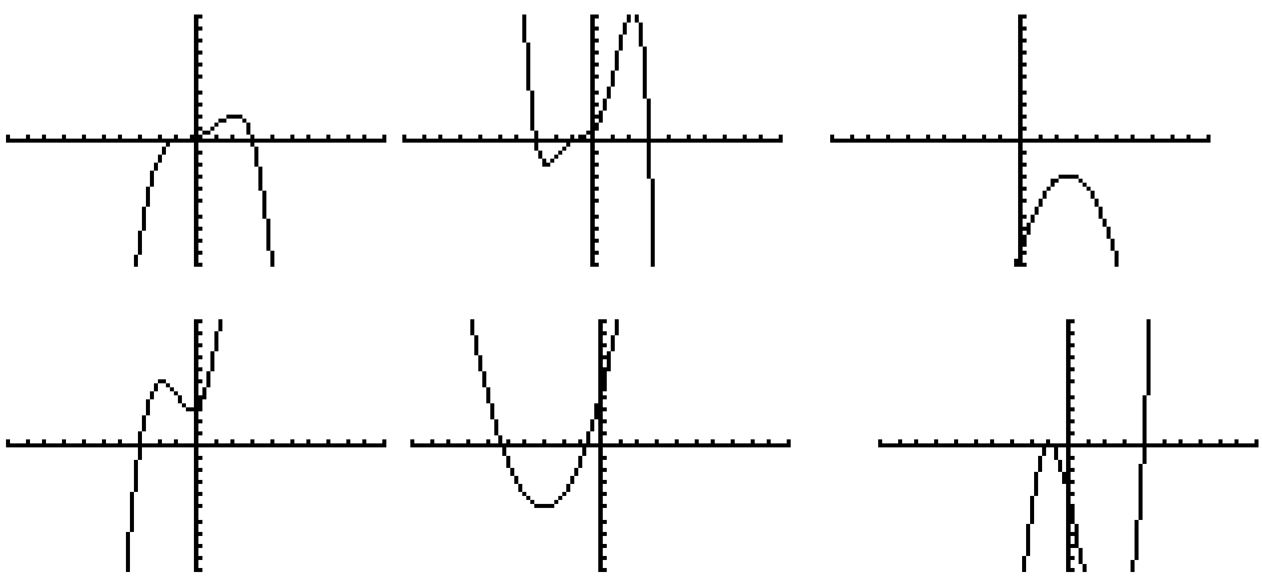
3. Find the following values exactly. (2 pts each)

a) $\cos\left(\dfrac{\pi}{3}\right)$ **b)** $\sin\left(\dfrac{-3\pi}{4}\right)$ **c)** $\tan \pi$ **d)** $\cot^{-1} 1$

4. Use the quadratic formula to solve this equation: $4x^2 = 16x - 19$ (3 pts)

5. Write the letter of the correct description below its graph. (1 pt each)

A. quadratic with irrational roots
B. quadratic with complex roots
C. cubic with a double root
D. quintic (5th power) with negative leading coefficient
E. quartic (4th power) with negative leading coefficient
F. Cubic with a root of i



6. For the equation $2x^3 - 11x^2 + 2x + 15 = 0$
a) List all of the possible rational solutions (1 pt)
b) Show that $x = 3$ is not a solution (1 pt)
c) Solve for all the solutions, using the fact that $x = 5$ is a solution (4 pts)

7. Factor completely. Use only integer coefficients and show all work. (3 pts each)
a) $x^3 - 3x^2 + 5x - 15$ **b)** $4y^4 - 9y^2$

8. Fill in the table of values such that the following criteria is true (1 pt each)

a) It has y-axis symmetry **b)** It is a function with no inverse **c)** It is not a function

x	-2	-1	0	1	2
y					

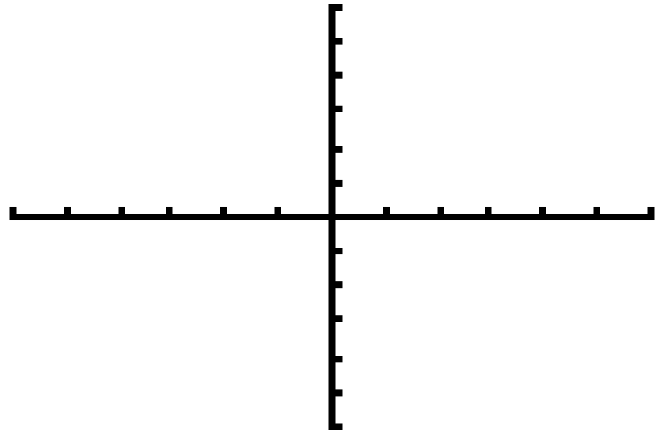
x	-2	-1	0	1	2
y					

x					
y					

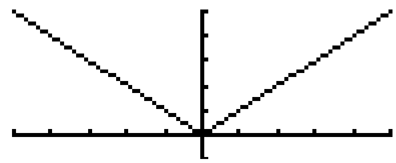
d) Extra Credit: Write an absolute value inequality that has $-12 \le x \le 4$ as a solution (1 pt)

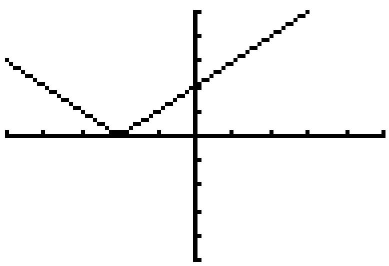
9. Graph the solution to this system of inequalities. Make your graphs neat and clear. (4 pts)

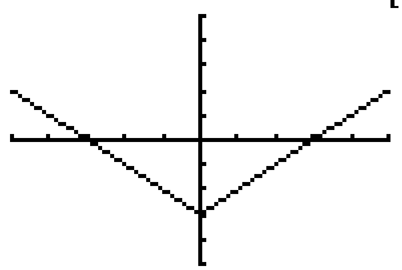
$$y - 2x > 1$$
$$x^2 + y^2 \le 16$$

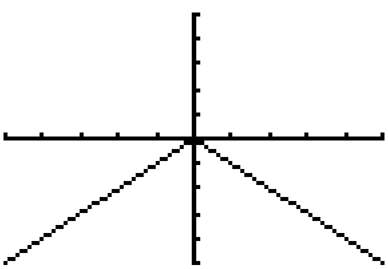


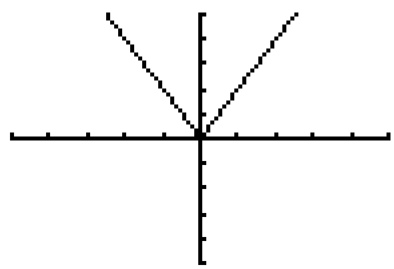
10. Given is the graph on the right of $f(x) = |x|$.
Use it to give an equation for the following:
Each tick mark is one unit. (1.5 pts each)











11. Solve these equations: (3 pts each)

a) $5^{3x-3} = 25^{x+2}$

b) $\dfrac{3}{x} - \dfrac{2}{5} = 4$

c) $\sqrt{x+7} - x = 5$

d) $\log_2 x - \log_2 3 = 2$

12. For the equation, give its period, average value (middle), amplitude and phase shift. Then graph it on an axis system that allows the entire graph to be seen for one period. Please label the coordinates of the first and last point of your graph. (5 pts)

$$y = -3 + 2\sin\left(2\left(x + \frac{\pi}{3}\right)\right)$$

13. An open house is happening in 18 hours and the house needs to be painted by then. Painting Company #1 can paint the house in 30 hours working alone. Painting Company #2 can do it alone in 24 hours. Company # 1 and Company # 2 start doing the job together. After both have worked for 5 hours, Company # 1 goes on strike and Company # 2 must finish alone. Will Company # 2 finish in time? Justify your answer, showing all your work. (4 pts)

14. Use the graph of $f(x)$ to answer the following. All of $f(x)$ is visible. (1 pt each)

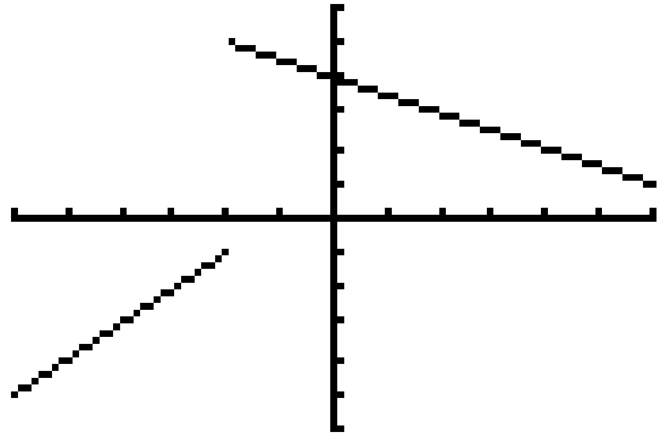
a) Find $f(2)$

b) Find $f(-3)$

c) What is the domain of $f(x)$?

d) What is the range of $f(x)$?

e) Honors: Give a piece-wise defined formula for $f(x)$



2006-2007 Algebra II Second Semester Final-A **Name: ______________________**

1. Use your graphing calculator all you can on this part, giving answers rounded to the nearest hundredth.

a) Solve $x^3 + 2x^2 - 7x = -3$ (2 pts)

b) Find the y- intercept of $y = x^3 + 2x^2 - 7x + 3$ (1 pt)

c) Solve $x^3 + 2x^2 - 7x + 3 > 0$, writing solutions in correct mathematical notation (2 pts)

d) Extra Credit: Give a polynomial with the same roots as in part c) but with a y-intercept of -5 (1 pt)

2. The cost of a can of coke was originally 10 cents in 1960 and has increased over time. The cost can be modeled by the formula, $C(t) = 0.10e^{0.057t}$ where C is the cost in dollars and t is the number of years after 1960.

a) Find $C(38)$ and explain, using correct units, what your answer represents (2 pts)

b) Solve $C(t)= 2.00$ and explain, using correct units, what your answer represents (2 pts)

c) Extra Credit: What is the average rate of change in C between the years 1968 and 1995? Explain your answer, using correct units (1 pt)

Honors Questions for the Final:

1. Find the value of cos ($\sin^{-1}$(¼)). Show work to get partial credit for wrong answers (2 pts)

2. For the function $f(x) = 3 + \sqrt{x+2}$ (1 pt each)

a) find $f(f(2))$

b) Find the domain

c) Find the range

d) Give a formula for $f^{-1}(x)$

e) What is the domain of the inverse?

3. Graph $y = \dfrac{x}{(x-1)(x+3)}$. Give the equations of all horizontal and vertical asymptotes and show them clearly on the graph. (3 pts)

Answers:

Part 1: Basic Calculator Section

1. $4 = \dfrac{y-6}{x+2}$ **2a.** $x\text{-int} = \dfrac{9}{8}$ **b.** answer varies but fit equation $y = \dfrac{8}{3}x - 3$

3a. $\dfrac{1}{2}$ **3b.** $\dfrac{-\sqrt{2}}{2}$ **3c.** 0 **3d.** $\dfrac{\pi}{4}$ **4.** $x = \dfrac{4 \pm i\sqrt{3}}{2}$

5. From left to right: (1st row) – E, D, B; (2nd row) – F, A, C

6a. Potential Rational Roots $= \pm 1, \pm 3, \pm 5, \pm 15, \pm\dfrac{1}{2}, \pm\dfrac{3}{2}, \pm\dfrac{5}{2}, \pm\dfrac{15}{2}$

6b. $2(3)^3 - 11(3)^2 + 2(3) + 15 \neq 0 \rightarrow \ -74 \neq 0$ **6c.** $x = -1,\ \dfrac{3}{2}$ and 5

7a. $(x-3)(x^2+5)$ **7b.** $y^2(2y-3)(2y+3)$

8a. answers may vary

x	-2	-1	0	1	2
y	b	a	any #	a	b

8b. answers may vary

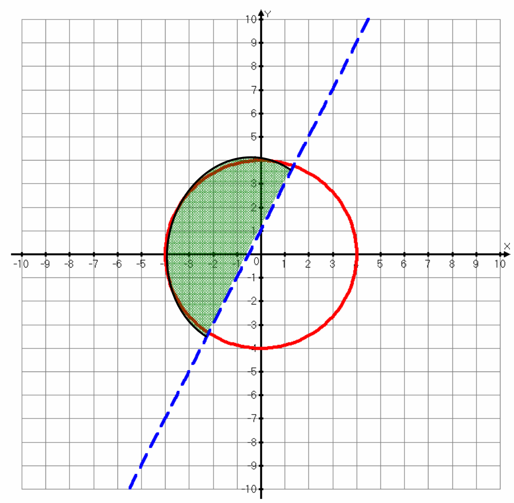
x	-2	-1	0	1	2
y	any two or more #'s being the same				

8c. answers may vary

x	-2	-1	0	1	2
y	any #'s with ± values				

8d. $|x+4| \leq 8$

9.



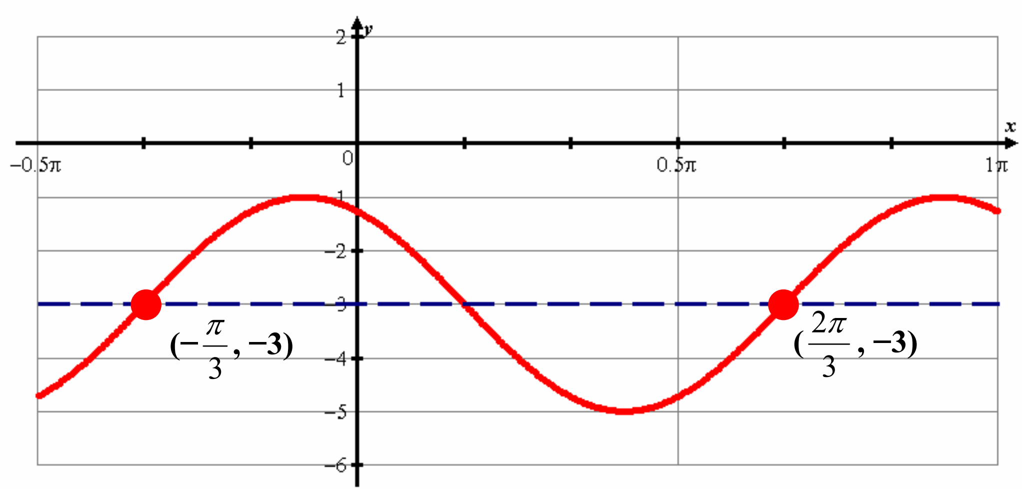
10. From left to right: (1st row) $y = |x+2|$ $y = |x| - 3$
(2nd row) $y = -|x|$ $y = 2|x|$

11a. $x = 7$ **11b.** $x = \dfrac{15}{22}$

11c. $x = -3$ **11d.** $x = 12$

12. Period $= \pi$
Avg (Mid) Value $= -3$
Amp $= 2$

Phase Shift $= \dfrac{\pi}{3}$ left



13. No. If they work together the whole time, it would take 13⅓ hours to complete the whole thing. That means for each hour, they can only finish 0.075 = 7.5% of the job. If Company #1 quits after 5 hours, there is still 0.625 = 62.5% of the job to be finished. Company #2 would have to take another 15 hours to do it alone. Total Time needed would be 20 hours (15 + 5). Hence, it would not be finished within the 18 hours allotted.

14a. $f(2) = 3$ **14b.** $f(-3) = -2$ **14c.** Domain: $-6 \le x \le 6$

14d. Range: $-5 \le y \le -1$ and $1 \le y \le 5$ **14e.** $f(x) = \begin{cases} -\frac{1}{2}x + 4 & \text{if } x \ge -2 \\ x + 1 & \text{if } x < -2 \end{cases}$

Part 2: Graphing Calculator Section

1a. $x = -3.96$, 0.53 and 1.43 **1b.** y-int = 3 **1c.** $-3.96 < x < 0.53$ or $x > 1.43$

1d. $y = \dfrac{-5}{3}(x^3 + 2x^2 - 7x + 3)$ **2a.** $C(38) = \$0.87$; This is the cost of a can of coke in 1998.

2b. In 2012 **2c.** \$0.02139/year

Honors

1. $\dfrac{\sqrt{15}}{4}$

2a. $3 + \sqrt{7}$ **2b.** Domain: $x \ge -2$ **2c.** Range: $y \ge 3 + \sqrt{5}$

2d. $f^{-1}(x) = (x-3)^2 - 2$ for $x \ge 3$ **2e.** Domain $x \ge 3$

3. Vertical Asymptotes: $x = 1$ and $x = -3$
Horizontal Asymptote: $y = 0$