Prerequisites Chapter: Algebra 1 Review

P-1: Modeling the Real World

Model: - a mathematical depiction of a real world condition.
- it can be a formula (equations with meaningful variables), a properly drawn graph, a clearly labelled diagram with quantitative measurements.

Modelling: - the process of discovering the mathematical model.

Example 1: To convert temperature measurements from degree Celsius to Fahrenheit, we can use the formula, $T_F = \frac{9}{5}T_C + 32$.

a. What is the temperature in Fahrenheit when the outside temperature is −10°C?
b. What is the temperature in degree Celsius for a patient with a temperature of 105 F?
c. At what temperature when its numerical value of degree Celsius is equivalent to that of Fahrenheit?

a. $T_F = \frac{9}{5}T_C + 32$ $\quad T_F = \frac{9}{5}(-10) + 32$

$T_F = -18 + 32$ $\quad$ **$T_F = 14$ F**

b. We can manipulate the formula first before substitution.

$$T_F = \frac{9}{5}T_C + 32$$

$$T_F - 32 = \frac{9}{5}T_C$$

$$\frac{5}{9}(T_F - 32) = T_C$$

$$\frac{5}{9}((105) - 32) = T_C$$

$T_C = 40.6$°C

c. At the same numerical value, we can set $x = T_F = T_C$

$$T_F = \frac{9}{5}T_C + 32$$

$$x = \frac{9}{5}x + 32$$

$$1x - \frac{9}{5}x = 32$$

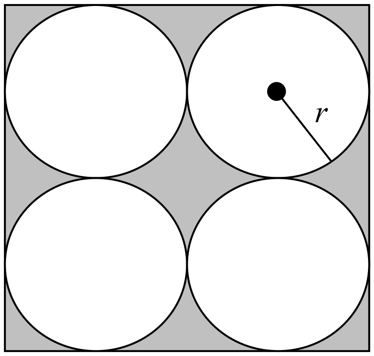
$$-\frac{4}{5}x = 32$$

$$x = \left(-\frac{5}{4}\right)32$$

$x = -40$ F $= -40$°C

Example 2: A rectangular box has a width measured twice its height and its length is three times its width.
a. Find the volume of the box if it has a height of 8 cm.
b. Write a formula for the volume V of this box in terms of its height x.
c. What are the dimensions of this box if it has a volume of 768 cubic feet?

Example 3: Four identical circles are enclosed by a square as shown below. Determine the cut out area A in terms of r as represents by the shaded area.



P-2: Real Numbers

Set: - a group of objects (called **elements** of the set).
- we commonly use fancy brackets, { }, to include elements of a set.

Natural Numbers (*N*): - counting numbers. $N = \{1, 2, 3, 4, 5, \ldots\}$

Whole Numbers (*W*): - counting numbers with 0. $W = \{0, 1, 2, 3, 4, 5, \ldots\}$

Integers (*I*): - positive and negative whole numbers. $I = \{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$

Set Notation (∈): - a symbol to indicate an object belongs in the a particular set.

Example: $0 \in W$ but $0 \notin N$ (0 belongs to in a set of whole numbers but not in a set of natural numbers.)

Set-Building Notation: - a set notation that involves a series of number.

Example: $Z = \{2, 3, 4, 5, 6, 7\}$ can be written as $Z = \{x \mid 2 \le x \le 7 \text{ and } x \in N\}$
(Z is a set such that the elements, represented by x, are between 2 to 7 and they are natural numbers)
(**Note**: when a set-building notation does not include the type of numbers it is assumed $x \in \Re$ real numbers)

Rational Numbers (*Q*): - numbers that can be turned into a fraction $\frac{a}{b}$, where $a, b \in I$, and $b \ne 0$.
- include all Terminating or Repeating Decimals.
- include all Natural Numbers, Whole Numbers and Integers.
- include any perfect roots (radicals).

a. Terminating Decimals: - decimals that stops. Examples: $0.25 = \frac{1}{4}$ $\quad -0.7 = -\frac{7}{10}$

b. Repeating Decimals: - decimals that repeats in a pattern and goes on.

Examples: $0.3\ldots = \frac{1}{3}$ $\quad -1.\overline{7} = -\frac{16}{9}$

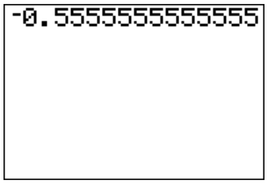
c. Perfect Roots: - radicals when evaluated will result in either Terminating or repeating decimals, or fractions $\frac{a}{b}$, where $a, b \in I$, and $b \ne 0$.

Examples: $\sqrt{0.16} = \pm 0.4$ $\quad \sqrt{0.111\ldots} = \pm 0.3\ldots = \pm\frac{1}{3}$ $\quad \sqrt{\frac{1}{25}} = \pm\frac{1}{5}$ $\quad \sqrt[3]{0.008} = 0.2$

To Convert a Decimal into Fraction using TI-83 Plus

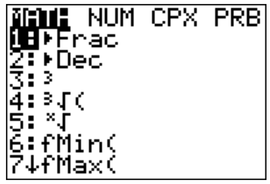
Example: Convert $-0.\overline{5}$ into a fraction.

(−)

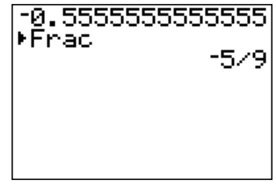


MATH

Select Option 1



ENTER



Repeat entering 5 to the edge of the screen

Irrational Numbers ($\overline{Q}$): - numbers that **CANNOT** be turned into a fraction $\frac{a}{b}$, where a, $b \in I$, and $b \neq 0$.
- include all non-terminating, non-repeating decimals.
- include any non-perfect roots (radicals).

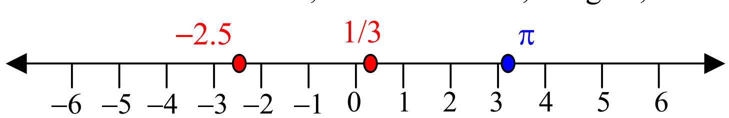
a. Non-terminating, Non-repeating Decimals: - decimals that do not repeat but go on and on.

Examples: $\pi = 3.141592654\ldots$ 0.123 123 312 333 123 333 …

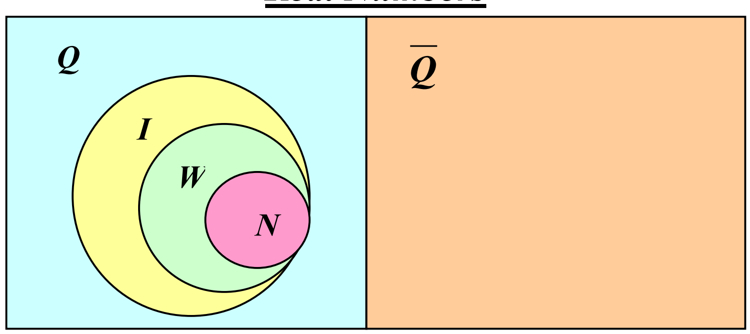
b. Non-Perfect Roots: radicals when evaluated will result in Non-Terminating, Non-Repeating decimals.

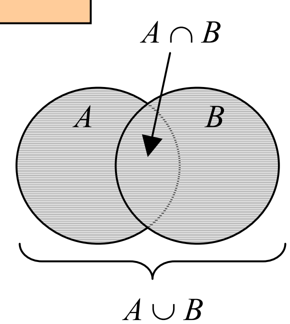
Examples: $\sqrt{5} = \pm 2.236067977...$ $\sqrt{0.52} = \pm 0.7211102551...$ $\sqrt[3]{-0.38} = -0.7243156443...$

Real Numbers ($\mathfrak{R}$): - any numbers that can be put on a number line.
- include all natural numbers, whole numbers, integers, rational and irrational numbers.



Real Numbers





Union ($\cup$): - the combined elements of two sets.
- for $A \cup B$, it means all elements in *A or B* (*or in both*).

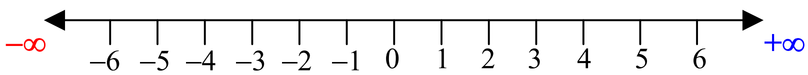
Intersection ($\cap$): - includes all elements that are in both sets.
- for $A \cap B$, it means all elements in *A and B*.

Empty Set (∅): - when the set consists of no elements.

Example 1: If $F = \{-2, -1, 0, 1, 2, 3, 4\}$, $G = \{0, 1, 2\}$, and $H = \{6, 7, 8\}$, find

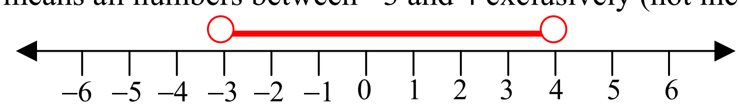
a. $F \cup G$ b. $F \cap G$ c. $G \cap H$

Infinity (∞): - use to denote that the patterns go on and on in a specific direction of the real number line.
- positive infinity (∞) means infinity towards the right of the number line.
- negative infinity (–∞) means infinity towards the left of the number line.



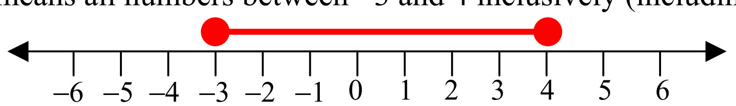
Open Interval: - when the boundary numbers are not included (exclusive).
- we use normal brackets for open intervals.
- on the number line, we use open circles at the endpoints.

Example: $(-3, 4)$ means all numbers between –3 and 4 exclusively (not including –3 and 4)

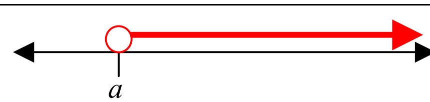
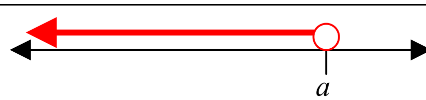
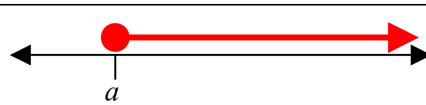
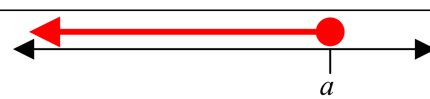


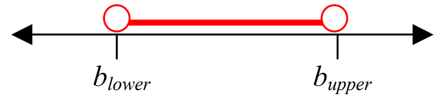
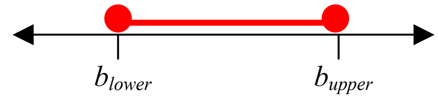
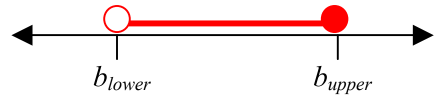
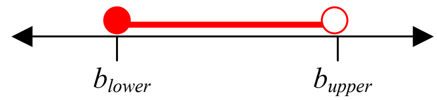
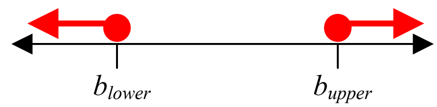
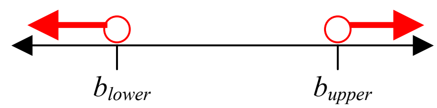
Closed Interval: - when the boundary numbers are included (inclusive).
- we use square brackets for open intervals.
- on the number line, we use closed (filled in) circles at the endpoints.

Example: $[-3, 4]$ means all numbers between –3 and 4 inclusively (including –3 and 4)



Inequalities and Intervals

Notation	Meaning and Set Description		Graphs
$>$ or (a, ∞)	Greater than	$\{x \mid x > a\}$	
$<$ or $(-\infty, a)$	Less than	$\{x \mid x < a\}$	
$\geq$ or $[a, \infty)$	Greater than or equal to	$\{x \mid x \geq a\}$	
$\leq$ or $(-\infty, a]$	Less than or equal to	$\{x \mid x \leq a\}$	

Notation	Meaning and Set Description	Graphs
$(b_{lower}\ ,\ b_{upper})$	x is between the lower and upper boundaries (exclusive). $\{x \mid b_{lower} < x < b_{upper}\}$	
$[b_{lower},\ b_{upper}]$	x is between the lower and upper boundaries (inclusive). $\{x \mid b_{lower} \le x \le b_{upper}\}$	
$(b_{lower}\ ,\ b_{upper}]$	x is between the lower (open) and upper (closed) boundaries. $\{x \mid b_{lower} < x \le b_{upper}\}$	
$[b_{lower}\ ,\ b_{upper})$	x is between the lower (closed) and upper (open) boundaries. $\{x \mid b_{lower} \le x < b_{upper}\}$	
$(-\infty,\ b_{lower}] \cup [b_{upper},\ \infty)$	x is less than the lower boundary and x is greater than the upper boundary (inclusive). $\{x \mid x \le b_{lower} \cup x \ge b_{upper}\}$	
$(-\infty,\ b_{lower}) \cup (b_{upper},\ \infty)$	x is less than the lower boundary and x is greater than the upper boundary (exclusive). $\{x \mid x < b_{lower} \cup\ x > b_{upper}\}$	

Example 2: Express each interval in terms of inequalities (set descriptions), and then graph the intervals.

a. $[-4, 9)$

b. $(-\infty, -2) \cup [3, \infty)$

Example 3: Graph each set.

a. $(1, 8] \cap [3, 4)$

b. $(1, 8] \cup [3, 4)$

P-1 Assignment: pg. 7–10 #5, 12, 25, 31, 38 and 41; Honours: #43
P-2 Assignment: pg. 19–21 #34, 35, 37, 39, 41, 45, 47, 49, 53, 57 and 75; Honour: #77

P-3: Integer Exponents

Integer Exponent: - an exponent that belongs in an integer set.
- an exponent indicates how many factors the base is multiplying itself.

***Note*: The exponent only applies to the immediate number, variable or bracket preceding it.**

"a to the n^{th} power"

exponent

power $\{ a^n = \underbrace{(a)(a)(a)(a)\ldots(a)}_{n \text{ factors}}$

base

Example 1: Evaluate the followings.

a. $(-2)^4$

$(-2)^4 = (-2)(-2)(-2)(-2)$
$(-2)^4 = 16$

b. -2^4

$-2^4 = -(2)(2)(2)(2)$
$-2^4 = -16$

Note that the exponent only applies to the immediate number preceding it and exclude the negative sign.

Laws of Exponents

Multiply Powers of the Same Base = Adding Exponents	$(a^m)(a^n) = a^{m+n}$
Divide Powers of the Same Base = Subtracting Exponents	$\dfrac{a^m}{a^n} = a^{m-n}$
Power Rule = Multiplying Exponents	$(a^m)^n = a^{m \times n}$
Zero Exponent = 1	$a^0 = 1$
Distribution of Exponent with Multiple Bases	$(ab)^n = a^n b^n$ $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
Negative Exponent = Reciprocal	$a^{-n} = \dfrac{1}{a^n}$ $\dfrac{a^{-m}}{b^{-n}} = \dfrac{b^n}{a^m}$
Distribution of Negative Exponent with Multiple Bases	$(ab)^{-n} = a^{-n}b^{-n} = \dfrac{1}{a^n b^n}$ $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n = \dfrac{b^n}{a^n}$

Example 2: Simplify. Express all answers in positive exponents only.

a. $(7c^{11}d^4)(-6c^8d^5)$

$= -42\,c^{11+9}\,d^{4+5}$

$-42c^{19}d^9$

b. $\dfrac{9a^5b^{10}}{-36a^{15}b^4}$

$= \dfrac{1a^{5-15}b^{10-4}}{-4}$

$= -\dfrac{a^{-10}b^6}{4}$ $\quad -\dfrac{b^6}{4a^{10}}$

c. $(3x^5y^2)^3$

$= (3)^3(x^{5\times3})(y^{2\times3})$

$27x^{15}y^6$

d. $\dfrac{\left(5x^3y^2\right)^3\left(3x^5y^9\right)^2}{\left(-6x^7y^3\right)^4}$

$= \dfrac{\left(125x^9y^6\right)\left(9x^{10}y^{18}\right)}{\left(1296x^{28}y^{12}\right)}$

$= \dfrac{1125x^{9+10-28}y^{6+18-12}}{1296}$

$= \dfrac{125x^{-9}y^{12}}{144}$ $\quad \dfrac{125y^{12}}{144x^9}$

e. $(4m^4n^{-7})^3\,(2m^3n^5)^{-4}$

$= \dfrac{\left(4m^4n^{-7}\right)^3}{\left(2m^3n^5\right)^4}$

When reciprocating an entire bracket, do NOT mess with its content.

$= \dfrac{64m^{12}n^{-21}}{16m^{12}n^{20}}$

$= 4\,m^{12-12}\,n^{-21-20}$

$= 4\,(1)\,n^{-41}$ $\quad \dfrac{4}{n^{41}}$

f. $\left(\dfrac{-5p^{-4}q^3}{4p^{-7}q^{-3}}\right)^{-2}$

$= \left(\dfrac{4p^{-7}q^{-3}}{-5p^{-4}q^3}\right)^2 = \dfrac{16p^{-14}q^{-6}}{25p^{-8}q^6}$

$= \dfrac{16p^{-14-(-8)}q^{-6-6}}{25}$

$= \dfrac{16p^{-6}q^{-12}}{25}$ $\quad \dfrac{16}{25p^6q^{12}}$

g. $\dfrac{3^{-1}-(-3)^2}{(-3)^{-3}+\left(-\frac{1}{3}\right)^{-4}}$

$= \dfrac{\left(\frac{1}{3}\right)-9}{\left(\frac{-1}{3}\right)^3+(-3)^4} = \dfrac{\left(\frac{1}{3}\right)-9}{\left(\frac{-1}{27}\right)+81} = \dfrac{\left(\frac{-26}{3}\right)}{\left(\frac{2186}{27}\right)}$

$= \left(\dfrac{-26}{3}\right) \div \left(\dfrac{2186}{27}\right)$ $\quad -\dfrac{117}{1093}$

h. $\dfrac{\left(-6h^{-2}k^3\right)^{-3}}{\left(9h^5k^{-1}\right)^{-2}\left(-3h^{-4}k^{-2}\right)^4}$

$= \dfrac{\left(9h^5k^{-1}\right)^2}{\left(-6h^{-2}k^3\right)^3\left(-3h^{-4}k^{-2}\right)^4}$

$= \dfrac{\left(81h^{10}k^{-2}\right)}{\left(-216h^{-6}k^9\right)\left(81h^{-16}k^{-8}\right)}$

$= \dfrac{h^{10-(-6)-(-16)}k^{-2-9-(-8)}}{-216}$ $\quad -\dfrac{h^{32}}{216k^3}$

Scientific Notation: - commonly used to state very big or very small numbers.

(1 to 9.999…) × 10^n where n is an integer
If $n < 0$, then the actual number was between 0 and 1
If $n > 0$, then the actual number was greater than 10

Example 3: Convert the following standard notations to scientific notations or vice versa.

a. Speed of Light = 3×10^5 km/s = **300,000 km/s** **(moved 5 decimal places to the right)**

b. Mass of an Electron = 9.11×10^{-31} kg = **0.000 000 000 000 000 000 000 000 000 000 911 kg** **(moved 31 decimal places to the left)**

c. Diameter of a Red Blood Cell = 0.000 007 5 m = **7.5×10^{-6} m** **(moved 6 decimal places to the right)**

d. 2003 US Debt = $6,804,000,000,000 = **$6.804 $\times 10^{12}$** **(moved 12 decimal places to the left)**

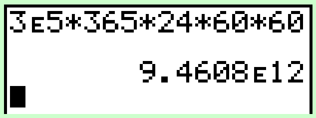
Example 4: In astronomy, one light year is the distance light can travel in one year. Light has a constant speed of 3×10^5 km/s in the vacuum of space.

a. Calculate the distance of one light year.

b. The closest star to the Sun, Alpha Centuri, is 3.78×10^{13} km. How many light years is it to our sun?

a. One Light Year = $(3 \times 10^5 \text{ km/}\cancel{\text{s}})(365\ \cancel{\text{days}}\text{/yr})(24\ \cancel{\text{hr}}/\cancel{\text{day}})(60\ \cancel{\text{min}}/\cancel{\text{hr}})(60\ \cancel{\text{s}}/\cancel{\text{min}})$

2nd **EE** **,**



One Light Year = 9.4608×10^{12} km/yr

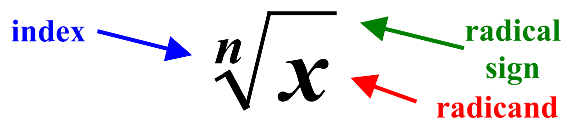
b. $\dfrac{3.78 \times 10^{13}\ \cancel{\text{km}}}{9.4608 \times 10^{12}\ \cancel{\text{km}}/\text{yr}}$

4 light years

P-3 Assignment: pg. 27–28 #9, 13, 17, 21, 27, 35, 39, 47, 49, 53, 63, 80; Honours: #82a

P-4: Rational Exponents and Radicals

Radicals: - the result of a number after a root operation.



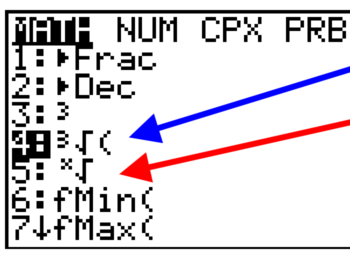
Radical Sign: - the mathematical symbol $\sqrt{\ }$.

Radicand: - the number inside a radical sign.

Index: - the small number to the left of the radical sign indicating how many times a number (answer to the radical) has to multiply itself to equal to the radicand.

$\sqrt{\ }$ square root	$\sqrt[3]{\ }$ cube root	$\sqrt[4]{\ }$ fourth root	$\sqrt[5]{\ }$ fifth root

To call up the cube root $\sqrt[3]{\ }$ or higher root functions $\sqrt[x]{\ }$, press **MATH**



Choose Option 4 for cube root

Choose Option 5 for higher root. But be sure to enter the number for the index first!

Example 1: Evaluate.

a. $\sqrt{25}$

$= \pm 5$

$5^2 = (5)(5) = 25$

$(-5)^2 = (-5)(-5) = 25$

b. $\sqrt[3]{-64}$

$= -4$

$(-4)^3 = (-4)(-4)(-4)$

$= -64$

c. $\sqrt[4]{16}$

$= \pm 2$

$2^4 = (2)(2)(2)(2) = 16$

$(-2)^4 = (-2)(-2)(-2)(-2)$

$= 16$

d. $\sqrt[5]{243}$

$= 3$

$(3)^5 = (3)(3)(3)(3)(3)$

$= 243$

A radical with an even index always has two answers (±), and can only have a radicand greater than or equal to 0 inside a radical sign.

A radical with an odd index always has one answer only and can have a negative radicand inside the radical sign.

Example 2: A formula $v_f^2 = v_i^2 + 2ad$ can be used to find the final velocity (speed) of an accelerated object, where v_f = final velocity, v_i = initial velocity, a = acceleration, and d = distance travelled. An apple is thrown from the tall building 300 m high with an initial velocity of 6 m/s. The acceleration due to gravity is 9.81 m/s^2. What is the final velocity of the apple as it reaches the ground?

v_f = ?
v_i = 6 m/s
d = 300 m
a = 9.81 m/s^2

Solve for v_f:

$$v_f^2 = v_i^2 + 2ad$$
$$v_f = \sqrt{v_i^2 + 2ad}$$
$$v_f = \sqrt{(6)^2 + 2(9.81)(300)}$$
$$v_f = \sqrt{36 + 5886}$$
$$v_f = \sqrt{5922}$$

v_f = 76.95 m/s

Example 3: Evaluate using only positive roots.

a. $\sqrt{36-25}$
$= \sqrt{11} \approx$ **3.31662**

b. $\sqrt{36} - \sqrt{25}$
$= 6 - 5 =$ **1**

c. $\sqrt{36 \times 25}$
$= \sqrt{900} =$ **30**

d. $\sqrt{36} \times \sqrt{25}$
$= 6 \times 5 =$ **30**

$$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b} \qquad \sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$
$$\sqrt{a-b} \neq \sqrt{a} - \sqrt{b} \qquad \sqrt{a \div b} = \sqrt{a} \div \sqrt{b}$$

Example 4: Evaluate using only positive roots. Verify by using a calculator.

a. $5\sqrt[3]{-64} + 2\sqrt[3]{27}$
$= 5(-4) + 2(3) = -20 + 6$
-14

```
5³√(-64)+2³√(27)
                -14
```

b. $\sqrt[4]{81} - 7\sqrt[4]{16}$
$= 3 - 7(2) = 3 - 14$
-11

```
4ˣ√(81)-7(4ˣ√16)
                -11
```

<u>Properties of Radicals</u>

Distribution of Radicals of the Same Index (where $a \geq 0$ and $b \geq 0$ if n is even)	$\sqrt[n]{ab} = \left(\sqrt[n]{a}\right)\left(\sqrt[n]{b}\right)$ $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$
Power Rule of Radicals = Multiplying Exponents	$\sqrt[m]{\sqrt[n]{a}} = \sqrt[(m \times n)]{a}$
Reverse Operations of Radicals and Exponents	$\sqrt[n]{a^n} = a$ **(if n is odd)** $\sqrt[n]{a^n} = \lvert a \rvert$ **(if n is even)**

<u>Entire Radicals</u>: - radicals that have no coefficient in front of them. Examples: $\sqrt{52}$ and $\sqrt[3]{48}$

<u>Mixed Radicals</u>: - radicals that have coefficients in front of them. Examples: $2\sqrt{13}$ and $2\sqrt[3]{6}$
- the coefficient is the n^{th} root of the radicand's perfect n^{th} factor.

To convert an entire radical to a mixed radical, find the <u>largest perfect n^{th} factor</u> of the radicand and <u>write its root as a coefficient</u> follow by the radicand factor that remains.

Example 5: Simplify. (Convert them to mixed radicals.)

a. $\sqrt[3]{192x^6y^5}$

$= \sqrt[3]{64}\sqrt[3]{x^6}\sqrt[3]{y^3}\left(\sqrt[3]{2y^2}\right)$

$= 4\sqrt[3]{\left(x^2\right)^3}\,y\left(\sqrt[3]{2y^2}\right)$

$4x^2y\sqrt[3]{2y^2}$

b. $\sqrt[4]{48a^9b^4}$

$= \sqrt[4]{16}\sqrt[4]{a^8}\sqrt[4]{b^4}\left(\sqrt[4]{3a}\right)$

$= 2\sqrt[4]{\left(a^2\right)^4}\,|b|\left(\sqrt[4]{3a}\right)$

$2a^2b\sqrt[4]{3a}$

c. $\dfrac{\sqrt{168p^7q^9}}{\sqrt{6p^2q^6}}$

$= \sqrt{\dfrac{168p^7q^9}{6p^2q^6}} = \sqrt{28p^5q^3}$

$= \sqrt{4}\sqrt{\left(p^2\right)^2}\sqrt{q^2}\left(\sqrt{7pq}\right)$

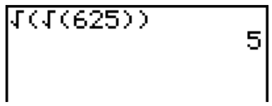
$2p^2q\sqrt{7pq}$

Example 6: Evaluate using only positive roots.

a. $\sqrt{\sqrt{625}}$

$= \sqrt[(2\times2)]{625}$

$= \sqrt[4]{625}$ $\quad 5$



b. $\sqrt{\sqrt[3]{729c^5d^6}}$

$= \sqrt[(2\times3)]{729c^5d^6} = \sqrt[6]{729c^5d^6}$

$= \sqrt[6]{729}\sqrt[6]{c^5}\sqrt[6]{d^6}$ $\quad 3d\sqrt[6]{c^5}$

To convert a mixed radical to an entire radical, raise the coefficient to the index, n^{th} power, and multiply the result to the radicand.

Example 7: Write the followings as entire radicals.

a. $5x^3\sqrt{8}$

$= \sqrt{\left(5x^3\right)^2(8)}$

$= \sqrt{\left(25x^6\right)(8)}$

$\sqrt{200x^6}$

b. $-4ab^2\sqrt[3]{7b^2}$

$= \sqrt[3]{\left(-4ab^2\right)^3\left(7b^2\right)}$

$= \sqrt[3]{\left(-64a^3b^6\right)\left(7b^2\right)}$

$\sqrt[3]{-448a^3b^8}$

Example 8: Order $9\sqrt{2}$, $5\sqrt{3}$, and $4\sqrt{13}$ from least to greatest.

$9\sqrt{2} = \sqrt{81\times2} = \sqrt{162}$

$5\sqrt{3} = \sqrt{25\times3} = \sqrt{75}$

$4\sqrt{13} = \sqrt{16\times13} = \sqrt{208}$

$\sqrt{75} < \sqrt{162} < \sqrt{208}$

$5\sqrt{3} < 9\sqrt{2} < 4\sqrt{13}$

Adding and Subtracting Radicals:

- Radicals can be added or subtracted ***if and only if*** they have the **same index and radicand**.
- Convert any entire radicals into mixed radicals first. Then, combine like terms (radicals with the same radicand) by adding or subtracting their coefficients.

Example 9: Simplify.

a. $\sqrt{32} - \sqrt{108} + \sqrt{27} - \sqrt{50}$

$= 4\sqrt{2} - 6\sqrt{3} + 3\sqrt{3} - 5\sqrt{2}$

$= 4\sqrt{2} - 5\sqrt{2} - 6\sqrt{3} + 3\sqrt{3}$

$-\sqrt{2} - 3\sqrt{3}$

b. $-3\sqrt[3]{24} + 2\sqrt[3]{40} - \sqrt[3]{375} + 3\sqrt[3]{135}$

$= -3\left(2\sqrt[3]{3}\right) + 2\left(2\sqrt[3]{5}\right) - 5\sqrt[3]{3} + 3\left(3\sqrt[3]{5}\right)$

$= -6\sqrt[3]{3} + 4\sqrt[3]{5} - 5\sqrt[3]{3} + 9\sqrt[3]{5}$

$-11\sqrt[3]{3} + 13\sqrt[3]{5}$

Rationalization: - turning radical denominator into a natural number denominator.

$$\textbf{For } m < n, \quad \frac{\sqrt[n]{a}}{\sqrt[n]{b^m}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b^m}} \times \left(\frac{\sqrt[n]{b^{(n-m)}}}{\sqrt[n]{b^{(n-m)}}}\right) = \frac{\sqrt[n]{ab^{(n-m)}}}{b}$$

Example 10: Simplify.

a. $\sqrt{\frac{8}{3}}$

$$= \frac{\sqrt{8}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{24}}{3} = \frac{\sqrt{4} \times \sqrt{6}}{3}$$

$$\frac{2\sqrt{6}}{3}$$

b. $\frac{2\sqrt[5]{4}}{\sqrt[5]{x^3}}$

$$= \frac{2\sqrt[5]{4}}{\sqrt[5]{x^3}} \times \left(\frac{\sqrt[5]{x^{(5-3)}}}{\sqrt[5]{x^{(5-3)}}}\right)$$

$$= \frac{2\sqrt[5]{4}}{\sqrt[5]{x^3}} \times \left(\frac{\sqrt[5]{x^2}}{\sqrt[5]{x^2}}\right)$$

$$= \frac{2\sqrt[5]{4x^2}}{\sqrt[5]{x^5}} \qquad \frac{2\sqrt[5]{4x^2}}{x}$$

Rational Exponents

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

The index of the radical is the denominator of the fractional exponent.

Example 11: Evaluate using a calculator.

a. $(-3)^{3.2}$

```
(-3)^3.2
          33.63473537
```

b. $12^{\frac{4}{3}}$

```
12^(4/3)
          27.47314182
```

c. $\left(\frac{-27}{8}\right)^{\frac{-2}{3}}$

```
(-27/8)^(-2/3)
          .4444444444
Ans▸Frac
                  4/9
```

2nd | ANS | (−)

Example 12: Simplify using only positive exponents.

a. $\sqrt{2a^3}$

$$= \left(2a^3\right)^{\frac{1}{2}}$$

$$2^{\frac{1}{2}}a^{\frac{3}{2}}$$

b. $\left(\sqrt[4]{x^3y^2}\right)^{-5}$

$$= \frac{1}{\left(\sqrt[4]{x^3y^2}\right)^5} = \frac{1}{\left(x^3y^2\right)^{\frac{5}{4}}} \qquad \frac{1}{x^{\frac{15}{4}}y^{\frac{5}{2}}}$$

c. $\sqrt[4]{\sqrt{256x^9}}$

$$= \sqrt[(4\times2)]{256x^9} = \left(256x^9\right)^{\frac{1}{8}}$$

$$= \left(256^{\frac{1}{8}}\right)\left(x^{\frac{9}{8}}\right) \qquad 2x^{\frac{9}{8}}$$

d. $\left(81a^{\frac{2}{5}}b^{-\frac{3}{2}}\right)^{-\frac{1}{4}}$

$$= \frac{1}{\left(81a^{\frac{2}{5}}b^{-\frac{3}{2}}\right)^{\frac{1}{4}}}$$

$$= \frac{1}{81^{\frac{1}{4}}a^{\left(\frac{2}{5}\times\frac{1}{4}\right)}b^{\left(-\frac{3}{2}\times\frac{1}{4}\right)}} = \frac{1}{3a^{\frac{1}{10}}b^{-\frac{3}{8}}} \qquad \frac{b^{\frac{3}{8}}}{3a^{\frac{1}{10}}}$$

e. $\left(\sqrt{x^5}\right)\left(\sqrt[4]{x^{-3}}\right)$

$$= \left(x^{\frac{5}{2}}\right)\left(x^{-\frac{3}{4}}\right) = x^{\frac{5}{2}+\left(-\frac{3}{4}\right)} = x^{-\frac{11}{20}} \qquad x^{\frac{7}{4}}$$

P-4 Assignment: pg. 33–35 #3, 11, 15, 17, 23, 27, 35, 39, 43, 49, 53, 57, 61, 65; Honours: #74

P-5: Algebraic Expressions

Expressions: - mathematical sentences with no equal sign. **Example**: $3x + 2$

Equations: - mathematical sentences that are equated with an equal sign. **Example**: $3x + 2 = 5x + 8$

Terms: - are separated by an addition or subtraction sign.
- each term begins with the sign preceding the variable or coefficient.

Monomial: - one term expression. **Example** $5x^2$

Numerical Coefficient

Exponent

Variable

Binomial: - two terms expression. **Example**: $5x^2 + 5x$

Trinomial: - three terms expression. **Example**: $x^2 + 5x + 6$

Polynomial: - many terms (more than one) expression with whole number exponents.

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_1 x + a_0$$

where $a_0, a_1, a_2, \ldots a_n$ are real number coefficients, and n is a whole number exponents to the n^{th} degree.

Degree: - the term of a polynomial that contains the largest sum of exponents

Example: $9x^5 + 4x^7 + 3x^4$ 7^{th} Degree Polynomial

Example 1: Fill in the table below.

Polynomial	Number of Terms	Classification	Degree	Classified by Degree
9	1	monomial	0	constant
$4x$	1	monomial	1	linear
$9x + 2$	2	binomial	1	linear
$x^2 - 4x + 2$	3	trinomial	2	quadratic
$2x^3 - 4x^2 + x + 9$	4	polynomial	3	cubic
$4x^4 - 9x + 2$	3	trinomial	4	quartic

Like Terms: - terms that have the same variables and exponents.

Examples: $2x^2y$ and $5x^2y$ are like terms $2x^2y$ and $5xy^2$ are NOT like terms

To Add and Subtract Polynomials:
- Combine like terms by adding or subtracting their numerical coefficients.

Example 2: Simplify.

a. $3x^2 + 5x - x^2 + 4x - 6$

$= \underline{3x^2} + \underline{5x} - \underline{x^2} + \underline{4x} - 6$

$\mathbf{2x^2 + 9x - 6}$

b. $(9x^2y^3 + 4x^3y^2) + (3x^3y^2 - 10x^2y^3)$

$= \underline{9x^2y^3} + \underline{4x^3y^2} + \underline{3x^3y^2} - \underline{10x^2y^3}$

$= \mathbf{-x^2y^3 + 7x^3y^2}$

c. $(9x^2y^3 + 4x^3y^2) - (3x^3y^2 - 10x^2y^3)$

$= \underline{9x^2y^3} + \underline{4x^3y^2} - \underline{3x^3y^2} + \underline{10x^2y^3}$

$= \mathbf{19x^2y^3 + x^3y^2}$

(drop brackets and switch signs in the bracket that had – sign in front of it)

Multiplying Monomials with Polynomials

Example 3: Simplify.

a. $2x(3x^2 + 2x - 4)$

$$= 2x\,(3x^2 + 2x - 4)$$
$$= 6x^3 + 4x^2 - 8x$$

b. $3x(5x + 4) - 4(x^2 - 3x)$

$$= 3x\,(5x + 4) - 4\,(x^2 - 3x)$$
$$= 15x^2 + 12x - 4x^2 + 12x$$
$$= 11x^2 + 24x$$

(only multiply brackets right after the monomial)

c. $8(a^2 - 2a + 3) - 4 - (3a^2 + 7)$

$$= 8\,(a^2 - 2a + 3) - 4 - (3a^2 + 7)$$
$$= 8a^2 - 16a + 24 - 4 - 3a^2 - 7$$
$$= 5a^2 - 16a + 13$$

Multiplying Polynomials with Polynomials

Example 4: Simplify.

a. $(3x + 2)(4x - 3)$

$$= (3x + 2)\,(4x - 3)$$
$$= 12x^2 - 9x + 8x - 6$$
$$= 12x^2 - x - 6$$

b. $(x + 3)(2x^2 - 5x + 3)$

$$= (x + 3)\,(2x^2 - 5x + 3)$$
$$= 2x^3 - 5x^2 + 3x + 6x^2 - 15x + 9$$
$$= 2x^3 + x^2 - 12x + 9$$

c. $3(x + 2)(2x + 3) - (2x - 1)(x + 3)$

$$= 3\,(x + 2)\,(2x + 3) - (2x - 1)\,(x + 3)$$
$$= 3\,(2x^2 + 3x + 4x + 6) - (2x^2 + 6x - x - 3)$$
$$= 3\,(2x^2 + 7x + 6) - (2x^2 + 5x - 3)$$
$$= 6x^2 + 21x + 18 - 2x^2 - 5x + 3$$
$$= 4x^2 + 16x + 21$$

d. $(x^2 - 2x + 1)(3x^2 + x - 4)$

$$= (x^2 - 2x + 1)\,(3x^2 + x - 4)$$
$$= 3x^4 + x^3 - 4x^2 - 6x^3 - 2x^2 + 8x + 3x^2 + x - 4$$
$$= 3x^4 - 5x^3 - 3x^2 + 9x - 4$$

Special Products

$(A + B)(A - B) = A^2 - B^2$

$(A + B)^2 = A^2 + 2AB + B^2$

$(A + B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$

$(A - B)^2 = A^2 - 2AB + B^2$

$(A - B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$

Example 5: Simplify.

a. $(2x + 3)^2$

Let $A = 2x$ and $B = 3$

$$(A + B)^2 = A^2 + 2AB + B^2$$
$$(2x + 3)^2 = (2x)^2 + 2(2x)(3) + (3)^2$$
$$= 4x^2 + 12x + 9$$

$$= (2x + 3)\,(2x + 3)$$
$$= 4x^2 + 6x + 6x + 9$$
$$= 4x^2 + 12x + 9$$

b. $(3x - 4)^3$

Let $A = 3x$ and $B = 4$

$$(A - B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$$
$$(3x - 4)^3 = (3x)^3 - 3(3x)^2(4) + 3(3x)(4)^2 - (4)^3$$
$$= 27x^3 - 108x^2 + 144x - 64$$

P-5 Assignment: pg. 39–40 #17, 21, 27, 31, 33, 37, 41, 47, 57, 61; Honours: #60

P-6: Factoring (Part 1)

Factoring: - a reverse operation of expanding (multiplying).
- in essence, we are dividing, with the exception that the factors can be polynomials.

Common Factors: - factors that are common in each term of a polynomial.

a. **Numerical GCF**: - Greatest Common Factor of all numerical coefficients and constant.
b. **Variable GCF**: - the lowest exponent of a particular variable.

After obtaining the GCF, use it to divide each term of the polynomial for the remaining factor.

Factor by Grouping (Common Brackets as GCF)

$$a\,(c+d) + b\,(c+d) = (c+d)\,(a+b)$$

Common Brackets **Take common bracket out as GCF**

Example 1: Factor each expression.

a. $4a^2b - 8ab^2 + 6ab$

$= \mathbf{2ab(2a - 4b + 3)}$ GCF $= 2ab$

b. $3x(2x - 1) + 4(2x - 1)$

$= \mathbf{(2x - 1)(3x + 4)}$ GCF $= (2x - 1)$

c. $2ab + 3ac + 4b^2 + 6bc$

$= (2ab + 3ac) + (4b^2 + 6bc)$

$= a\,(2b + 3c) + 2b\,(2b + 3c)$ GCF $= (2b + 3c)$

$= \mathbf{(2b + 3c)(a + 2b)}$

d. $3x^2 - 6y^2 + 9x - 2xy^2$

$= (3x^2 - 6y^2) + (9x - 2xy^2)$

$= 3\,(x^2 - 2y^2) + x\,(9 - 2y^2)$

Brackets are NOT the same! We might have to first rearrange terms.

Try again after rearranging terms!

$= 3x^2 + 9x - 2xy^2 - 6y^2$

$= (3x^2 + 9x) - (2xy^2 + 6y^2)$

$= 3x\,(x + 3) - 2y^2\,(x + 3)$

$= \mathbf{(x + 3)(3x - 2y^2)}$

Switch Sign in Second Bracket!
We have put a minus sign in front of a new bracket!

Factoring $x^2 + bx + c$ (Leading Coefficient is 1)

$$x^2 + bx + c$$

What two numbers multiply to give c, but add up to be b?

Example 2: Completely factor each expression.

a. $x^2 - 3x - 10$

$= \mathbf{(x + 2)(x - 5)}$

Factor Pairs of −10:

(-1×10)	(1×-10)
(-2×5)	$\mathbf{(2 \times -5)}$

$(2 + -5) =$ sum of -3

b. $a^2 - 8a + 15$

$= \mathbf{(a - 3)(a - 5)}$

Factor Pairs of 15:

(1×15)	(-1×-15)
(3×5)	$\mathbf{(-3 \times -5)}$

$(-3 + -5) =$ sum of -8

c. $x^2 - 7xy + 12y^2$

$= (x - 3y)(x - 4y)$

Factor Pairs of 12:
(1 × 12) (−1 × −12)
(2 × 6) (−2 × −6)
(3 × 4) **(−3 × −4)**

(−3) + (−4) = sum of −7

d. $14 - 5w - w^2$

$= -w^2 - 5w + 14$ Rearrange in Descending Degree.
$= -(w^2 + 5w - 14)$ Take out −1 as common factor.
$= -(w + 7)(w - 2)$

(+7)(−2) = −14
(+7) + (−2) = 5

e. $3ab^2 - 3ab - 60a$

$= 3a(b^2 - b - 20)$ Take out GCF
$= 3a(b + 4)(b - 5)$

(+4)(−5) = −20
(+4) + (−5) = −1

f. $x^4 + 14x^2 - 32$

$= (x^2 + 16)(x^2 - 2)$

(+16)(−2) = −32
(+16) + (−2) = 14

Assume $x^4 + bx^2 + c$ as the same as $x^2 + bx + c$ and factor. The answer will be $(x^2\quad)(x^2\quad)$.

Factoring $ax^2 + bx + c$ (Leading Coefficient is not 1, a ≠ 1)

For factoring trinomial with the form $ax^2 + bx + c$, we will have to factor by grouping.

Example 3: Factor $6x^2 + 11x + 4$

$6x^2 + \mathbf{11x} + 4$ Multiply a and c.

Factor Pairs of 24:
(1 × 24) (−1 × −24)
(2 × 12) (−2 × −12)
(3 × 8) (−3 × −8)
(4 × 6) (−4 × −6)

(3 + 8) = sum of 11

$= 6x^2 + \mathbf{3x} + \mathbf{8x} + 4$ **Split the *bx* term into two separate terms.**
$= (6x^2 + 3x) + (8x + 4)$ Group by brackets
$= 3x(2x + 1) + 4(2x + 1)$ Take out GCF for each bracket.
$= (2x + 1)(3x + 4)$ Factor by Common Bracket!

Example 4: Factor completely.

a. $6x^3 - 14x^2 + 4x$

$= 2x(3x^2 - 7x + 2)$ GCF = $2x$
$= 2x(3x^2 - x - 6x + 2)$
$= 2x[(3x^2 - x) - (6x - 2)]$
$= 2x[x(3x - 1) - 2(3x - 1)]$
$= 2x(3x - 1)(x - 2)$

(−1)(−6) = 6
(−1) + (−6) = −7

switch sign!
(− sign in front of bracket)

b. $8m^2 - 6mn - 9n^2$

$= 8m^2 + 6mn - 12mn - 9n^2$
$= (8m^2 + 6mn) - (12mn + 9n^2)$
$= 2m(4m + 3n) - 3n(4m + 3n)$
$= (4m + 3n)(2m - 3n)$

8 × −9 = −72
(6)(−12) = −72
(6) + (−12) = −6

switch sign!
(− sign in front of bracket)

c. $4(3x - 2)^2 + 13(3x - 2) + 9$

$4(3x - 2)^2 + 13(3x - 2) + 9$ **Let $A = (3x - 2)$**
$= 4A^2 + 13A + 9$
$= 4A^2 + 4A + 9A + 9$
$= (4A^2 + 4A) + (9A + 9)$
$= 4A(A + 1) + 9(A + 1)$
$= (A + 1)(4A + 9)$
$= [(3x - 2) + 1][4(3x - 2) + 9]$ **Substitute $(3x - 2)$ back into A**
$= (3x - 1)(12x + 1)$

4 × 9 = 36
(4)(9) = 36
(4) + (9) = 13

d. $18x^4 - 27x^2y + 4y^2$

$= 18x^4 - 3x^2y - 24x^2y + 4y^2$
$= (18x^4 - 3x^2y) - (24x^2y - 4y^2)$
$= 3x^2(6x^2 - y) - 4y(6x^2 - y)$
$= (6x^2 - y)(3x^2 - 4y)$

18 × 4 = 72
(−3)(−24) = 72
(−3) + (−24) = 72

switch sign!
(− sign in front of bracket)

P-6 (Part 1) Assignment: pg. 46–47 #5, 9, 13, 15, 37, 43

P-6: Factoring (Part 2)

Special Expressions

Difference of Squares	$A^2 - B^2 = (A + B)(A - B)$
Perfect Trinomial Squares	$A^2 + 2AB + B^2 = (A + B)^2$
Perfect Trinomial Squares	$A^2 - 2AB + B^2 = (A - B)^2$
Sum of Cubes	$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$
Difference of Cubes	$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

Example 1: Factor completely.

a. $x^2 + 9$

(NOT Factorable Sum of Squares)

b. $3x^2 - 300$

$= 3(x^2 - 100)$ GCF = 3

$= \mathbf{3(x - 10)(x + 10)}$

c. $x^4 - 81$

$= (x^2 - 9)(x^2 + 9)$

$= \mathbf{(x - 3)(x + 3)(x^2 + 9)}$

d. $9x^2 - 64y^2$

$= \mathbf{(3x - 8y)(3x + 8y)}$

e. $(2x + 3)^2 - (3x - 1)^2$

$= [(2x + 3) - (3x - 1)]\ [(2x + 3) + (3x - 1)]$ Look at $(2x + 3)$ and $(3x - 1)$ as individual items!

Watch Out!
Subtracting a bracket!

$= [-x + 4]\ [5x + 2]$

$= \mathbf{-(x - 4)(5x + 2)}$ Take out negative sign from the first bracket!

Perfect Trinomial Square

$$ax^2 + bx + c = (\sqrt{a}x + \sqrt{c})^2$$
$$ax^2 - bx + c = (\sqrt{a}x - \sqrt{c})^2$$

where a, c are square numbers, and $b = 2(\sqrt{a})(\sqrt{c})$

Example 2: Expand $(3x + 2)^2$.

$(3x + 2)^2 = (3x + 2)(3x + 2)$
$= 9x^2 + 6x + 6x + 4$
$= \mathbf{9x^2 + 12x + 4}$

$\boxed{\sqrt{9} = 3}$ $\boxed{2(\sqrt{9})(\sqrt{4}) = 12}$ $\boxed{\sqrt{4} = 2}$

Example 3: Factor completely.

a. $9x^2 + 30x + 25$

$9x^2$ $+ 30x$ $+ 25$

$\boxed{\sqrt{9} = 3}$ $\boxed{2(\sqrt{9})(\sqrt{25}) = 30}$ $\boxed{\sqrt{25} = 5}$

$= \mathbf{(3x + 5)^2}$

b. $4x^2 - 28x + 49$

$4x^2$ $-28x$ $+ 49$

$\boxed{\sqrt{4} = 2}$ $\boxed{-2(\sqrt{4})(\sqrt{49}) = -28}$ $\boxed{\sqrt{49} = 7}$

$= \mathbf{(2x - 7)^2}$

c. $x^6 - 20x^3 + 100$

Assumes $x^6 + bx^3 + c$ is the same as $x^2 + bx + c$.
But the answer will be in the form of $(x^3 + \quad)(x^3 + \quad)$.

x^6 $-20x^3$ $+ 100$

$\boxed{\sqrt{x^6} = x^3}$ $\boxed{-2(\sqrt{x^6})(\sqrt{100}) = -20x^3}$ $\boxed{\sqrt{100} = 10}$

$= \mathbf{(x^3 - 10)^2}$

Sum of Cubes	$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$
Difference of Cubes	$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

Example 4: Factor completely.

a. $27x^3 + 8y^3$

Let $A^3 = 27x^3$ and $B^3 = 8y^3$
Hence, $A = 3x$ and $B = 2y$

$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$
$$27x^3 + 8y^3 = (3x + 2y)((3x)^2 - (3x)(2y) + (2y)^2)$$
$$= \mathbf{(3x + 2y)(9x^2 - 6xy + 4y^2)}$$

b. $9a^3b - 72b$

$9a^3b - 72b = 9b(a^3 - 8)$ GCF = $9b$

Let $A^3 = a^3$ and $B^3 = 8$
Hence, $A = a$ and $B = 2$

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$
$$a^3 - 8 = (a + 2)((a)^2 + (a)(2) + (2)^2)$$
$$9b(a^3 - 8) = \mathbf{9b(a + 2)(a^2 + 2a + 4)}$$

Factoring Non-Polynomial Expressions
- always take out the GCF with the lowest exponents of any common variables.
- divide each term by the GCF. Be careful with fractional exponents.

Example 5: Factor completely.

a. $y^{\frac{4}{3}} - 5y^{\frac{1}{3}} - 24y^{-\frac{2}{3}}$

$$\frac{y^{\frac{4}{3}}}{y^{-\frac{2}{3}}} = y^{\frac{4}{3}-\left(-\frac{2}{3}\right)} = y^2$$

$$\frac{-5y^{\frac{1}{3}}}{y^{-\frac{2}{3}}} = -5\,y^{\frac{1}{3}-\left(-\frac{2}{3}\right)} = -5y$$

$= y^{-\frac{2}{3}}(y^2 - 5y - 24)$ GCF = $y^{-\frac{2}{3}}$ (lowest exponent)

$= \mathbf{y^{-\frac{2}{3}}(y - 8)(y + 3)}$ Factor form $x^2 + bx + c$

b. $r(4r+1)^{\frac{1}{2}} - 3(4r+1)^{-\frac{1}{2}}$

$$\frac{rA^{\frac{1}{2}}}{A^{-\frac{1}{2}}} = r\,A^{\frac{1}{2}-\left(-\frac{1}{2}\right)} = rA \qquad \frac{3A^{-\frac{1}{2}}}{A^{-\frac{1}{2}}} = -5\,A^{-\frac{1}{2}-\left(-\frac{1}{2}\right)} = 3$$

Let $A = (4r + 1)$

$r(4r+1)^{\frac{1}{2}} - 3(4r+1)^{-\frac{1}{2}} = rA^{\frac{1}{2}} - 3A^{-\frac{1}{2}}$

$= A^{-\frac{1}{2}}[rA - 3]$ GCF = $A^{-\frac{1}{2}}$ (lowest exponent)

$= (4r+1)^{-\frac{1}{2}}[r(4r + 1) - 3]$ Substitute $(4r + 1)$ back into A

$= (4r+1)^{-\frac{1}{2}}[4r^2 + r - 3]$ Factor form $ax^2 + bx + c$

$= \mathbf{(4r+1)^{-\frac{1}{2}}(4r - 3)(r + 1)}$

Factoring Cubic Polynomials by Grouping
- for cubic polynomials consists of four terms, we can ***sometimes*** factor them by grouping.

Example 6: Factor $x^3 - 5x^2 - 4x + 20$ completely.

$= (x^3 - 5x^2) - (4x - 20)$ switch sign! (– sign in front of bracket)

$= x^2(x - 5) - 4(x - 5)$ Factor GCF from each group

$= (x - 5)(x^2 - 4)$ GCF = $(x - 5)$

$= \mathbf{(x - 5)(x + 2)(x - 2)}$ Factor Difference of Squares

P-6 (Part 2) Assignment: pg. 46–48 #17, 19, 21, 25, 29, 33, 47, 51, 53, 57, 61, 65, 69, 79, 93, 98a and 98c; Honours: #71, 75

P-7: Rational Expressions

Fractional Expression: - a quotient of two algebraic expressions.
- the variable(s) can have negative and fractional exponents (or in radical form).

Examples: $\dfrac{2x^2+3}{x^2+4x-2}$ $\dfrac{\sqrt{x}+6}{\sqrt{x^2-4}}$ $\dfrac{x^{3/2}+2x^{1/2}-3x^{-1/2}}{x+2}$

Rational Expression: - fractional expressions with polynomials as denominator and / or numerator.

Examples: $\dfrac{2x^2+3}{x^2+4x-2}$ $\dfrac{x^3+4x^2-6x+7}{3x-1}$ $\dfrac{7x}{3x^2-8x+2}$

Domain: - all possible x-values from an algebraic expression.
- some algebraic expressions have a certain "***no go zone***". This might involve ***not being able to divide by zero*** or ***x has to be positive because it is in an even indexed radical***.

Examples: $\dfrac{1}{x}$ Domain is $\{x \mid x \neq 0\}$ $\sqrt{x}$ Domain is $\{x \mid x \geq 0\}$ $\dfrac{1}{\sqrt{x}}$ Domain is $\{x \mid x > 0\}$

Example 1: Find the domain of the following expressions.

a. $2x^2-4x+7$

There is *no restriction* on x as *x can be anything in the real number set*. Hence, the domain is $x \in \Re$.

b. $\dfrac{x+3}{x^2-x-12}$

Since there is a **polynomial** expression in the **denominator**, we need to **solve** it when it is **not equal to zero by factoring** to find the domain.

$$x^2-x-12 \neq 0$$
$$(x-3)(x+4) \neq 0$$
$$x-3 \neq 0 \text{ or } x+4 \neq 0$$

Domain: $x \neq 3$ or $x \neq -4$

c. $\dfrac{\sqrt{x}}{2x-5}$

We need to find the **domain of the numerator (radical)** as well as the **denominator (polynomial)**.

$2x-5 \neq 0$

$x \neq \dfrac{5}{2}$

For $\sqrt{x}$, $x \geq 0$

Combine Domain

Domain: $x \geq 0$ and $x \neq \dfrac{5}{2}$

Simplifying Rational Expressions:
- factor both the numerator and denominator and cancel out the common factors / brackets between them.
- this is similar to reducing a numerical fraction by cancelling out the common factors between the numerator and denominator.
- **the final domain is the domain of the original rational expression**, not the domain of the reduced form.

Examples: $\dfrac{30}{24} = \dfrac{\cancel{6}\times 5}{\cancel{6}\times 4} = \dfrac{5}{4}$ $\dfrac{x^2-6x+9}{x^2-9} = \dfrac{\cancel{(x-3)}(x-3)}{\cancel{(x-3)}(x+3)} = \dfrac{(x-3)}{(x+3)}$ **Domain: $x \neq 3$ or $x \neq -3$**

Note: we **cannot** cancel $\dfrac{x-3}{x+3} \neq \dfrac{\cancel{x}-3}{\cancel{x}+3} \to -1$

This is because $\dfrac{x-3}{x+3}$ really means $\dfrac{(x-3)}{(x+3)}$ and we have do the parenthesis first before division.

Example 2: Simplify the following expressions and state their domains.

a. $\dfrac{3x}{x^2+6x}$

$= \dfrac{3\cancel{x}}{\cancel{x}(x+6)}$

$= \dfrac{3}{x+6}$

$x^2 + 6x \neq 0$
$x(x+6) \neq 0$
$x \neq 0$ or $x + 6 \neq 0$
Domain: $x \neq 0$ or $x \neq -6$

b. $\dfrac{2x^2-7x+3}{x^2-5x+6}$

$= \dfrac{\cancel{(x-3)}(2x-1)}{(x-2)\cancel{(x-3)}}$

$= \dfrac{(2x-1)}{(x-2)}$

$x^2 - 5x + 6 \neq 0$
$(x-2)(x-3) \neq 0$
$x - 2 \neq 0$ or $x - 3 \neq 0$
Domain: $x \neq 2$ or $x \neq 3$

Multiplying and Dividing Rational Expressions:

- much like multiplying and dividing fractions, we factor all numerators and denominators and reduce common bracket(s) / factors between them.
- for division, we must "flip" (take the reciprocal) of the fraction behind the ÷ sign.
- **the final domain is the domain of both the original rational expressions**, not the domain of the reduced answer.

Examples: $\dfrac{3}{5} \times \dfrac{10}{21} = \dfrac{\cancel{3}}{\cancel{5}} \times \dfrac{\cancel{5}\times 2}{\cancel{3}\times 7} = \dfrac{2}{7}$ $\quad \dfrac{24}{7} \div \dfrac{15}{28} = \dfrac{24}{7} \div \dfrac{28}{15} = \dfrac{8\times\cancel{3}}{\cancel{7}} \times \dfrac{\cancel{7}\times 4}{\cancel{3}\times 5} = \dfrac{32}{5}$

Example 3: Perform the indicated operations, simplify and state their domains.

a. $\dfrac{x^2-1}{x^2+x-6} \times \dfrac{2x^2+7x+3}{2x^2-x-1}$

$= \dfrac{(x+1)\cancel{(x-1)}}{(x-2)\cancel{(x+3)}} \times \dfrac{\cancel{(2x+1)}\cancel{(x+3)}}{\cancel{(2x+1)}\cancel{(x-1)}}$

$= \dfrac{(x+1)}{(x-2)}$

$(x-2) \neq 0$ or $(x+3) \neq 0$
$(2x-1) \neq 0$ or $(x-1) \neq 0$

Domain: $x \neq 2$ or $x \neq -3$ or $x \neq \dfrac{1}{2}$ or $x \neq 1$

b. $\dfrac{x^2+6x+9}{3x^2-4x-4} \div \dfrac{x^2-9}{3x^2+8x+4}$

$= \dfrac{x^2+6x+9}{3x^2-4x-4} \times \dfrac{3x^2+8x+4}{x^2-9}$

$= \dfrac{\cancel{(x+3)}(x+3)}{\cancel{(3x+2)}(x-2)} \times \dfrac{\cancel{(3x+2)}(x+2)}{(x-3)\cancel{(x+3)}}$

$= \dfrac{(x+3)(x+2)}{(x-2)(x-3)}$

Domain is taken from the numerator **and** the denominator of the fraction after the ÷ sign.

$(3x+2) \neq 0$ or $(x-2) \neq 0$
$(3x+2) \neq 0$ or $(x+2) \neq 0$
$(x-3) \neq 0$ or $(x+3) \neq 0$

Domain: $x \neq -\dfrac{2}{3}$, 2, −2, 3 or −3

Lowest Common Denominator (LCM) of Monomials:

- LCD of monomial coefficient, and the variable(s) with its / their **highest exponent(s)**.

Example: LCD of $3a^2$, $5a$, $6a^3$

LCD = $30a^3$

LCM of 3, 5, 6 = 30 Variable with Highest Exponent = a^3

Lowest Common Denominator (LCM) of Polynomials:
- common factor(s) (written once) along with any uncommon (leftover) factor(s).

Example: LCD of $x^2 - 2x - 3$ and $x^2 - x - 2$

Factors of $x^2 - 2x - 3 = (x - 3)\,(x + 1)$ and Factors of $x^2 - x - 2 = (x + 1)\,(x - 2)$

Common Factor Leftovers

LCD = $(x + 1)\,(x - 3)\,(x - 2)$

Adding and Subtracting Rational Expressions:
- much like adding and subtracting fractions, we first find the LCD of the denominators. Then, we convert each fraction into their equivalent fractions before adding or subtracting the numerators.
- **the final domain is the domain of both the original rational expressions**, not the domain of the reduced answer.

Example: $\dfrac{3}{4} + \dfrac{5}{6}$ (LCD = 12) $\dfrac{3\times 3}{4\times 3} + \dfrac{5\times 2}{6\times 2} = \dfrac{9}{12} + \dfrac{10}{12} = \dfrac{19}{12}$

Example 4: Perform the indicated operations, simplify and state their domains.

a. $\dfrac{5}{x+2} + \dfrac{3x+1}{3x+6}$

$$= \frac{5}{x+2} + \frac{3x+1}{3(x+2)} \qquad \text{LCD} = 3(x+2)$$

$$= \frac{(5)(3) + (3x+1)}{3(x+2)} = \frac{15 + 3x + 1}{3(x+2)}$$

$$= \mathbf{\frac{3x+16}{3(x+2)}}$$

$(x + 2) \neq 0$

Domain: $x \neq -2$

b. $\dfrac{2x}{9x^2 - 4} - \dfrac{3x}{9x^2 - 12x + 4}$ LCD = $(3x - 2)\,(3x - 2)\,(3x + 2)$

Common Factor Leftovers

$$= \frac{2x}{(3x-2)(3x+2)} - \frac{3x}{(3x-2)(3x-2)}$$

$$= \frac{(2x)(3x-2) - (3x)(3x+2)}{(3x-2)(3x-2)(3x+2)}$$

$$= \frac{6x^2 - 4x - 9x^2 - 6x}{(3x-2)^2(3x+2)} = \frac{-3x^2 - 10x}{(3x-2)^2(3x+2)}$$

$$= \mathbf{\frac{-x(3x+10)}{(3x-2)^2(3x+2)}}$$

$(3x - 2) \neq 0$ or $(3x + 2) \neq 0$

Domain: $x \neq \dfrac{2}{3}$ or $-\dfrac{2}{3}$

Compound Fraction: - a fraction where the numerator and / or denominator themselves contain fraction(s).

Simplifying Compound Fractions:
- simplify each of the numerator and denominator into single fractions. Then, divide the numerator's fraction by the denominator's fraction.

Example: Simplify $\dfrac{1 + x^{-1}}{1 - x^{-1}}$

$$= \frac{\left(1 + \frac{1}{x}\right)}{\left(1 - \frac{1}{x}\right)} = \frac{\left(\frac{x+1}{x}\right)}{\left(\frac{x-1}{x}\right)} = \left(\frac{x+1}{x}\right) \div \left(\frac{x-1}{x}\right) = \left(\frac{x+1}{x}\right) \times \left(\frac{x}{x-1}\right) = \frac{x+1}{x-1}$$

Example 5: Simplify $\dfrac{y^2 + \frac{y-3}{2}}{y^2 - \frac{5y-2}{3}}$.

$$= \frac{\frac{2(y^2) + y - 3}{2}}{\frac{3(y^2) - (5y-2)}{3}}$$

$$= \frac{\frac{2y^2 + y - 3}{2}}{\frac{3y^2 - 5y + 2}{3}} = \frac{\frac{(2y+3)(y-1)}{2}}{\frac{(3y-2)(y-1)}{3}}$$

$$= \frac{(2y+3)\cancel{(y-1)}}{2} \times \frac{3}{(3y-2)\cancel{(y-1)}} = \mathbf{\frac{3(2y+3)}{2(3y-2)}}$$

Conjugates: - binomials that have the exact same terms by opposite signs in between.

Examples: $(a + b)$ and $(a - b)$ $\qquad$ $\left(a\sqrt{b} + c\sqrt{d}\right)$ and $\left(a\sqrt{b} - c\sqrt{d}\right)$

Multiplying Conjugate Radicals:
- multiplying conjugate radicals will **always** give a **Rational Number** (radical terms would cancel out).

Example 6: Simplify $\left(\sqrt{5} + 3\sqrt{6}\right)\left(\sqrt{5} - 3\sqrt{6}\right)$.

$$= \left(\sqrt{5} + 3\sqrt{6}\right)\left(\sqrt{5} - 3\sqrt{6}\right)$$

$$= \sqrt{25} - 3\sqrt{30} + 3\sqrt{30} - 9\sqrt{36}$$

$$= 5 - 9(6) = -49$$

Notice the middle two radical terms always cancel out!

Rationalizing Binomial Radical Denominator:
- multiply the radical expression by a fraction that consists of the conjugate of the denominator over itself.

Example 7: Simplify $\dfrac{3}{5+\sqrt{7}}$.

$$= \frac{3}{\left(5+\sqrt{7}\right)} \times \frac{\left(5-\sqrt{7}\right)}{\left(5-\sqrt{7}\right)} = \frac{3\left(5-\sqrt{7}\right)}{25 - 5\sqrt{7} + 5\sqrt{7} - \sqrt{49}}$$

$$= \frac{3\left(5-\sqrt{7}\right)}{25-7} = \frac{3\left(5-\sqrt{7}\right)}{18} = \mathbf{\frac{\left(5-\sqrt{7}\right)}{6}}$$

P-7 Assignment: pg. 55–57 #9, 15, 17, 21, 25, 29, 31, 39, 45, 51, 55, 59, 77, 81 and 99; Honours: #97