Essential Formulas for Algebra 2 Final Exam

Laws of Exponents

Multiply Powers of the Same Base = Adding Exponents	$(a^m)(a^n) = a^{m+n}$
Divide Powers of the Same Base = Subtracting Exponents	$\frac{a^m}{a^n} = a^{m-n}$
Power Rule = Multiplying Exponents	$(a^m)^n = a^{m \times n}$
Zero Exponent = 1	$a^0 = 1$
Distribution of Exponent with Multiple Bases	$(ab)^n = a^n b^n$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
Negative Exponent = Reciprocal	$a^{-n} = \frac{1}{a^n}$ $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$
Distribution of Negative Exponent with Multiple Bases	$(ab)^{-n} = a^{-n}b^{-n} = \frac{1}{a^n b^n}$ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$	$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$
$\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$	$\sqrt{a \div b} = \sqrt{a} \div \sqrt{b}$

Properties of Radicals

Distribution of Radicals of the Same Index (where $a \geq 0$ and $b \geq 0$ if n is even)	$\sqrt[n]{ab} = \left(\sqrt[n]{a}\right)\left(\sqrt[n]{b}\right)$ $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
Power Rule of Radicals = Multiplying Exponents	$\sqrt[m]{\sqrt[n]{a}} = \sqrt[(m \times n)]{a}$
Reverse Operations of Radicals and Exponents	$\sqrt[n]{a^n} = a$ (if n is odd) $\sqrt[n]{a^n} = \lvert a \rvert$ (if n is even)

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

The index of the radical is the denominator of the fractional exponent.

Special Products

$(A+B)^2 = A^2 + 2AB + B^2$

$(A-B)^2 = A^2 - 2AB + B^2$

$(A+B)(A-B) = A^2 - B^2$

$(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$

$(A-B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$

Special Expressions

Difference of Squares	$A^2 - B^2 = (A+B)(A-B)$
Perfect Trinomial Squares	$A^2 + 2AB + B^2 = (A+B)^2$
Perfect Trinomial Squares	$A^2 - 2AB + B^2 = (A-B)^2$
Sum of Cubes	$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$
Difference of Cubes	$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant $= b^2 - 4ac$

When Discriminant is *Positive*, $b^2 - 4ac > 0 \rightarrow$ ***Two Distinct Real Roots***

When Discriminant is *Zero*, $b^2 - 4ac = 0 \rightarrow$ ***One Distinct Real Root (or Two Equal Real Roots)***

When Discriminant is *Negative*, $b^2 - 4ac < 0 \rightarrow$ ***No Real Roots***

***Note the pattern*:**

$i^1 = i \quad i^2 = -1 \quad i^3 = -i \quad i^4 = 1$

$i^5 = i \quad i^6 = -1 \quad i^7 = -i \quad i^8 = 1$

$i^9 = i \quad i^{10} = -1 \quad \ldots\ldots$

Pattern repeats every 4th power of *i*.

Product of Conjugate Complex Numbers

$(a+bi)(a-bi) = a^2 - b^2i^2 = a^2 - b^2(-1)$

$(a+bi)(a-bi) = a^2 + b^2$

Midpoint of a Line Segment

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Distance of a Line Segement

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Standard Equation for Circles

$$(x-h)^2 + (y-k)^2 = r^2$$

$P\ (x, y)$ = any point on the path of the circle

$C\ (h, k)$ = centre of the circle

r = length of the radius

Point-Slope form: - a form of a linear equation when given a slope (m) and a point (x_1, y_1) on the line

$\frac{y - y_1}{x - x_1} = m$ (slope formula) $y - y_1 = m(x - x_1)$ **(Point-Slope form)**

If we rearrange the equations so that all terms are on one side, it will be in **standard (general) form**:

$Ax + By + C = 0$ **(Standard or General form)**

($A \geq 0$, the leading coefficient for the x term must be positive)

When given a slope (m) and the y-intercept (0, b) of the line, we can find the equation of the line using the **slope and y-intercept form**:

$y = mx + b$ where m **= slope** and b **= y-intercept**

Parallel Lines	**Perpendicular Lines**
slope of line 1 = slope of line 2	**slope of line 1 = negative reciprocal slope of line 2**
$m_1 = m_2$	$m_{l_1} = \frac{-1}{m_{l_2}}$

$y \propto x$ (y is directly proportional to x)

$$y = kx$$

where k = constant of variation (constant of proportionality – rate of change)

$y \propto \frac{xz}{w}$ **(y is jointly proportional to x, z and w)**

$$y = k\frac{xz}{w}$$

where k = constant of variation (constant of proportionality)

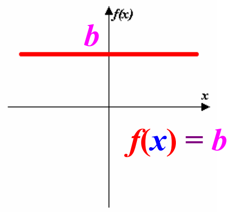
Average Rate of Change $= m = \frac{\Delta y}{\Delta x}$

Average Rate of Change $= \frac{f(b) - f(a)}{b - a}$

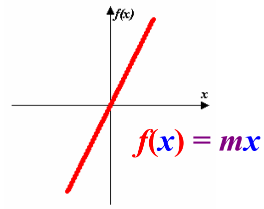
It is the slope of the secant line between the points (a, $f(a)$) and (b, $f(b)$)

Summary of Types of Functions: (see page 226 of textbook)

Linear Functions $f(x) = mx + b$



Domain: $x \in R$
Range: $f(x) \in R$



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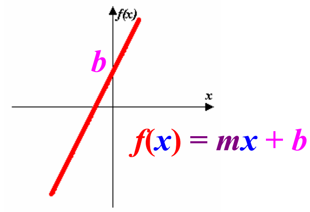
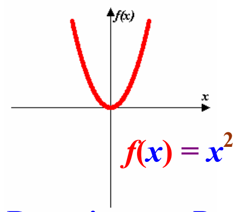
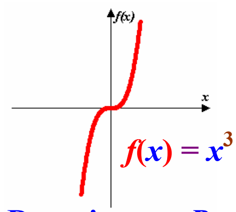


Domain: $x \in R$
Range: $f(x) \in R$

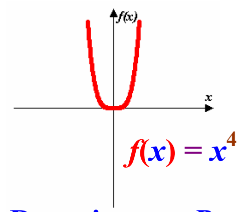
Power Functions $f(x) = x^n$ where $n > 1$ and $n \in N$



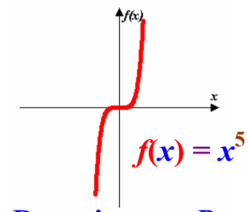
Domain: $x \in R$
Range: $f(x) \geq 0$



Domain: $x \in R$
Range: $f(x) \in R$

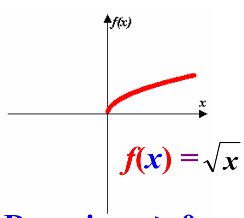


Domain: $x \in R$
Range: $f(x) \geq 0$

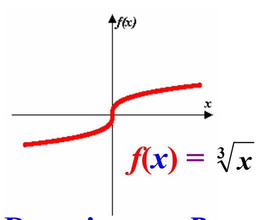


Domain: $x \in R$
Range: $f(x) \in R$

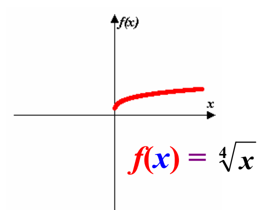
Root Functions $f(x) = \sqrt[n]{x}$ where $n \geq 2$ and $n \in N$



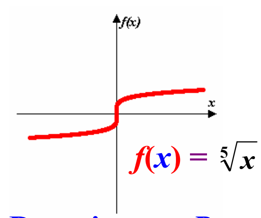
Domain: $x \geq 0$
Range: $f(x) \geq 0$



Domain: $x \in R$
Range: $f(x) \in R$

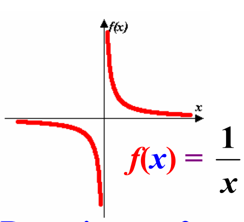


Domain: $x \geq 0$
Range: $f(x) \geq 0$

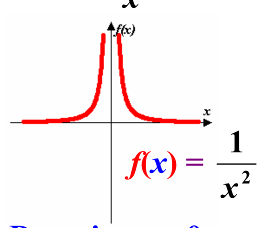


Domain: $x \in R$
Range: $f(x) \in R$

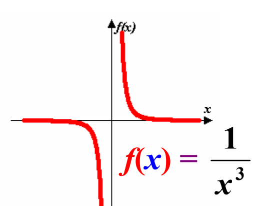
Reciprocal Functions $f(x) = \dfrac{1}{x^n}$ where $n \in N$



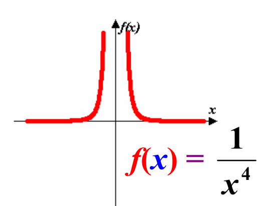
Domain: $x \neq 0$
Range: $f(x) \neq 0$



Domain: $x \neq 0$
Range: $f(x) > 0$

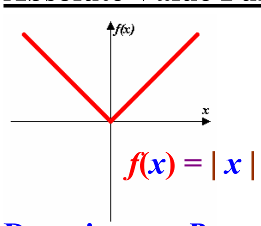


Domain: $x \neq 0$
Range: $f(x) \neq 0$



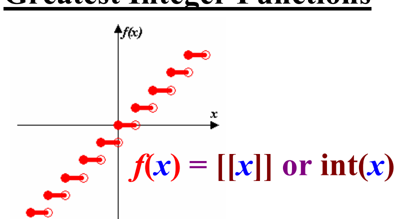
Domain: $x \neq 0$
Range: $f(x) > 0$

Absolute Value Functions



Domain: $x \in R$
Range: $f(x) \geq 0$

Greatest Integer Functions



Domain: $x \in R$
Range: $f(x) \in I$

$g(x) = f(x + h) + k$

h = amount of horizontal movement — **$h > 0$ (move left); $h < 0$ (move right)**

k = amount of vertical movement — **$k > 0$ (move up); $k < 0$ (move down)**

Reflection off the x-axis

$g(x) = -f(x)$

All values of y has to switch signs but all values of x remain unchanged.

Reflection off the y-axis

$g(x) = f(-x)$

All values of x has to switch signs but all values of y remain unchanged.

Vertical Stretching and Shrinking

$g(x) = af(x)$

a is the Vertical Stretch Factor

$a > 1$ (Stretches Vertically by a factor of a)

$0 < a < 1$ (Shrinks Vertically by a factor of a)

Horizontal Stretching and Shrinking

$g(x) = f(bx)$

b is the Horizontal Stretch Factor

$0 < b < 1$ (Stretches Horizontally by a factor of $1/b$)

$b > 1$ (Shrinks Horizontally by a factor of $1/b$)

For Quadratic Functions in Standard Form of $f(x) = a(x - h)^2 + k$

Vertex at (h, k) — **Axis of Symmetry at $x = h$** — **Domain: $x \in R$**

a = Vertical Stretch Factor

$a > 0$ Vertex at Minimum (Parabola opens UP) — **Range: $y \geq k$ (Minimum)**

$a < 0$ Vertex at Maximum (Parabola opens DOWN) — **Range: $y \leq k$ (Maximum)**

$|a| > 1$ Stretched out Vertically — **$|a| < 1$ Shrunken in Vertically**

h = Horizontal Translation (Note the standard form has $x - h$ in the bracket!)

$h > 0$ Translated Right — **$h < 0$ Translated Left**

k = Vertical Translation

$k > 0$ Translated Up — **$k < 0$ Translated Down**

For Quadratic Functions in General Form: $f(x) = ax^2 + bx + c$

y-intercept at $(0, c)$ by letting $x = 0$ (Note: Complete the Square to change to *Standard Form*)

x-intercepts at $\left(\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0\right)$ if $b^2 - 4ac \geq 0$. No x-intercepts when $b^2 - 4ac < 0$

Vertex locates at $x = -\dfrac{b}{2a}$ $\quad y = f\left(-\dfrac{b}{2a}\right)$ **Minimum when $a > 0$; Maximum when $a < 0$**

$f(x)$ = One-to-One Function — **$f^{-1}(x)$ = Inverse Function**

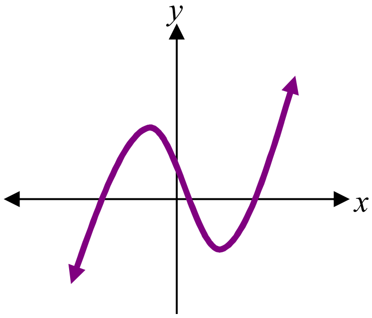
(x, y) — **(y, x)**

Domain of $f(x) \rightarrow$ Range of $f^{-1}(x)$

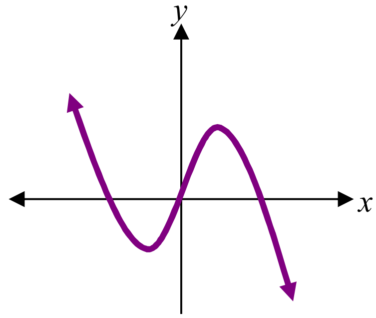
Range of $f(x) \rightarrow$ Domain of $f^{-1}(x)$

Note: $f^{-1}(x) \neq \dfrac{1}{f(x)}$ (Inverse is DIFFERENT than Reciprocal)

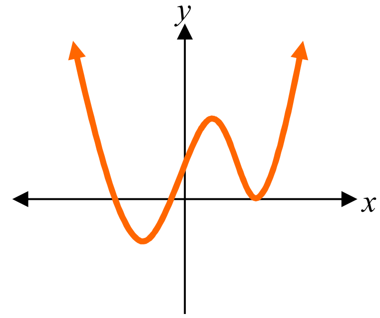
End Behaviours and Leading Terms



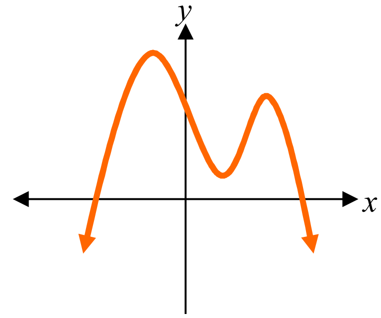
Odd Degree Polynomial Function and Positive Leading Coefficient, $a > 0$



Odd Degree Polynomial Function and Negative Leading Coefficient, $a < 0$



Even Degree Polynomial Function and Positive Leading Coefficient, $a > 0$



Even Degree Polynomial Function and Negative Leading Coefficient, $a < 0$

Odd Degree Polynomial Functions

When $a > 0$, Left is Downward ($y \to -\infty$ as $x \to -\infty$) and Right is Upward ($y \to \infty$ as $x \to \infty$).

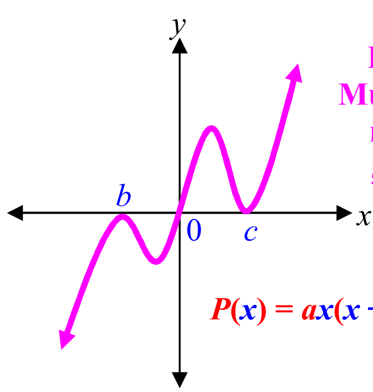
When $a < 0$, Left is Upward ($y \to \infty$ as $x \to -\infty$) and Right is Downward ($y \to -\infty$ as $x \to \infty$).

Even Degree Polynomial Functions

When $a > 0$, Left is Upward ($y \to \infty$ as $x \to -\infty$) and Right is Upward ($y \to \infty$ as $x \to \infty$).

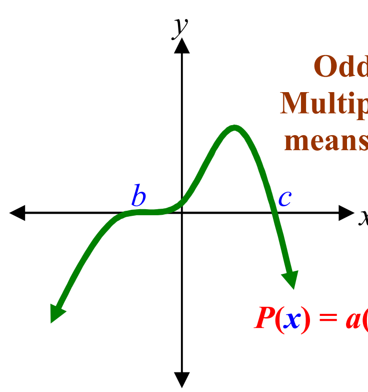
When $a < 0$, Left is Downward ($y \to -\infty$ as $x \to -\infty$) and Right is Downward ($y \to -\infty$ as $x \to \infty$).

Multiplicity: - when a factored polynomial expression has exponents on the factor that is greater than 1.



Even Multiplicity or Multiplicities of 2, 4, 6… means x-intercept is *tangent to the x-axis*

$$P(x) = ax(x-b)^{2,\,4,\,6\,\ldots}(x-c)^{2,\,4,\,6,\ldots}$$



Odd Multiplicity or Multiplicities of 3, 5, 7… means *inflection* at x-int

$$P(x) = a(x-b)^{3,\,5,\,7\,\ldots}(x-c)$$

Polynomial Function **Divisor Function**

In general, for $P(x) \div D(x)$, we can write

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R}{D(x)} \quad \text{or} \quad P(x) = D(x)\,Q(x) + R$$

Restriction: $D(x) \neq 0$

Quotient Function **Remainder**

If $R = 0$ when $\frac{P(x)}{(x-b)}$, then $(x - b)$ is a factor of $P(x)$ and $P(b) = 0$.

$$P(x) = D(x) \times Q(x)$$

$P(x)$ = Original Polynomial $D(x)$ = Divisor (Factor) $Q(x)$ = Quotient

If $R \neq 0$ when $\frac{P(x)}{(x-b)}$, then $(x - b)$ is NOT a factor of $P(x)$.

$$P(x) = D(x) \times Q(x) + R(x)$$

The Remainder Theorem:

To find the remainder of $\frac{P(x)}{x-b}$: Substitute b from the Divisor, $(x - b)$, into the Polynomial, $P(x)$.

In general, when $\frac{P(x)}{x-b}$, $P(b)$ = Remainder.

To find the remainder of $\frac{P(x)}{ax-b}$: Substitute $\left(\frac{b}{a}\right)$ from the Divisor, $(ax - b)$, into the Polynomial, $P(x)$.

In general, when $\frac{P(x)}{ax-b}$, $P\left(\frac{b}{a}\right)$ = Remainder.

The Factor Theorem:

1. If $\frac{P(x)}{x-b}$ gives a Remainder of 0, then $(x - b)$ is the Factor of $P(x)$.

   OR

   If $P(b) = 0$, then $(x - b)$ is the Factor of $P(x)$.

2. If $\frac{P(x)}{ax-b}$ gives a Remainder of 0, then $(ax - b)$ is the Factor of $P(x)$.

   OR

   If $P\left(\frac{b}{a}\right) = 0$, then $(ax - b)$ is the Factor of $P(x)$.

Rational Roots Theorem:

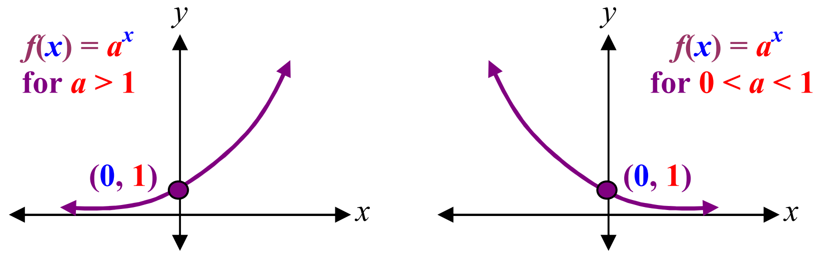
For a polynomial $P(x)$, a List of POTENTIAL Rational Roots can be generated by Dividing ALL the Factors of its Constant Term by ALL the Factors of its Leading Coefficient.

$$\text{Potential Rational Zeros of } P(x) = \frac{\text{ALL Factors of the Constant Term}}{\text{ALL Factors of the Leading Coefficient}}$$

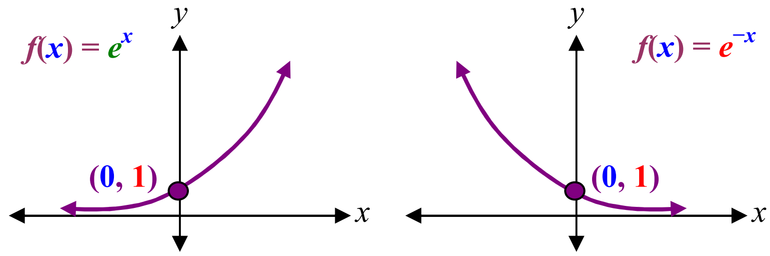
The Zero Theorem

There are n number of solutions (complex, real or both) for any n^{th} degree polynomial function accounting that that a zero with multiplicity of k is counted k times.

Graphs of Exponential Functions



Graphs of Natural Exponential Functions



$$y = a^x \longleftrightarrow x = \log_a y$$

Simple Properties of Logarithms

$\log_a 1 = 0$ because $a^0 = 1$

$\log_a a = 1$ because $a^1 = a$

$a^{\log_a x} = x$ because exponent and logarithm are inverse of one another

$\log_a a^x = x$ because logarithm and exponent are inverse of one another

Common and Natural Logarithm

Common Logarithm: $\log x = y \longleftrightarrow 10^y = x$

Natural Logarithm: $\ln x = y \longleftrightarrow e^y = x$

Exponential Laws	Logarithmic Laws
$(a^m)(a^n) = a^{m+n}$	$\log_a x + \log_a y = \log_a (xy)$
$\dfrac{a^m}{a^n} = a^{m-n}$	$\log_a x - \log_a y = \log_a \left(\dfrac{x}{y}\right)$
$(a^m)^n = a^{m \times n}$	$\log_a x^y = y \log_a x$
$a^0 = 1$	$\log_a 1 = 0$

Common Logarithm Mistakes

$\log_a(x+y) \neq \log_a x + \log_a y$

Example: $\log(2+8) \neq \log 2 + \log 8$

$1 \neq 0.3010 + 0.9031$

$\log_a(x-y) \neq \log_a x - \log_a y$

Example: $\log(120-20) \neq \log 120 + \log 20$

$2 \neq 2.0792 + 1.3010$

$\log_a\left(\frac{x}{y}\right) \neq \frac{\log_a x}{\log_a y}$

Example: $\log\left(\frac{1}{10}\right) \neq \frac{\log 1}{\log 10}$

$-1 \neq \frac{0}{1}$

$(\log_a x)^y \neq y\log_a x$

Example: $(\log 100)^3 \neq 3 \log 100$

$2^3 \neq 3(2)$

$a^x = y \qquad x = \frac{\log y}{\log a}$

$A = P\left(1+\frac{r}{n}\right)^{nt}$

A = Final Amount after t years

r = Interest Rate per year

P = Principal

n = Number of Terms per year

$A(t) = A_0\left(1+\frac{r}{n}\right)^{nt} \xrightarrow{n \to \infty} A(t) = A_0 e^{rt}$

$A(t)$ = Final Amount after t years

A_0 = Initial Amount

r = Rate of Increase ($+r$) / Decrease ($-r$) per year

$A(t) = A_0 e^{rt}$

$A(t)$ = Final Amount after t years

A_0 = Initial Amount

r = Rate of Increase ($+r$) / Decrease ($-r$) per year

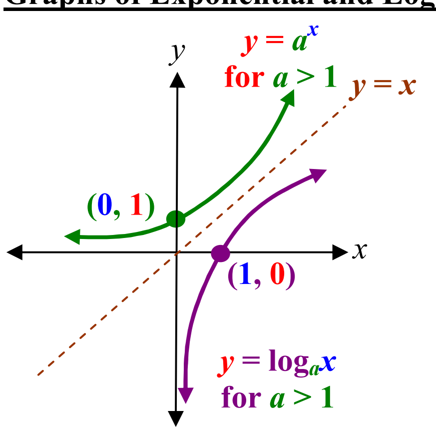
$N(t) = N_0 e^{rt}$

$N(t)$ = Final Population after t years, hours, minutes, or seconds

N_0 = Initial Population

r = Rate of Increase per year, hour, minute, or second

Graphs of Exponential and Logarithmic Functions



Exponential Function

y-int = 1 No x-intercept

Domain $x \in R$; Range $y > 0$

Logarithmic Function

x-int = 1 No y-intercept

Domain $x > 0$; Range $y \in R$

To obtain equation for the inverse of an exponential function, we start with

$y = a^x$

$x = a^y$ (switch x and y for inverse)

$y = \log_a x$ (rearrange to solve for y)

$$\pi \text{ rad} = 180^\circ \quad \text{OR} \quad \frac{\pi}{180} \text{ rad} = 1^\circ$$

$y = a \sin k(x + b) + c$ $\qquad$ $y = a \cos k(x + b) + c$

$|a|$ = Amplitude $\qquad$ c = Vertical Displacement (how far away from the x–axis)

b = Horizontal Displacement (Phase Shift) $\qquad$ $b > 0$ (shifted left) $\qquad$ $b < 0$ (shifted right)

k = number of complete cycles in 2π $\qquad$ Period $= \frac{2\pi}{k} = \frac{360^\circ}{k}$

Range = Minimum $\leq y \leq$ Maximum

$y = a \sin [\omega(t + b)] + c$ $\qquad$ $y = a \cos [\omega(t + b)] + c$

$|a|$ = Amplitude $\qquad$ c = Vertical Displacement (distance between *mid-line* and t-axis)

b = Horizontal Displacement (Phase Shift) $\qquad$ $b > 0$ (shifted left) $\qquad$ $b < 0$ (shifted right)

ω = number of complete cycles in 2π $\qquad$ Period $= \frac{2\pi}{\omega}$ $\qquad$ Frequency $= \frac{\omega}{2\pi}$

Range = Minimum $\leq y \leq$ Maximum

Note: $\sin^{-1}(x) \neq \frac{1}{\sin(x)}$ $\qquad$ $\sin^{-1}(x) \neq (\sin x)^{-1}$ $\qquad$ $(\sin x)^{-1} = \frac{1}{\sin(x)} = \csc x$

$y = \sin^{-1} x$

Domain: $-1 \leq x \leq 1$ $\qquad$ Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$y = \cos^{-1} x$

Domain: $-1 \leq x \leq 1$ $\qquad$ Range: $0 \leq x \leq \pi$

$\sin(\sin^{-1} x) = x$ $\qquad$ for $-1 \leq x \leq 1$

$\sin^{-1}(\sin x) = x$ $\qquad$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$\cos(\cos^{-1} x) = x$ $\qquad$ for $-1 \leq x \leq 1$

$\cos^{-1}(\cos x) = x$ $\qquad$ for $0 \leq x \leq \pi$

$y = \tan^{-1} x$

Domain: $x \in R$ $\qquad$ Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$\tan(\tan^{-1} x) = x$ $\qquad$ for $x \in R$

$\tan^{-1}(\tan x) = x$ $\qquad$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

<u>Some Basic Trigonometric Definitions and Identities (proven equations)</u>

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\qquad$ $\csc\theta = \frac{1}{\sin\theta}$ $\qquad$ $\sec\theta = \frac{1}{\cos\theta}$ $\qquad$ $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

$$\cos^2\theta + \sin^2\theta = 1$$