Introduction to Sequences and Series

Our book covers this topic in Chapter 12. It is not a core idea of math (until you get to Calculus and beyond) but it can be on the SAT so we will give an introduction into the basic concepts of the topic. You can look at the book (sections 12-1 to 12-3) as a reference for these ideas. We will just be hitting the highlights of these sections.

A **sequence** is a list of numbers, where each number has a well defined place (the 2nd number or the 12th number or …). A sequence can be given as a list. For example, the sequence: 1, 3, 7, 14, 18, … has 1 as its 1st term and 7 as its 3rd term. Sometimes a sequence is given by the formula that its terms follow (not every sequence has a nice pattern but if there is no pattern, the sequence is usually not very interesting to us) An example of this is $a_n = \frac{1}{n+3}$. Notice that this looks a lot like function notation where *a* is the name of the sequence and *n* represents which term we are looking at. The first term ($n = 1$) would be $\frac{1}{4}$ and the 5th term ($n = 5$) would be $\frac{1}{8}$. These ideas are found on pages 870-872 in the book.

1. Do 12-1 # 3, 5, 7

One special type of sequence is called an **arithmetic sequence.** It is a sequence that has any number as its first term and each term is the previous term plus a certain number (called the **common difference,** it could be negative). An example of this is 8, 13, 18, 23, … This arithmetic sequence has a first term of 8 and a common difference of 5. It can be written using the formula $a_n = 8 + 5(n-1)$. A general formula is $a_n = a_1 + d(n-1)$ where a_1 is the first term, *d* is the common difference and *n* is the term you are trying to find ($n = 7$ for the 7th term). This idea is in the book on pages 881-882.

2. Do 12-2 # 3ab, 15, 25, 31

A second special type of sequence is called a **geometric sequence**. It is a sequence that has any number as its first term and then each term is the previous term multiplied by a certain number. The number that the terms are multiplied by is called the **common ratio**. An example is 4, 12, 36, 108, … The first term is 4 and the common ratio is 3. A formula for this sequence is $a_n = 4(3)^{n-1}$. In general the formula for a geometric

sequence is $a_n = a_1(r)^{n-1}$ where *r* is the common difference. This idea is in the book on pages 886- 888.

3. Do 12-3 # 3ab, 11, 19, 29

The last idea that we will look at is the concept of a **series**. A series is a number, it is the sum (addition) of certain terms of a sequence. It often has some funky notation associated with it, called Summation or Sigma notation. The best way to illustrate this is with a couple of examples. $\sum_{k=1}^{3} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$ or $\sum_{j=2}^{6} j^2 = 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = 90$.
Notice that the letter used does not matter (like it does not matter what variable you use to represent the domain in a function) but where to start and stop in the sequence is given by the lower and upper limit on the summation symbol (from the 1st to 3rd term in the first example and from the 2nd to 6th term in the second). The format of the terms is given by the expression to the right of the sigma symbol. (This is notation that will be used extensively in Calculus) Look in the book on pages 876 (after example 6) to 877 for more help.

4. Do 12-1 # 39, 43, 45, 61, 65

3. On the first day of school, Mr. Vilani gave his third-grade students 5 new words to spell. On each day of school after that, he gave the students 3 new words to spell. In the first 20 days of school, how many new words had he given the students to spell?

A. 28
B. 62
C. 65
D. 68
E. 152

(ACT test problem)

4, 11, 18, . . .

1. In the sequence above, the first term is 4 and each term after the first is 7 more than the previous term. What is the 12th term of the sequence?

(A) 77
(B) 81
(C) 84
(D) 86
(E) 92

(SAT test problem)