Unit 4: Finance and Spreadsheet

4-1A: Investing Money

Compound Interest: - interest is incurred on the **existing** balance.
- usually interest incurs at each term. (A term can be annually, semi-annually, quarterly, monthly, or daily.)
- interest rate is always quoted per annum unless otherwise stated.
- it is used to calculate interest on different types of savings and loans.

Saving: - putting money away to earn interest for a future purpose.
- there are three types of saving.

Three Types of Saving:

1) **Lump Sum**: - an amount of money is put away initially with no additional contributions.

2) **Annuity**: - no initial principal but a specific amount of money is saved every period for a larger sum.

3) **Combination**: - putting away an initial amount of money at first and make addition contributions every period.

Lump Sum Saving/Loan

$$A = P(1 + r)^n$$

A = Final Amount
P = Initial Investment (Principal)
r = rate per term
n = total number of terms

Compound Term	r	n
Annually	$\frac{\text{Annual Interest Rate}}{1}$	Number of Years × 1
Semi-annually	$\frac{\text{Annual Interest Rate}}{2}$	Number of Years × 2
Quarterly	$\frac{\text{Annual Interest Rate}}{4}$	Number of Years × 4
Monthly	$\frac{\text{Annual Interest Rate}}{12}$	Number of Years × 12
Daily	$\frac{\text{Annual Interest Rate}}{365}$	Number of Years × 365

There are many **purposes for saving**:

a) Educational: College Fund or RESP (Registered Education Saving Plan).

b) Insurance: Term Life and Disability Insurance.

c) Retirement: RRSP (Registered Retirement Saving Plan), RSP (Registered Saving Plan), Company Pension Plan.

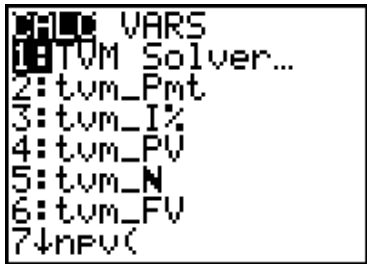
d) Personal Savings: (for a down payment of a purchase, a travel trip, or wedding expenses).

Using the TVM Solver (Time Value Money) by TI-83 Plus

To access the TVM Solver

1. Press APPS

2. Select Option 1



3. Select Option 1 again

N= Number of Years × Number of Payments per year
I%= Interest Rate per year
PV= Principal Value (Present Value)
PMT= Payment Amount per Period
FV= Future Value
P/Y= Number of Payments per Year (minimum value is 1)
C/Y= Number of Compound Period per Year
PMT: END BEGIN (payment made at the end or beginning of the period)

For the TVM Solver:

Money is POSITIVE when it is GOING INTO the pocket.
Money is NEGATIVE when it is LEAVING the pocket.

To solve for any parameters using the TVM Solver

1. **Enter all other known parameters.**
2. **Take the cursor to the parameter that needs to be solved.**

3. Press ALPHA ENTRY SOLVE

ENTER

Example 1: Using the lump sum formula and the TVM Solver, calculate the final amount and the interest earned on the followings.

a. Investment of $2400 at 6%/a compounded annually for 5 years.

$P = \$2400$ $r = 0.06$ $n = 5$

$A = P(1 + r)^n$

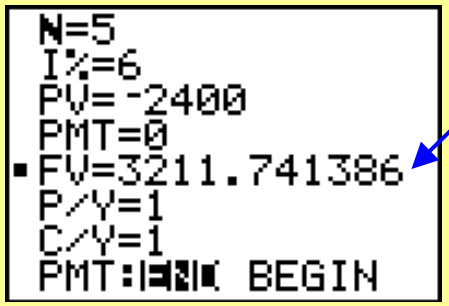
$A = \$2400(1 + 0.06)^5$ **$A = \$3211.74$**

Interest Earned = Final Amount – Principle

$= \$3211.74 - \$2400$

Interest Earned = $811.74

TVM Solver



Press ALPHA ENTRY SOLVE ENTER after entering all other parameters.

b. Investment of $2400 at 6%/a compounded semi-annually for 5 years.

$P = \$2400$ $r = \frac{0.06}{2}$ $n = 5 \times 2 = 10$

$A = P(1 + r)^n$

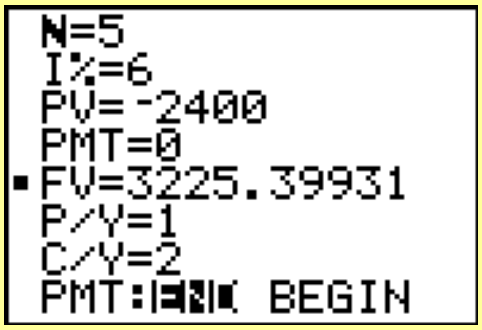
$A = \$2400\left(1 + \frac{0.06}{2}\right)^{10}$ **$A = \$3225.40$**

Interest Earned = Final Amount – Principle

$= \$3225.40 - \$2400$

Interest Earned = $825.40

TVM Solver



c. Investment of $2400 at 6%/a compounded quarterly for 5 years.

$P = \$2400$ $r = \frac{0.06}{4}$ $n = 5 \times 4 = 20$

$A = P(1 + r)^n$

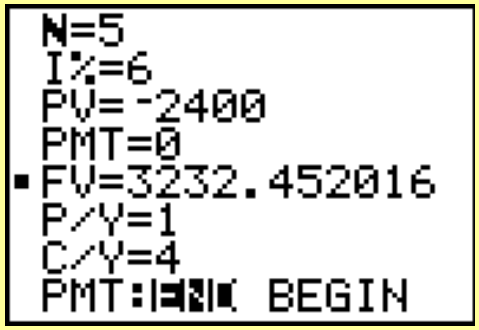
$A = \$2400\left(1 + \frac{0.06}{4}\right)^{20}$ **$A = \$3232.45$**

Interest Earned = Final Amount – Principle

$= \$3232.45 - \$2400$

Interest Earned = $832.45

TVM Solver



d. Investment of $2400 at 6%/a compounded monthly for 5 years

$P = \$2400$ $r = \frac{0.06}{12}$ $n = 5 \times 12 = 60$

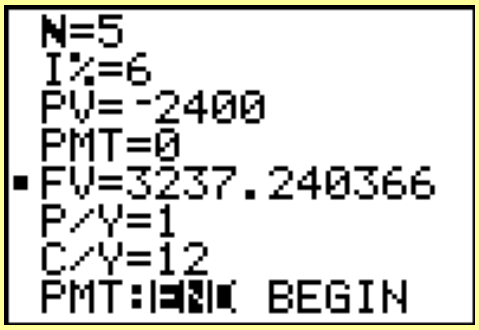
$A = P(1 + r)^n$

$A = \$2400\left(1 + \frac{0.06}{12}\right)^{60}$ **$A = \$3237.24$**

Interest Earned = Final Amount – Principle
= $3237.24 – $2400

Interest Earned = $837.24

TVM Solver

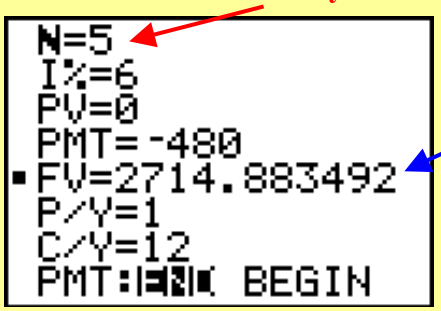


Example 2: Using the TVM Solver, calculate the final amount and the interest earned on the followings.

a. Investment of $480 at the **end** of each year for 5 years at 6%/a compounded monthly.

TVM Solver

N = 5 years × 1 payment /yr



Final Amount = $2714.88

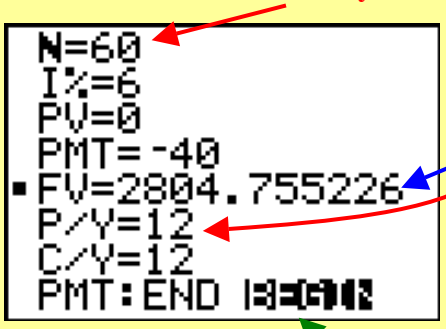
Interest Earned = Final Amount – Principle
= $2714.88 – (5 × $480)

Interest Earned = $314.88

b. Investment of $40 at the **beginning** of each month for 5 years at 6%/a compounded monthly.

TVM Solver

N = 5 years × 12 payments /yr



Final Amount = $2804.76

Interest Earned = Final Amount – Principle
= $2804.76 – (5 × 12 × $40)

Interest Earned = $404.76

Invested at the beginning of each month.

Present Value: - amount of money needed now for regular withdrawals in the future.

Example 3: A scholarship committee would like to set up a scholarship fund of $1000 at the end of every half a year for the next ten years, how much money does the committee need to invest now if the present value gets 8%/a compound monthly. Determine the total interest this present value will accumulate over the ten years.

TVM Solver

N = 10 years × 2 payments /yr

```
N=20
I%=8
PV=-13509.73968
PMT=1000
FV=0
P/Y=2
C/Y=12
PMT:END BEGIN
```

Present Value = $13509.74

No money left in scholarship fund after 10 years

Interest Earned = Total Payouts – Present Value
= (2 × 10 × $1000) – $13509.74

Interest Earned = $6490.26

Scholarship paid at the end of each half a year.

Example 4: Mary won the LOTTO 649 jackpot of $2.2 million. If she invested the entire sum in an account that earns 8%/a compound quarterly, how much can she withdraw at the end of each month for the next 70 years of her life? Determine the amount of interest she will earn.

TVM Solver

N = 70 years × 12 payments /yr

```
N=840
I%=8
PV=-2200000
PMT=14627.12348
FV=0
P/Y=12
C/Y=4
PMT:END BEGIN
```

Monthly Payment = $14627.12

No money left in scholarship fund after 10 years

Interest Earned = Total Payouts – Present Value
= (12 × 70 × $14627.12) – $2,200,000

Interest Earned = $10,086,780.80

Each Withdrawal is at the end of each month.

Rule of 72: - the **number of year it takes to double** a lump sum investment $\approx \dfrac{72}{\text{Annual Interest Rate}}$

Example 5: Determine the number of years it will take to double an initial investment of $2400 at 6%/a compounded annually using the Rule of 72. Verify the answer with the TVM Solver.

$t \approx \dfrac{72}{6}$

$t \approx 12$ years

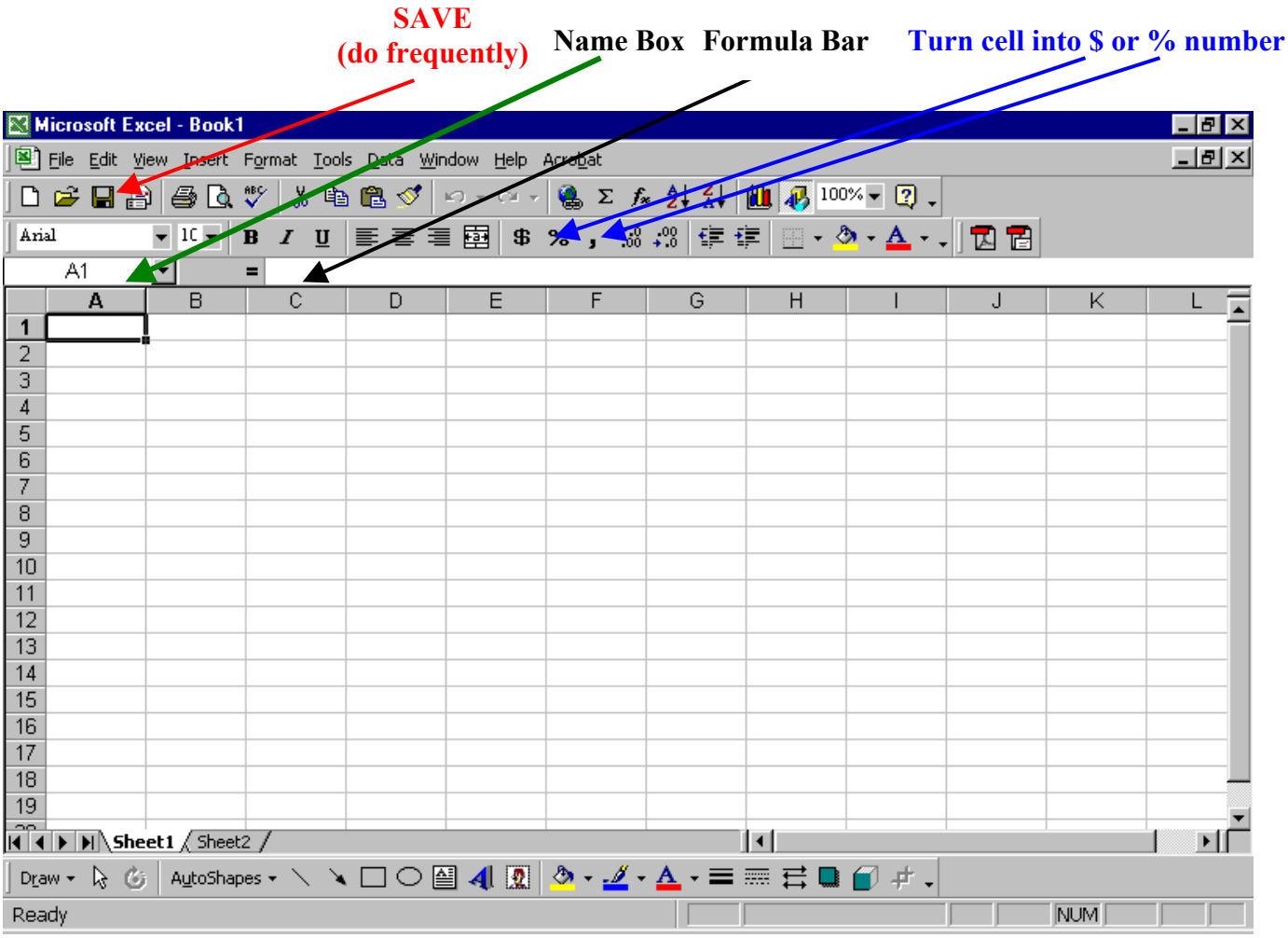
```
N=11.89566105
I%=6
PV=-2400
PMT=0
FV=4800
P/Y=1
C/Y=1
PMT:END BEGIN
```

4-1 Assignment
pg. 158-159 # 1 to 3, 6

4-1B: Creating and Using Spreadsheet

Spreadsheet: - a computer table software that allows the user to manage and calculate on columns and rows of data.

Using Microsoft EXCEL



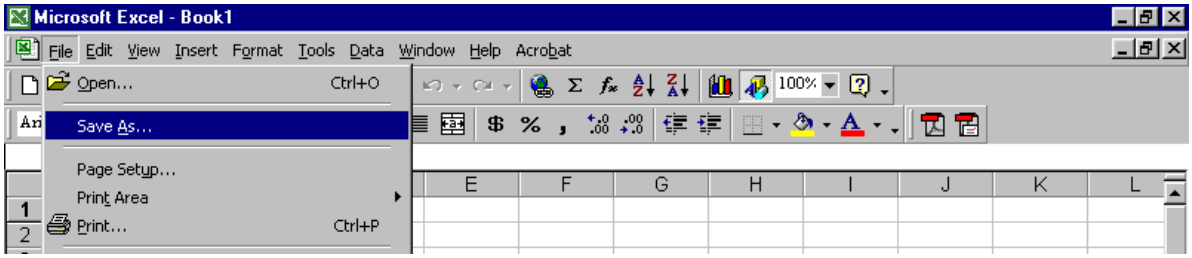
An advantage of using the spreadsheet over the TVM Solver is that the user can see the entire schedule of the investment or loan. This includes amount of interest received or charge, and the balance at every compounding period. In addition, it can show the amount of taxes taken out of the account at every compounding period if the investment is not a tax shelter (tax exempted).

We will use the following example to understand how to create and format a spreadsheet.

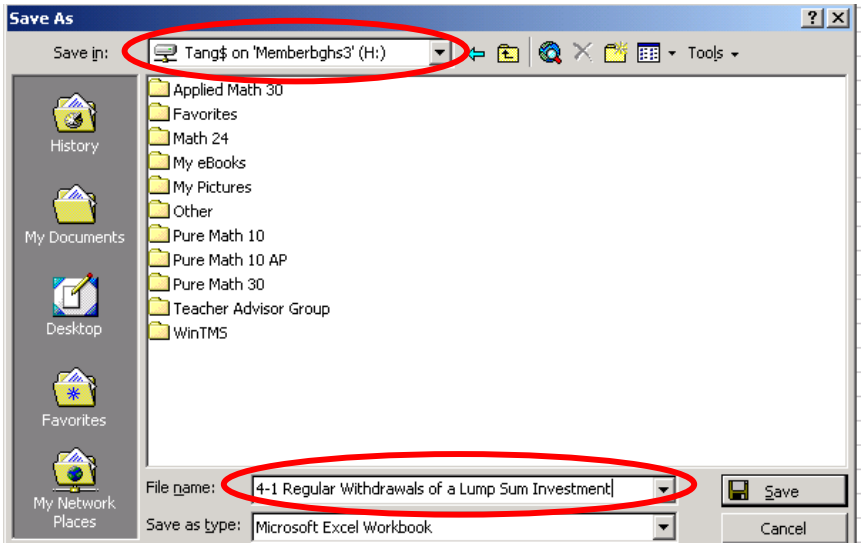
Example 1: John has won $100,000 in a lottery. He invest it at 8%/a compound semi-annually. How much can he withdraw at the end of each compounding period if he wants the money to last 10 years?

Step 1: Saving a Spreadsheet

a. Go to the Menu Bar, click on File and select Save As



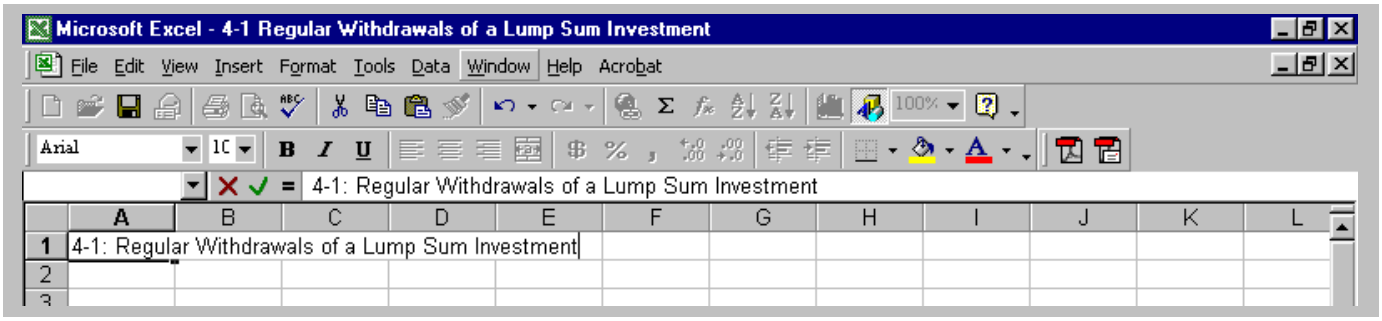
b. Select the **H:** drive and Name it "**4-1 Regular Withdrawals of a Lump Sum Investment**"



Step 2: Creating a Title

A spreadsheet should have an appropriate title for future users. It should also be clear of what the spreadsheet can do.

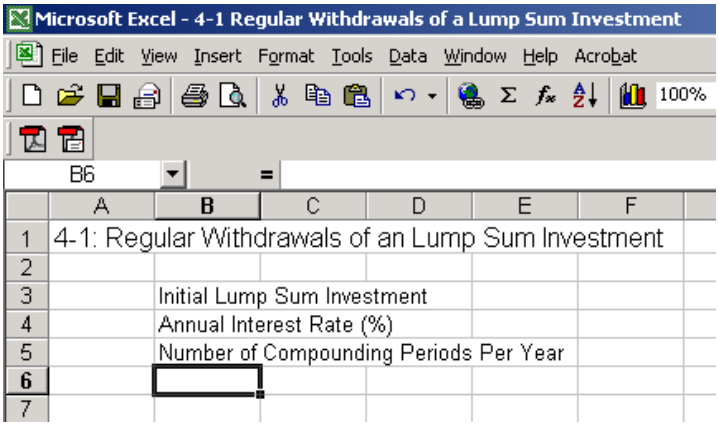
In cell A1, type in the following title "**4-1: Regular Withdrawals of a Lump Sum Investment**"



Step 3: Defining the Variables

It is a good practice to set up all the variables right underneath the title. If any users need to change the values for any of these variables, they can do so quickly and not have to bother messing around with the formulas in the actual table below.

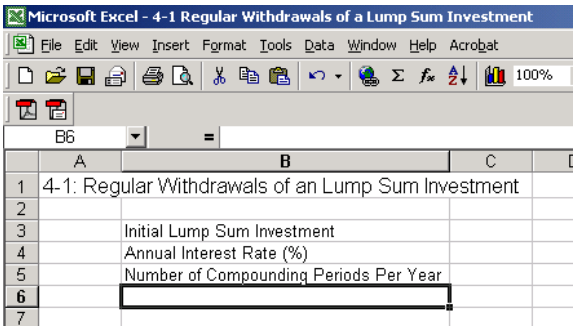
For this example, we have three variables: Initial Lump Sum Investment, Annual Interest Rate (%), and Number of Compounding Periods Per Year. Enter the headings of these variables in cells B3, B4 and B5.

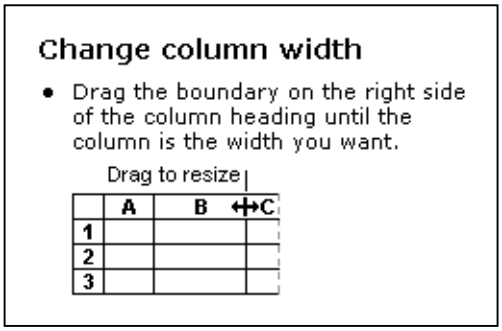


Step 4: Changing the Width of a Column

You can adjust the width of any column by dragging the boundary on the right side of the column. If you would like to have the computer automatically adjust the column, place the cursor on the boundary on the right side of the column and double click on the mouse.

Adjust column B, by dragging the line between column B and C.



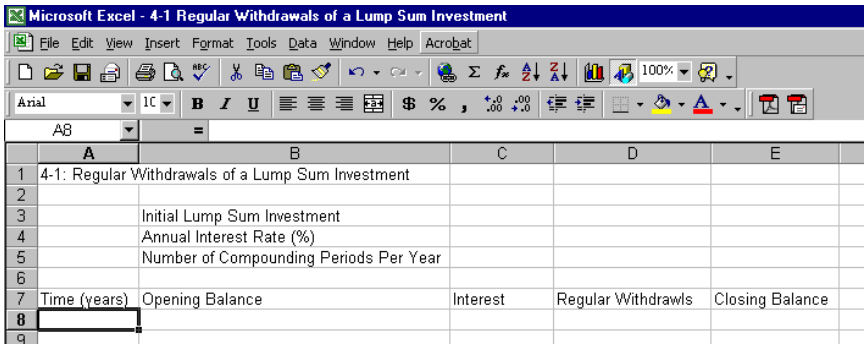


Note: If the cell shows #####. It means the width of the column is too small to show the numbers. You need to adjust the width of the column to view the number.

Step 5: Setting up the Column Headings for the Table

Before we set up the column headings, we have to think about what are the different information that need to be displayed.

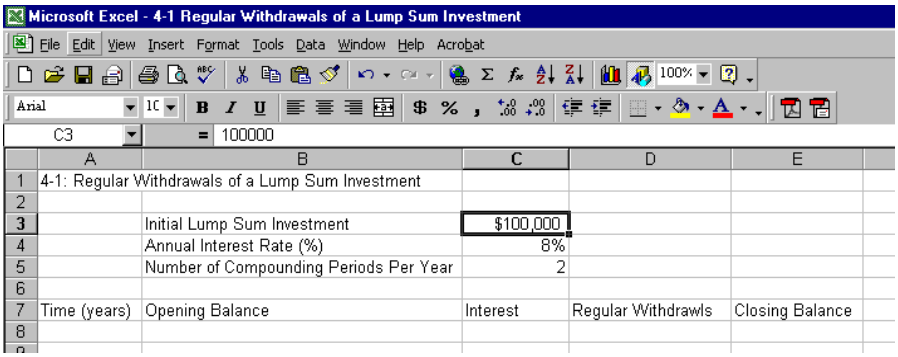
For this example, we will need Time (years), Opening Balance, Interest, Regular Withdrawals, and Closing Balance. Type these headings from A7 to E7. Adjust the column widths afterwards.



Step 6: Entering the Values for the Variables

We now need to enter the values for the variables listed below the title. These values can be changed when the same table is used to answer a same type of question but with different numbers.

For this example, the Initial Lump Sum Investment is $100,000 **(Do NOT put commas when entering numbers).** The Interest Rate is 8% and the Number of Compounding Periods in a Year is 2.



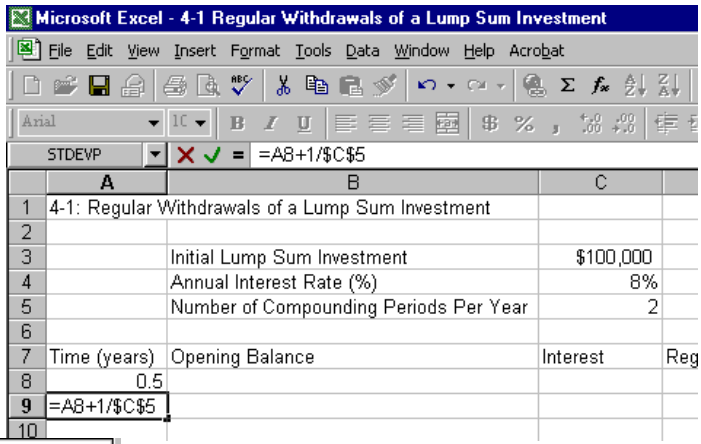
Step 7: Decide on the Formulas to use in the Table

The entire purpose of using a spreadsheet is to have the computer do the repetitive calculations. However, we need to tell the computer how to do the initial calculations. This is why we use formulas and **cell references (locations of any cells).**

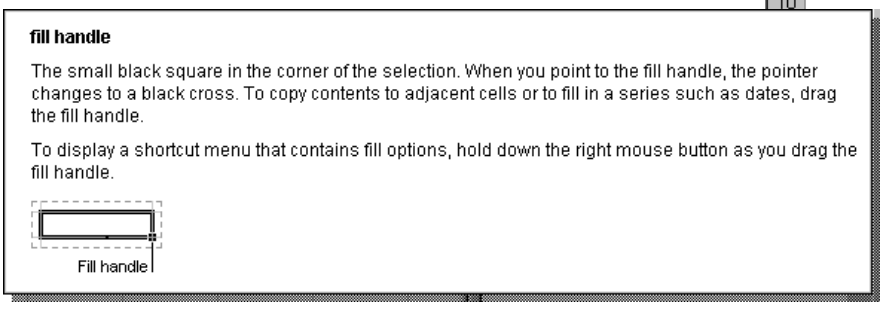
Relative Cell Reference: - the reference of the cell can be changed throughout the table.
(Example D8=B8–C8 If D9 has a similar calculation, the computer will automatically calculate D9=B9–C9 as the column is extended downward)

Absolute Cell Reference: - the reference of the cell will remain fixed throughout the table.
(Example D8=$B$8–C8 If column D is to be extended downward, the computer will automatically calculate D9=$B$8–C9. It will keep on referring to Cell B8)

For the Time column, type **0.5** in **A8.** To continue the pattern, we need to set up a formula for the remaining cells in the time column. Since each withdrawal is made right after each compounding period, in **A9**, type the formula **=A8+1/$C$5**
(All formulas in EXCEL must starts with an equal sign.)



Drag down the **Fill Handle** from A9 to **A27** (see below).

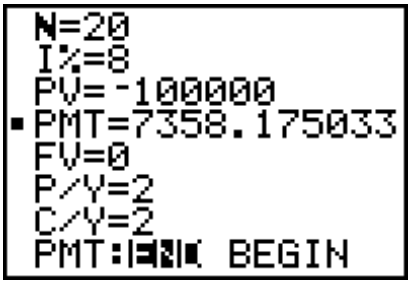


For the Opening Balance column, in **B8**, type **=$C$3**.

Since $I = Prt$, the formula in **C8** is **=B8*($C$4/$C$5)** First Interest Payment = \$100,000 (0.08/2) = \$4000 (Time is factored into the 4% interest rate every half year).

Using the TVM Solver, we can calculate what the withdrawal amount is without guessing and checking (see below). In **D8,** type **$7358.18**

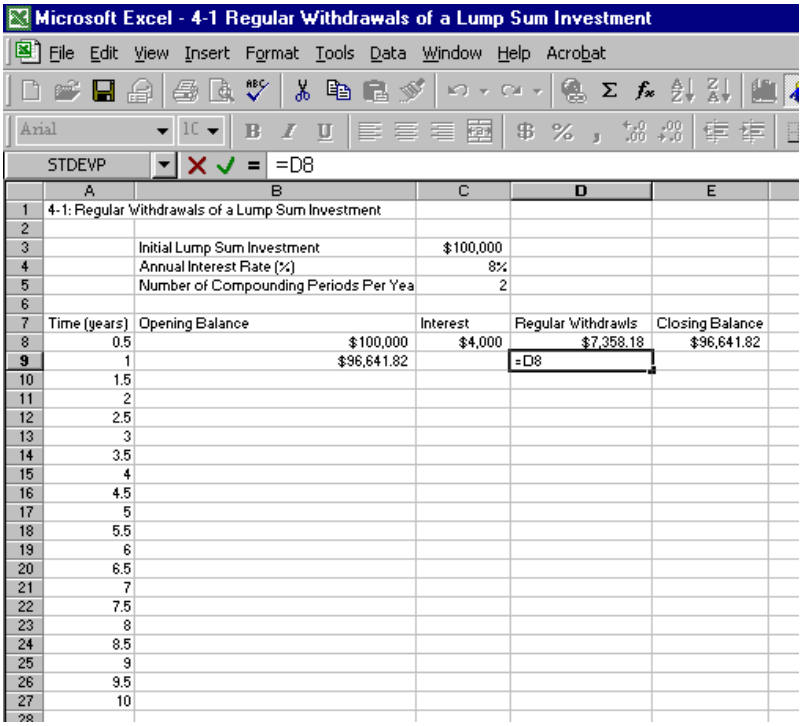
The Closing Balance =Opening Balance + Interest – Withdrawal. Therefore, in **E8**, type **=B8+C8–D8**



The Opening Balance for the next row (Row 9) should be the Closing Balance of the previous row (Row 8).

In **B9**, type **=E8**.

Since the regular withdrawal amount for row 9 is the same as row 8, in **D9**, type **=D8**. (This way, we can just change D8 if the values of a similar question are different than this one.)



	A	B	C	D	E
1	4-1: Regular Withdrawals of a Lump Sum Investment				
2					
3		Initial Lump Sum Investment	$100,000		
4		Annual Interest Rate (%)	8%		
5		Number of Compounding Periods Per Yea	2		
6					
7	Time (years)	Opening Balance	Interest	Regular Withdrawls	Closing Balance
8	0.5	$100,000	$4,000	$7,358.18	$96,641.82
9	1	$96,641.82		=D8	
10	1.5				
11	2				
12	2.5				
13	3				
14	3.5				
15	4				
16	4.5				
17	5				
18	5.5				
19	6				
20	6.5				
21	7				
22	7.5				
23	8				
24	8.5				
25	9				
26	9.5				
27	10				

Fill columns B to E straight down to row 27 using the Fill Handle. The table should look like the one below.

E27 = =B27+C27-D27

	A	B	C	D	E	F
1	4-1: Regular Withdrawals of a Lump Sum Investment					
2						
3		Initial Lump Sum Investment	$100,000			
4		Annual Interest Rate (%)	8%			
5		Number of Compounding Periods Per Yea	2			
6						
7	Time (years)	Opening Balance	Interest	Regular Withdrawls	Closing Balance	
8	0.5	$100,000	$4,000	$7,358.18	$96,641.82	
9	1	$96,641.82	$3,866	$7,358	$93,149.31	
10	1.5	$93,149.31	$3,726	$7,358	$89,517.11	
11	2	$89,517.11	$3,581	$7,358	$85,739.61	
12	2.5	$85,739.61	$3,430	$7,358	$81,811.01	
13	3	$81,811.01	$3,272	$7,358	$77,725.27	
14	3.5	$77,725.27	$3,109	$7,358	$73,476.11	
15	4	$73,476.11	$2,939	$7,358	$69,056.97	
16	4.5	$69,056.97	$2,762	$7,358	$64,461.07	
17	5	$64,461.07	$2,578	$7,358	$59,681.33	
18	5.5	$59,681.33	$2,387	$7,358	$54,710.40	
19	6	$54,710.40	$2,188	$7,358	$49,540.64	
20	6.5	$49,540.64	$1,982	$7,358	$44,164.09	
21	7	$44,164.09	$1,767	$7,358	$38,572.47	
22	7.5	$38,572.47	$1,543	$7,358	$32,757.19	
23	8	$32,757.19	$1,310	$7,358	$26,709.30	
24	8.5	$26,709.30	$1,068	$7,358	$20,419.49	
25	9	$20,419.49	$817	$7,358	$13,878.09	
26	9.5	$13,878.09	$555	$7,358	$7,075.03	
27	10	$7,075.03	$283	$7,358	($0.15)	

Should be ($0.00). We will fix it later!

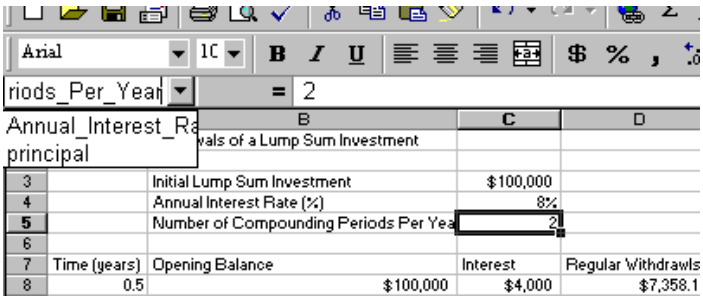
Step 8: Assigning Names to Variables

Sometimes it is difficult to verify formulas by always looking at the Absolute Cell Reference. We can assign Special Names of these variables in the **Name Box**.

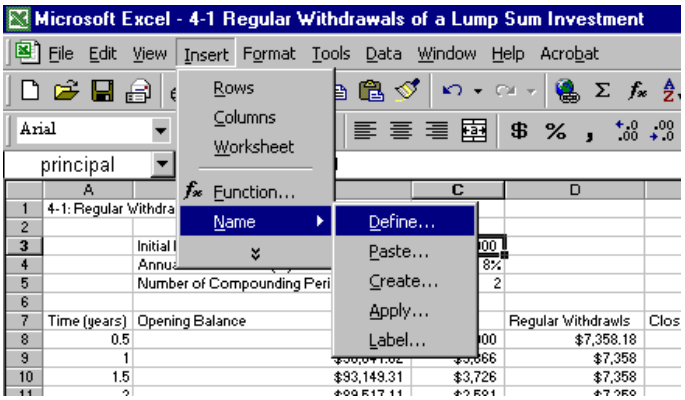
Click on **C3**. Type **principal** in the Name Box and hit Enter.

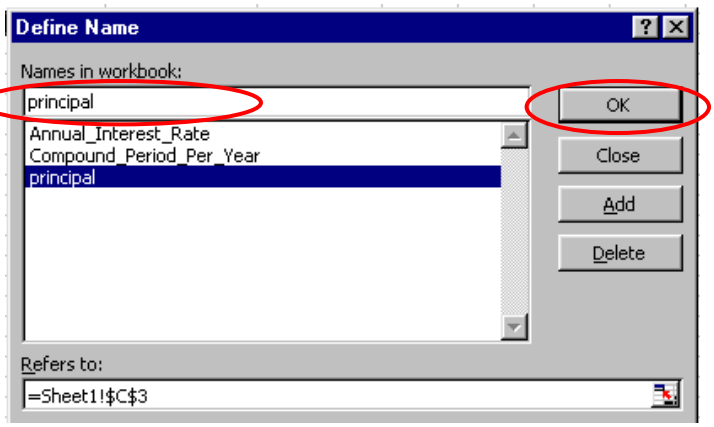
Click on **C4**. Type **Annual_Interest_Rate** in the Name Box and hit Enter

Click on **C5**. Type **Compound_Periods_Per_Year** in the Name Box and hit Enter.



Note: **Be Careful how you type in the Names in the Name Box!!** Once you have hit the ENTER key, you cannot change the name in the name box. To change the name in a name box, click on INSERT, NAME, DEFINE. After you have modified the selected Name, click OK





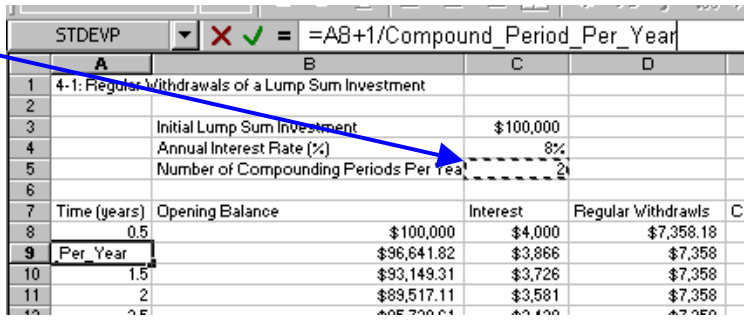
For the example, make the following changes.

a. In **A9**, highlight $C$5 and click on cell C5. Hit Enter. (Notice that you do not have to type in the long name, it will replace the absolute cell reference with the name assigned to the cell).

b. In **B8**, highlight $C$3 and click on cell C3. Hit Enter.

c. In **C8**, highlight $C$4 and click on cell C4. Highlight $C$5 and click on cell C5. Hit Enter.

d. Use the Fill Handle, drag A9 to A27 and C9 to C27.



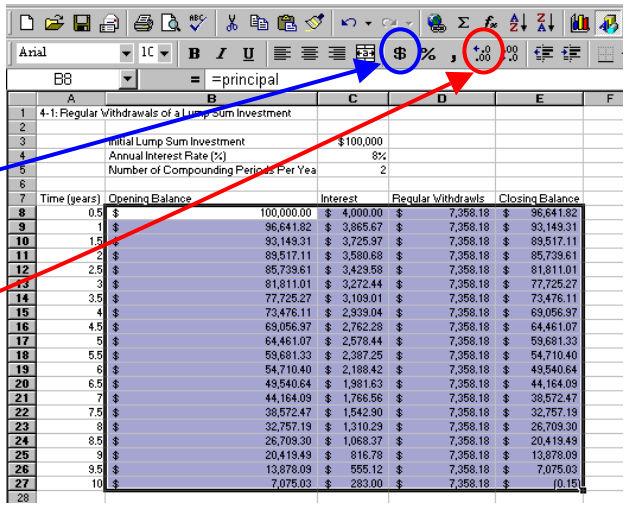
Step 9: Formatting Spreadsheet and Final Touches

In order to capture the attention of the user, we should format the spreadsheet so it looks more appealing.

For this example:

a. Assigning Currency Unit:
Columns B to E deals with money.
Highlight B8 to E27. Click on the **$ icon**.

Increase the decimal places to two places by using the **Increase Decimal icon.**

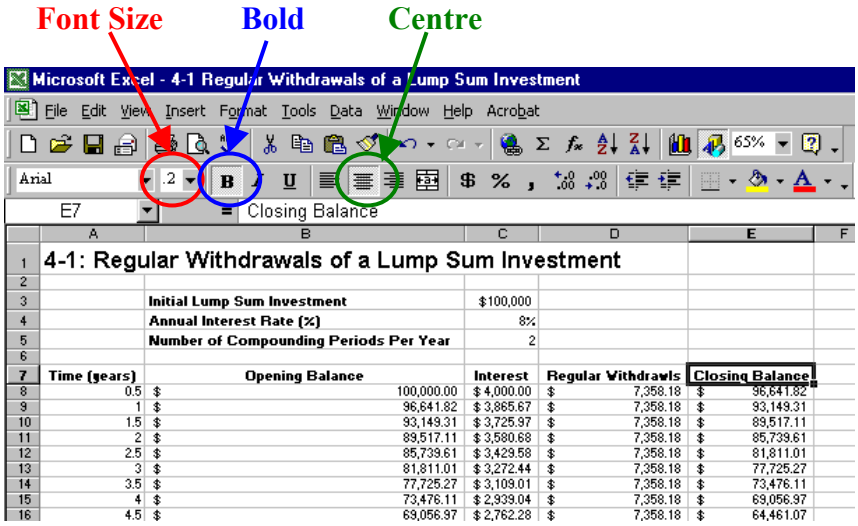


b. Formatting Fonts:
Title should be a bigger font size.
Highlight the title and change it to font size 20 and Bold.

Column Headings should be Bolded, font size 12, and Centre.

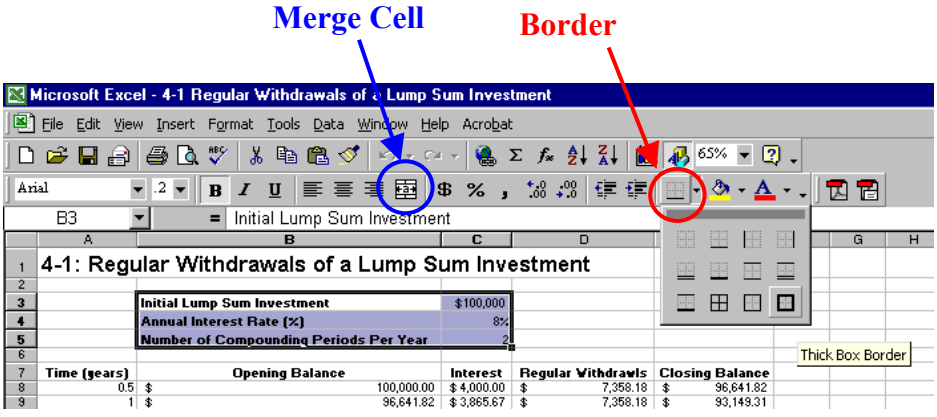
Variables should be Bolded and font size 12.

Adjust Column Widths so nothing is cut off.



c. Adding Borders
There should be a border around the variables to make them stand out. Users will know to enter these values for a different question.

Highlight B3 to C5. Click on the **Border icon** and select the last option.



d. Merge Cells
A group of cells can be merged into one to emphasize a title.
Highlight A1 to D1. Click on the **Merge Cell** icon.

e. Adjusting the Final Balance

The last final balance should have $0. Due to the rounding off of the result from the TVM Solver, it is now showing a −$0.15 ($0.15 overdrawn). If we adjust the last withdrawal amount (in **D27**) by $0.15 less, the final balance in year 10 should be $0.00.

=7358.03

4-1: Regular Withdrawals of a Lump Sum Investment

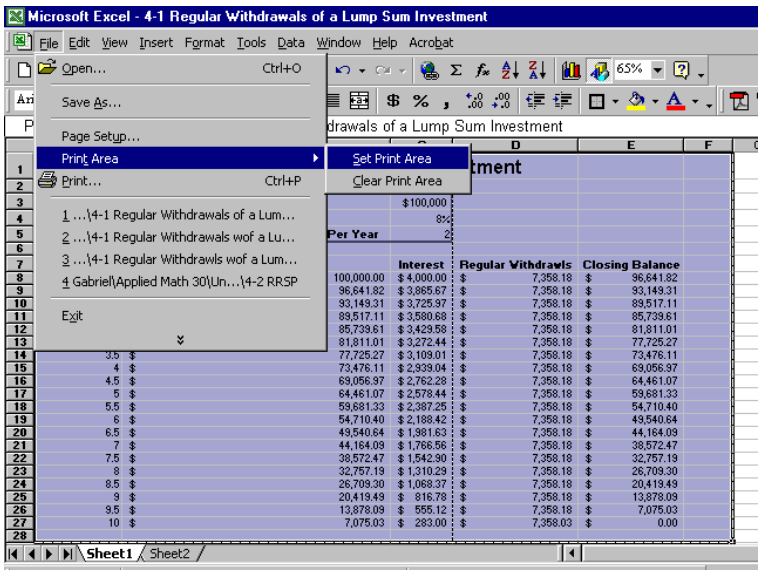
Initial Lump Sum Investment	$100,000
Annual Interest Rate (%)	8%
Number of Compounding Periods Per Year	2

Time (years)	Opening Balance	Interest	Regular Withdrawls	Closing Balance
0.5	$ 100,000.00	$ 4,000.00	$ 7,358.18	$ 96,641.82
1	$ 96,641.82	$ 3,865.67	$ 7,358.18	$ 93,149.31
1.5	$ 93,149.31	$ 3,725.97	$ 7,358.18	$ 89,517.11
2	$ 89,517.11	$ 3,580.68	$ 7,358.18	$ 85,739.61
2.5	$ 85,739.61	$ 3,429.58	$ 7,358.18	$ 81,811.01
3	$ 81,811.01	$ 3,272.44	$ 7,358.18	$ 77,725.27
3.5	$ 77,725.27	$ 3,109.01	$ 7,358.18	$ 73,476.11
4	$ 73,476.11	$ 2,939.04	$ 7,358.18	$ 69,056.97
4.5	$ 69,056.97	$ 2,762.28	$ 7,358.18	$ 64,461.07
5	$ 64,461.07	$ 2,578.44	$ 7,358.18	$ 59,681.33
5.5	$ 59,681.33	$ 2,387.25	$ 7,358.18	$ 54,710.40
6	$ 54,710.40	$ 2,188.42	$ 7,358.18	$ 49,540.64
6.5	$ 49,540.64	$ 1,981.63	$ 7,358.18	$ 44,164.09
7	$ 44,164.09	$ 1,766.56	$ 7,358.18	$ 38,572.47
7.5	$ 38,572.47	$ 1,542.90	$ 7,358.18	$ 32,757.19
8	$ 32,757.19	$ 1,310.29	$ 7,358.18	$ 26,709.30
8.5	$ 26,709.30	$ 1,068.37	$ 7,358.18	$ 20,419.49
9	$ 20,419.49	$ 816.78	$ 7,358.18	$ 13,878.09
9.5	$ 13,878.09	$ 555.12	$ 7,358.18	$ 7,075.03
10	$ 7,075.03	$ 283.00	7358.03	$ (0.15)

Step 10: Printing out the Spreadsheet and its Formulas

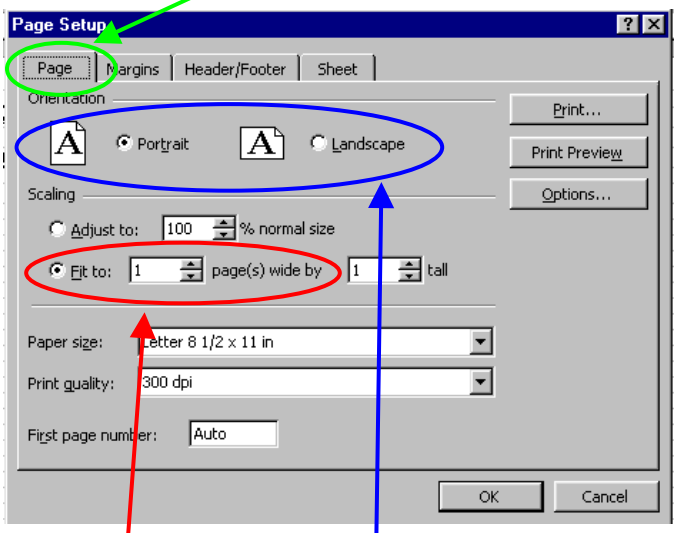
a. To print out a spreadsheet nicely, you must first highlight the area that needs to be printed and set the Print Area (FILE, PRINT AREA, SET PRINT AREA)

Highlight A1 to F28 and Set Print Area.



b. Usually, it is much easier to read the entire spreadsheet on one page. To have the printout fit in one page, we go to FILE, PAGE SETUP

Click on Page Tab

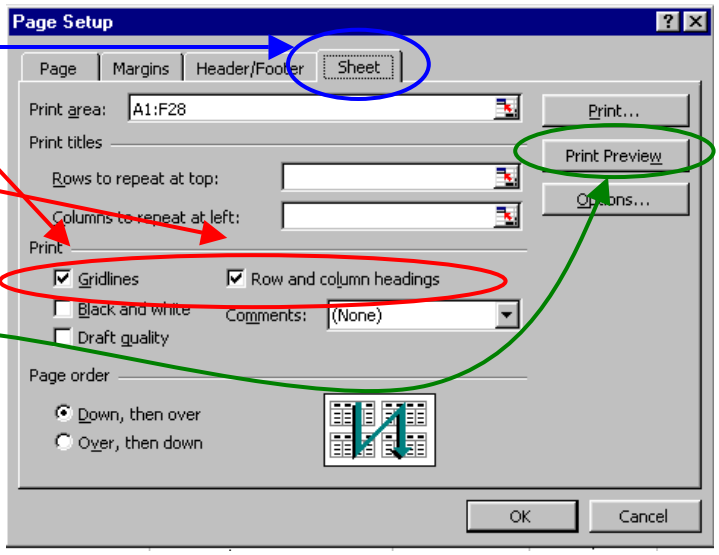


Select Fit to 1 Page

Portrait or Landscape Orientation based on the width of the Table

c. Under the **Sheet Tab**, select **Gridlines** with **Row and Column Heading**. (It is much easier to see the spreadsheet this way.)

d. Check the **Print Preview** to see if the table has gridlines, row and column headings and fit into one page before the actual printing.



e. **Printing Formulas**

To print formulas, **press the Ctrl key and the ~ key together**. (The ~ key is on the left of the 1 key.) Adjust the widths of the columns. Repeat Step 10 a. to d. and print.

	A	B	C	D	E
1	4-1: Regular Withdrawals of a Lump Sum Investment				
2					
3		Initial Lump Sum Investment	100000		
4		Annual Interest Rate (%)	0.08		
5		Number of Compounding Periods Per Y	2		
6					
7	Time (years)	Opening Balance	Interest	Regular Withdrawl	Closing Balance
8	0.5	principal	=B8*(Annual_Interest_Rate/Compound_Period_Per_Year)	7358.18	=B8+C8-D8
9	=A8+1/Compound_Period_Per_Ye	=E8	=B9*(Annual_Interest_Rate/Compound_Period_Per_Year)	=D8	=B9+C9-D9
10	=A9+1/Compound_Period_Per_Ye	=E9	=B10*(Annual_Interest_Rate/Compound_Period_Per_Year)	=D9	=B10+C10-D10
11	=A10+1/Compound_Period_Per_Y	=E10	=B11*(Annual_Interest_Rate/Compound_Period_Per_Year)	=D10	=B11+C11-D11
12	=A11+1/Compound_Period_Per_Y	=E11	=B12*(Annual_Interest_Rate/Compound_Period_Per_Year)	=D11	=B12+C12-D12
13	=A12+1/Compound_Period_Per_Y	=E12	=B13*(Annual_Interest_Rate/Compound_Period_Per_Year)	=D12	=B13+C13-D13
14	=A13+1/Compound_Period_Per_Y	=E13	=B14*(Annual_Interest_Rate/Compound_Period_Per_Year)	=D13	=B14+C14-D14
15	=A14+1/Compound_Period_Per_Y	=E14	=B15*(Annual_Interest_Rate/Compound_Period_Per_Year)	=D14	=B15+C15-D15
16	=A15+1/Compound_Period_Per_Y	=E15	=B16*(Annual_Interest_Rate/Compound_Period_Per_Year)	=D15	=B16+C16-D16
17	=A16+1/Compound_Period_Per_Y	=E16	=B17*(Annual_Interest_Rate/Compound_Period_Per_Year)	=D16	=B17+C17-D17
18	=A17+1/Compound_Period_Per_Y	=E17	=B18*(Annual_Interest_Rate/Compound_Period_Per_Year)	=D17	=B18+C18-D18
19	=A18+1/Compound_Period_Per_Y	=E18	=B19*(Annual_Interest_Rate/Compound_Period_Per_Year)	=D18	=B19+C19-D19
20	=A19+1/Compound_Period_Per_Y	=E19	=B20*(Annual_Interest_Rate/Compound_Period_Per_Year)	=D19	=B20+C20-D20
21	=A20+1/Compound_Period_Per_Y	=E20	=B21*(Annual_Interest_Rate/Compound_Period_Per_Year)	=D20	=B21+C21-D21
22	=A21+1/Compound_Period_Per_Y	=E21	=B22*(Annual_Interest_Rate/Compound_Period_Per_Year)	=D21	=B22+C22-D22
23	=A22+1/Compound_Period_Per_Y	=E22	=B23*(Annual_Interest_Rate/Compound_Period_Per_Year)	=D22	=B23+C23-D23
24	=A23+1/Compound_Period_Per_Y	=E23	=B24*(Annual_Interest_Rate/Compound_Period_Per_Year)	=D23	=B24+C24-D24
25	=A24+1/Compound_Period_Per_Y	=E24	=B25*(Annual_Interest_Rate/Compound_Period_Per_Year)	=D24	=B25+C25-D25
26	=A25+1/Compound_Period_Per_Y	=E25	=B26*(Annual_Interest_Rate/Compound_Period_Per_Year)	=D25	=B26+C26-D26
27	=A26+1/Compound_Period_Per_Y	=E26	=B27*(Annual_Interest_Rate/Compound_Period_Per_Year)	7358.03	=B27+C27-D27

SAVE your File!!

4-1B Assignment

pg. 159 # 7 (Use the Spreadsheet created above to answer this question.)
Redo pg. 159 #2 (Use the Spreadsheet Template "4-1 Lump Sum with Annuities".)
(You may download from the website or type the template in from the next page)
pg. 159 # 5 (Create a new Spreadsheet and Save it as "4-1 Animal Shelter".)
Print Both Spreadsheets (Figures and Formulas)

4-1: Lump Sum with Annuities

	Principal	$5,000.00
	Interest Rate	10.00%
	Compunding Periods Per Year	2
	Additional Regular Contribution	$1,200.00

Year	Opening Balance	Interest	Contribution	Closing Balance
1/2	$ 5,000.00	$ 250.00	$ 1,200.00	$ 6,450.00
1 1/2	$ 6,450.00	$ 322.50	$ 1,200.00	$ 7,972.50
2 1/2	$ 7,972.50	$ 398.63	$ 1,200.00	$ 9,571.13
3 1/2	$ 9,571.13	$ 478.56	$ 1,200.00	$ 11,249.68
4 1/2	$ 11,249.68	$ 562.48	$ 1,200.00	$ 13,012.17
5 1/2	$ 13,012.17	$ 650.61	$ 1,200.00	$ 14,862.77
6 1/2	$ 14,862.77	$ 743.14	$ 1,200.00	$ 16,805.91
7 1/2	$ 16,805.91	$ 840.30	$ 1,200.00	$ 18,846.21
8 1/2	$ 18,846.21	$ 942.31	$ 1,200.00	$ 20,988.52
9 1/2	$ 20,988.52	$ 1,049.43	$ 1,200.00	$ 23,237.94
10	$ 23,237.94	$ 1,161.90	$ 1,200.00	$ 25,599.84
11	$ 25,599.84	$ 1,279.99	$ 1,200.00	$ 28,079.83
12	$ 28,079.83	$ 1,403.99	$ 1,200.00	$ 30,683.83
13	$ 30,683.83	$ 1,534.19	$ 1,200.00	$ 33,418.02
14	$ 33,418.02	$ 1,670.90	$ 1,200.00	$ 36,288.92
15	$ 36,288.92	$ 1,814.45	$ 1,200.00	$ 39,303.36
16	$ 39,303.36	$ 1,965.17	$ 1,200.00	$ 42,468.53
17	$ 42,468.53	$ 2,123.43	$ 1,200.00	$ 45,791.96
18	$ 45,791.96	$ 2,289.60	$ 1,200.00	$ 49,281.56
19	$ 49,281.56	$ 2,464.08	$ 1,200.00	$ 52,945.63

Total Interest	$ 23,945.63
Total Amount Invested	$ 29,000.00
Final Closing Balance	$ 52,945.63

Special Name Boxes
C3 = principal
C4 = rate
C5 = n
C6 = regular_payment

4-1: Lump Sum with Annuities

	Principal	5000
	Interest Rate	0.1
	Compunding Periods Per Year	2
	Additional Regular Payment	1200

Year	Opening Balance	Interest	Payment	Closing Balance
=1/n	=principal	=B9*rate/n	=regular_contribution	=B9+C9+D9
=A9+1/n	=E9	=B10*rate/n	=regular_contribution	=B10+C10+D10
=A10+1/n	=E10	=B11*rate/n	=regular_contribution	=B11+C11+D11
=A11+1/n	=E11	=B12*rate/n	=regular_contribution	=B12+C12+D12
=A12+1/n	=E12	=B13*rate/n	=regular_contribution	=B13+C13+D13
=A13+1/n	=E13	=B14*rate/n	=regular_contribution	=B14+C14+D14
=A14+1/n	=E14	=B15*rate/n	=regular_contribution	=B15+C15+D15
=A15+1/n	=E15	=B16*rate/n	=regular_contribution	=B16+C16+D16
=A16+1/n	=E16	=B17*rate/n	=regular_contribution	=B17+C17+D17
=A17+1/n	=E17	=B18*rate/n	=regular_contribution	=B18+C18+D18
=A18+1/n	=E18	=B19*rate/n	=regular_contribution	=B19+C19+D19
=A19+1/n	=E19	=B20*rate/n	=regular_contribution	=B20+C20+D20
=A20+1/n	=E20	=B21*rate/n	=regular_contribution	=B21+C21+D21
=A21+1/n	=E21	=B22*rate/n	=regular_contribution	=B22+C22+D22
=A22+1/n	=E22	=B23*rate/n	=regular_contribution	=B23+C23+D23
=A23+1/n	=E23	=B24*rate/n	=regular_contribution	=B24+C24+D24
=A24+1/n	=E24	=B25*rate/n	=regular_contribution	=B25+C25+D25
=A25+1/n	=E25	=B26*rate/n	=regular_contribution	=B26+C26+D26
=A26+1/n	=E26	=B27*rate/n	=regular_contribution	=B27+C27+D27
=A27+1/n	=E27	=B28*rate/n	=regular_contribution	=B28+C28+D28

Total Interest	=SUM(C9:C28)
Total Amount Invested	=principal+SUM(D9:D28)
Final Closing Balance	=SUM(C30:C31)

Special Name Boxes
C3 = principal
C4 = rate
C5 = n
C6 = regular_contribution

4-2: Investing for the Future

1. CPP (Canada Pension Plan)

- Government sponsored.
- Maximum contribution: about 3% of your gross income, up to $1186.50.
- Government pays any Canadian over the age of 65 who had contributed.
- Very little pay out.
- Government strongly suggests supplementation of retirement income with your own RRSP.

2. RRSP (Registered Retirement Saving Plan)

- RRSP is a **self-directed** investment.
- You can invest up to 18% of your gross income up to a preset maximum.
  (Maximum RRSP contribution for 2001 is $13500).
- No Tax on RRSP amount until actual withdrawals during retirement.
  (RRSP is **NOT** subject to any income tax or **capital gain tax**)
- Cannot take out until retirement. (Penalty is an immediate 30% Tax on withdraw amount.)
  [The only exceptions to this penalty is if the money is used to buy a house – Home Buyer Plan or for further education Lifelong Learning Plan. Both plans allow a one-lifetime withdrawal of a maximum of $20,000 to be repaid back to the RRSP within the next 15 years.]
- Minimum of 70% of the investment must be on Canadian companies; Maximum of 30% can be foreign investment.
- You can transfer your RRSP amount to your spouse if you have exceeded your maximum amount for that year.
- Any unused RRSP amount may be used in the next seven years.

3. RPP (Registered Pension Plan)

- RPP is usually set up through private **company's pension plan (self-directed)** or **trade union group pension plan (not self-directed)**.
- It is subjected to the same rule as RRSP. The total amount of the RRSP and RPP contributions must **NOT** exceed 18% of the gross income.
- Some companies **matches RPP contribution dollar by dollar (vested)**, some don't.
  Usually an employee has to work continuously for a number of years before vesting take place.
  (All RPP contributions will be matched retroactively.)
- If the RPP is self-directed, upon leaving the company before retirement, the worker can roll it into his or her own RRSP.
- If a trade union group directs the RSP, the worker will receive a prorated pension amount upon retirement.

Capital Gain Tax: - tax on **non-registered investment** incomes from stocks, mutual funds, real estate properties …etc.

Capital Gain Tax = **Non-Registered Investment Earnings** × **Capital Gain Portion** × **Marginal Tax Rate**

(The Canadian Capital Gain Portion for 2001 is 50%.)

Marginal Tax: - an **average income tax** based on different tax brackets.

Example 1: John made a taxable income of $50,000 in 2001. Using the 2001 Federal Tax Return below, calculate his marginal tax rate.

Tax Brackets on Taxable Income	Federal Income Tax Rate
First $0 to $30,753	16%
Next $30,754 to $61,508	22%
Next $61,509 to $100,000	26%
Next $100,001 or more	29%

Total Tax Paid = ($30,753 × 16%) + ($50,000 – $30753) × 22%

First $30753 at 16% and **the next $19247 at 22%**

= $4920.48 + $4234.34

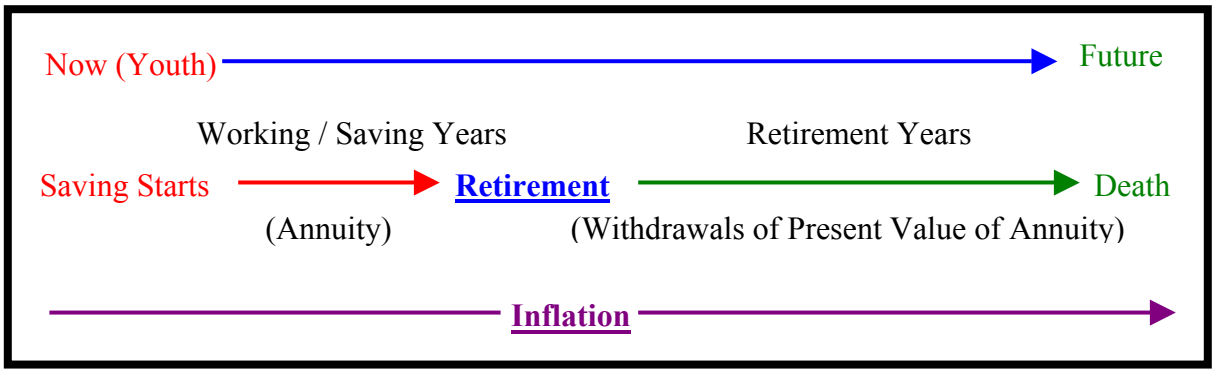
Total Tax Paid = **$9154.82**

$$\textbf{Marginal Tax Rate} = \frac{\textbf{Total Tax Paid}}{\textbf{Taxable Income}} \times \textbf{100\%} = \frac{\$9154.82}{\$50000} \times 100\%$$

Marginal Tax Rate = 18.31%

Retirement Planning: - CANNOT rely on the government
- want to retire early, better **plan NOW**! When you are **still YOUNG**!
- let the power of compound interest make money for you.

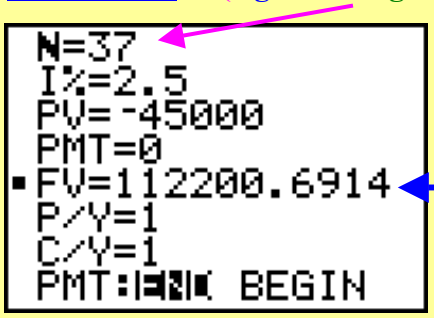
RETIREMENT TIME LINE



Example 2: Mary is 18 years old, and she thinks that she will live till she is 80 years old. She would like to retire at the age of 55. Knowing the power of compound interest, she starts saving for her retirement now. From research, she knows that the average annual inflation rate is 2.5%. Her father suggested a RRSP mutual fund account that earns an average 8%/a compounded monthly. Mary would like to have the same living standard as someone who earns $45,000 per year by the time she retires.

a. Accounting for inflation, what should Mary be earning in the year at the beginning of her retirement?

TVM Solver **(Age 55 – Age 18)**

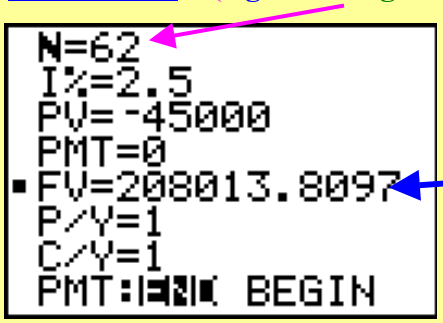


This is similar to a lump sum saving problem, except we will use **the annual inflation rate of 2.5%/a (compound annually) as our interest rate,** and **the earning potential of $45000 as our principal**.

$112,200.69 will have the same value 37 years from now as $45,000 has today.

b. Accounting for inflation, what should Mary be earning in the very last year of her retirement (just before she die)?

TVM Solver **(Age 80 – Age 18)**



Again, this is similar to a lump sum saving problem. We will use **the annual inflation rate of 2.5%/a (compound annually) as our interest rate,** and **the earning potential of $45000 as our principal**.

$208,013.81 will have the same value 62 years from now as $45,000 has today.

c. Determine the total amount that Mary needs for her retirement using the average amount calculated for part a. and b. above.

$$\textbf{Average Annual Withdrawal During Retirement} = \frac{(\$112{,}200.69 + \$208{,}013.80)}{2}$$

Average Annual Withdrawal During Retirement = $160,107.25

During 25 years of retirement (Age 55 to Age 80), she will need on average $160,107.25 per year.

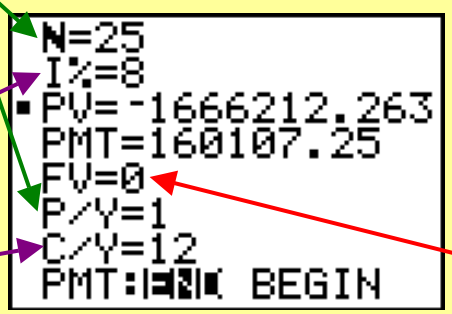
Total Withdrawals = 25 years × $160,107.25 / year

Total Withdrawals over 25 years of Retirement = $4,002,681.25

d. Calculate Mary's **retirement goal** (the amount of money needed to be in the account at the beginning of her retirement).

Assuming ONE payment per year during retirement

TVM Solver



RRSP account Interest Rate is 8%/a compound monthly

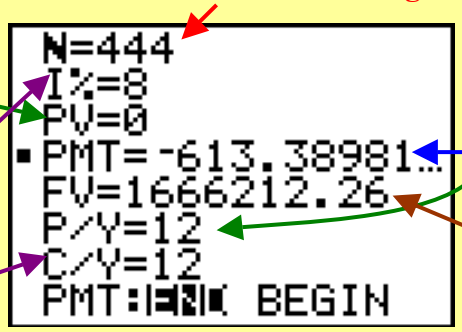
This is a **Present Value problem**. We expect the **Retirement Goal (Present Value)** is going to be much lower than the Total Withdrawals for the entire Retirement Period.

Retirement Goal at Age 55 = $1,666,212.26

At the end of her retirement (at age 80), all the money would have been used up.

e. Finally, determine how much Mary needs to contribute into her RRSP account every month to reach her retirement goal.

TVM Solver **N = 37 Working Years × 12 payments / yr**



Initially, there is NO Money in RRSP account

RRSP account Interest Rate is 8%/a compound monthly

Number of Working Years = Age 55 – Age 18
Number of Working Years = 37 years

RRSP Contribution = $613.39/month

Future Value is the Retirement Goal

f. Calculate the amount of interest she will make on her retirement plan.

Total Withdrawals by Age 80 = $4,002,681.25

Total RRSP Contributions = $613.39/month × 37 Working Years × 12 months / year
Total RRSP Contributions = $272,345.16

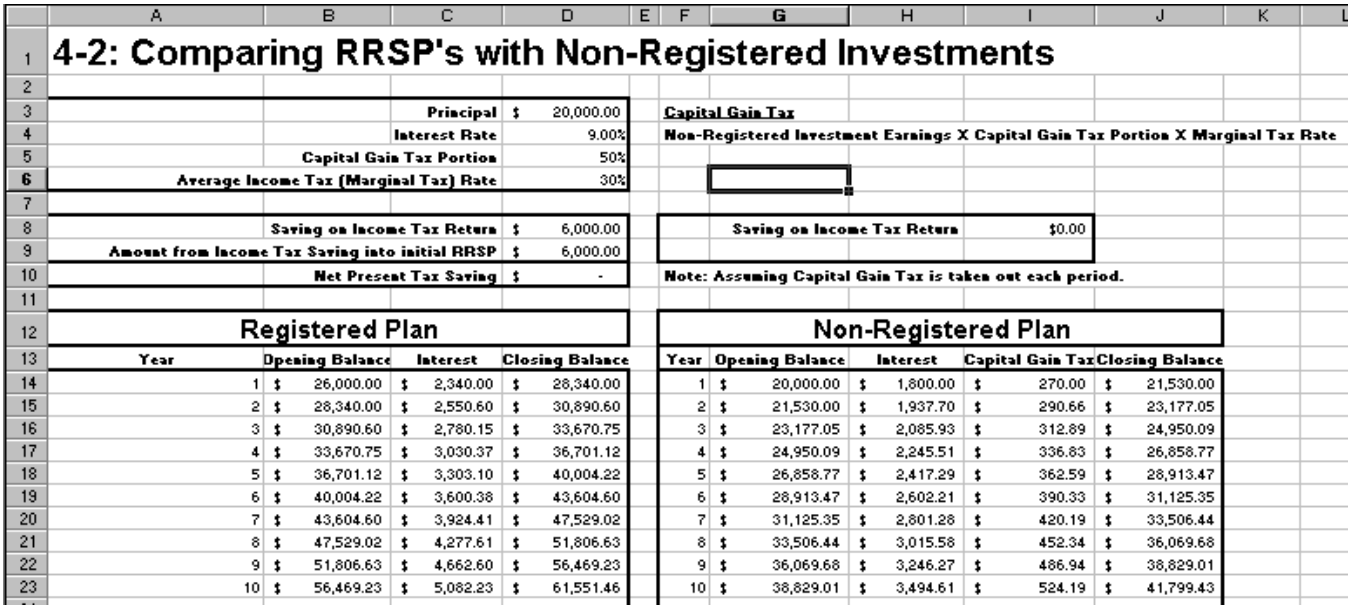
Overall Interest Earned = Total Withdrawals – Total RRSP Contributions
= $4,002,681.25 – $272,345.16

Overall Interested Earned Between Age 18 to Age 80 = $3,730,336.09

Example 3: Using the spreadsheet template "4-2 RRSP" (available on the website), complete the following table for an investment of $20,000 at 9%/a compound annually over 30 years with a marginal tax rate of 30%.

Investment of $20,000 for 30 years at a Marginal Tax Rate of 30%

Registered Plan		Non- Registered Plan			
Closing Balance	Average Rate of Return	Closing Balance	Tax paid	Net Return	Average Annual Rate of Return
$344,959.64	**9.00%**	**$182,579.08**	**$28,690.43**	**$162,579.08**	**7.65%**



4-2: Comparing RRSP's with Non-Registered Investments

Principal	$ 20,000.00
Interest Rate	9.00%
Capital Gain Tax Portion	50%
Average Income Tax (Marginal Tax) Rate	30%
Saving on Income Tax Return	$ 6,000.00
Amount from Income Tax Saving into initial RRSP	$ 6,000.00
Net Present Tax Saving	$ -

Capital Gain Tax
Non-Registered Investment Earnings X Capital Gain Tax Portion X Marginal Tax Rate

Saving on Income Tax Return	$0.00

Note: Assuming Capital Gain Tax is taken out each period.

Registered Plan

Year	Opening Balance	Interest	Closing Balance
1	$ 26,000.00	$ 2,340.00	$ 28,340.00
2	$ 28,340.00	$ 2,550.60	$ 30,890.60
3	$ 30,890.60	$ 2,780.15	$ 33,670.75
4	$ 33,670.75	$ 3,030.37	$ 36,701.12
5	$ 36,701.12	$ 3,303.10	$ 40,004.22
6	$ 40,004.22	$ 3,600.38	$ 43,604.60
7	$ 43,604.60	$ 3,924.41	$ 47,529.02
8	$ 47,529.02	$ 4,277.61	$ 51,806.63
9	$ 51,806.63	$ 4,662.60	$ 56,469.23
10	$ 56,469.23	$ 5,082.23	$ 61,551.46
20	$ 133,683.19	$ 12,031.49	$ 145,714.68
21	$ 145,714.68	$ 13,114.32	$ 158,829.00
22	$ 158,829.00	$ 14,294.61	$ 173,123.61
23	$ 173,123.61	$ 15,581.13	$ 188,704.74
24	$ 188,704.74	$ 16,983.43	$ 205,688.16
25	$ 205,688.16	$ 18,511.93	$ 224,200.10
26	$ 224,200.10	$ 20,178.01	$ 244,378.11
27	$ 244,378.11	$ 21,994.03	$ 266,372.14
28	$ 266,372.14	$ 23,973.49	$ 290,345.63
29	$ 290,345.63	$ 26,131.11	$ 316,476.73
30	$ 316,476.73	$ 28,482.91	$ 344,959.64

Non-Registered Plan

Year	Opening Balance	Interest	Capital Gain Tax	Closing Balance
1	$ 20,000.00	$ 1,800.00	$ 270.00	$ 21,530.00
2	$ 21,530.00	$ 1,937.70	$ 290.66	$ 23,177.05
3	$ 23,177.05	$ 2,085.93	$ 312.89	$ 24,950.09
4	$ 24,950.09	$ 2,245.51	$ 336.83	$ 26,858.77
5	$ 26,858.77	$ 2,417.29	$ 362.59	$ 28,913.47
6	$ 28,913.47	$ 2,602.21	$ 390.33	$ 31,125.35
7	$ 31,125.35	$ 2,801.28	$ 420.19	$ 33,506.44
8	$ 33,506.44	$ 3,015.58	$ 452.34	$ 36,069.68
9	$ 36,069.68	$ 3,246.27	$ 486.94	$ 38,829.01
10	$ 38,829.01	$ 3,494.61	$ 524.19	$ 41,799.43
20	$ 81,151.52	$ 7,303.64	$ 1,095.55	$ 87,359.61
21	$ 87,359.61	$ 7,862.36	$ 1,179.35	$ 94,042.62
22	$ 94,042.62	$ 8,463.84	$ 1,269.58	$ 101,236.88
23	$ 101,236.88	$ 9,111.32	$ 1,366.70	$ 108,981.50
24	$ 108,981.50	$ 9,808.34	$ 1,471.25	$ 117,318.58
25	$ 117,318.58	$ 10,558.67	$ 1,583.80	$ 126,293.46
26	$ 126,293.46	$ 11,366.41	$ 1,704.96	$ 135,954.91
27	$ 135,954.91	$ 12,235.94	$ 1,835.39	$ 146,355.46
28	$ 146,355.46	$ 13,171.99	$ 1,975.80	$ 157,551.65
29	$ 157,551.65	$ 14,179.65	$ 2,126.95	$ 169,604.35
30	$ 169,604.35	$ 15,264.39	$ 2,289.66	$ 182,579.08

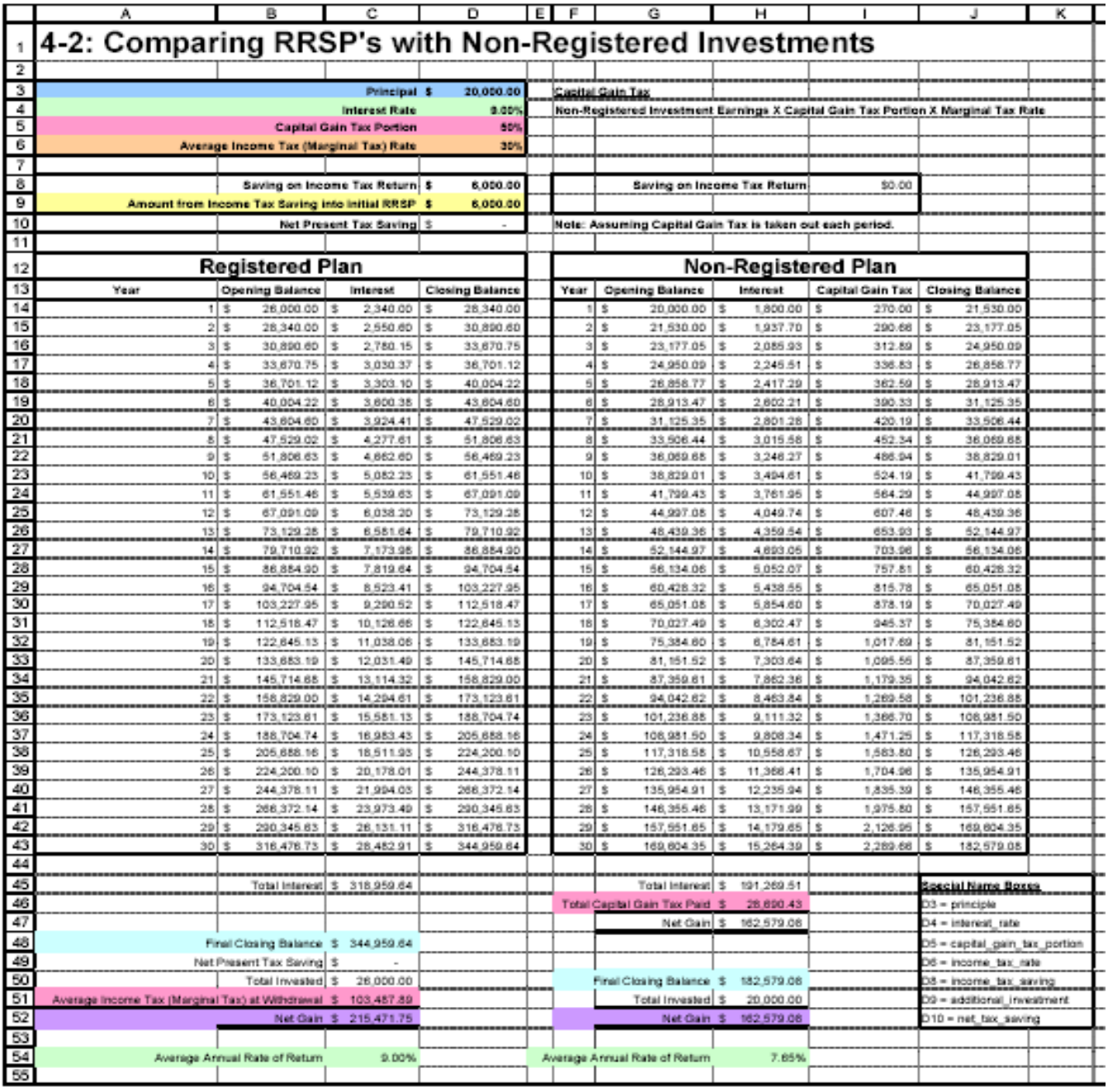
Total Interest	$ 318,959.64
Final Closing Balance	$ 344,959.64
Net Present Tax Saving	$ -
Total Invested	$ 26,000.00
Average Income Tax (Marginal Tax) at Withdrawal	$ 103,487.89
Net Gain	$ 215,471.75
Average Annual Rate of Return	9.00%

Total Interest	$ 191,269.51
Total Capital Gain Tax Paid	$ 28,690.43
Net Gain	$ 162,579.08
Final Closing Balance	$ 182,579.08
Total Invested	$ 20,000.00
Net Gain	$ 162,579.08
Average Annual Rate of Return	7.65%

Special Name Boxes

D3 = principal
D4 = interest_rate
D5 = capital_gain_tax_portion
D6 = income_tax_rate
D8 = income_tax_saving
D9 = additional_investment
D10 = net_tax_saving

4-2 Assignment: pg. 163-165 # 1, 3 to 7
(Question #1 – Use Spreadsheet "4-2 RRSP") (Question #3 to 7 – Use TVM Solver)



4-2: Comparing RRSP's with Non-Registered Investments

Principal	$ 20,000.00
Interest Rate	9.00%
Capital Gain Tax Portion	50%
Average Income Tax (Marginal Tax) Rate	30%

Saving on Income Tax Return	$ 6,000.00
Amount from Income Tax Saving into Initial RRSP	$ 6,000.00
Net Present Tax Saving	$ -

Capital Gain Tax

Non-Registered Investment Earnings X Capital Gain Tax Portion X Marginal Tax Rate

Saving on Income Tax Return	$0.00

Note: Assuming Capital Gain Tax is taken out each period.

Registered Plan

Year	Opening Balance	Interest	Closing Balance
1	$ 26,000.00	$ 2,340.00	$ 28,340.00
2	$ 28,340.00	$ 2,550.60	$ 30,890.60
3	$ 30,890.60	$ 2,780.15	$ 33,670.75
4	$ 33,670.75	$ 3,030.37	$ 36,701.12
5	$ 36,701.12	$ 3,303.10	$ 40,004.22
6	$ 40,004.22	$ 3,600.38	$ 43,604.60
7	$ 43,604.60	$ 3,924.41	$ 47,529.02
8	$ 47,529.02	$ 4,277.61	$ 51,806.63
9	$ 51,806.63	$ 4,662.60	$ 56,469.23
10	$ 56,469.23	$ 5,082.23	$ 61,551.46
11	$ 61,551.46	$ 5,539.63	$ 67,091.09
12	$ 67,091.09	$ 6,038.20	$ 73,129.28
13	$ 73,129.28	$ 6,581.64	$ 79,710.92
14	$ 79,710.92	$ 7,173.98	$ 86,884.90
15	$ 86,884.90	$ 7,819.64	$ 94,704.54
16	$ 94,704.54	$ 8,523.41	$ 103,227.95
17	$ 103,227.95	$ 9,290.52	$ 112,518.47
18	$ 112,518.47	$ 10,126.66	$ 122,645.13
19	$ 122,645.13	$ 11,038.06	$ 133,683.19
20	$ 133,683.19	$ 12,031.49	$ 145,714.68
21	$ 145,714.68	$ 13,114.32	$ 158,829.00
22	$ 158,829.00	$ 14,294.61	$ 173,123.61
23	$ 173,123.61	$ 15,581.13	$ 188,704.74
24	$ 188,704.74	$ 16,983.43	$ 205,688.16
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26	$ 224,200.10	$ 20,178.01	$ 244,378.11
27	$ 244,378.11	$ 21,994.03	$ 266,372.14
28	$ 266,372.14	$ 23,973.49	$ 290,345.63
29	$ 290,345.63	$ 26,131.11	$ 316,476.73
30	$ 316,476.73	$ 28,462.91	$ 344,959.64
Total Interest		$ 318,959.64	

Final Closing Balance	$ 344,959.64
Net Present Tax Saving	$ -
Total Invested	$ 26,000.00
Average Income Tax (Marginal Tax) at Withdrawal	$ 103,487.89
Net Gain	$ 215,471.75
Average Annual Rate of Return	9.00%

Non-Registered Plan

Year	Opening Balance	Interest	Capital Gain Tax	Closing Balance
1	$ 20,000.00	$ 1,800.00	$ 270.00	$ 21,530.00
2	$ 21,530.00	$ 1,937.70	$ 290.66	$ 23,177.05
3	$ 23,177.05	$ 2,085.93	$ 312.89	$ 24,950.09
4	$ 24,950.09	$ 2,245.51	$ 336.83	$ 26,858.77
5	$ 26,858.77	$ 2,417.29	$ 362.59	$ 28,913.47
6	$ 28,913.47	$ 2,602.21	$ 390.33	$ 31,125.35
7	$ 31,125.35	$ 2,801.28	$ 420.19	$ 33,506.44
8	$ 33,506.44	$ 3,015.58	$ 452.34	$ 36,069.68
9	$ 36,069.68	$ 3,246.27	$ 486.94	$ 38,829.01
10	$ 38,829.01	$ 3,494.61	$ 524.19	$ 41,799.43
11	$ 41,799.43	$ 3,761.95	$ 564.29	$ 44,997.08
12	$ 44,997.08	$ 4,049.74	$ 607.46	$ 48,439.36
13	$ 48,439.36	$ 4,359.54	$ 653.93	$ 52,144.97
14	$ 52,144.97	$ 4,693.05	$ 703.96	$ 56,134.06
15	$ 56,134.06	$ 5,052.07	$ 757.61	$ 60,428.32
16	$ 60,428.32	$ 5,438.55	$ 815.78	$ 65,051.08
17	$ 65,051.08	$ 5,854.60	$ 878.19	$ 70,027.49
18	$ 70,027.49	$ 6,302.47	$ 945.37	$ 75,384.60
19	$ 75,384.60	$ 6,784.61	$ 1,017.69	$ 81,151.52
20	$ 81,151.52	$ 7,303.64	$ 1,095.55	$ 87,359.61
21	$ 87,359.61	$ 7,862.36	$ 1,179.35	$ 94,042.62
22	$ 94,042.62	$ 8,463.84	$ 1,269.58	$ 101,236.88
23	$ 101,236.88	$ 9,111.32	$ 1,366.70	$ 108,981.50
24	$ 108,981.50	$ 9,808.34	$ 1,471.25	$ 117,318.58
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29	$ 157,551.65	$ 14,179.65	$ 2,126.95	$ 169,604.35
30	$ 169,604.35	$ 15,264.39	$ 2,289.66	$ 182,579.08
Total Interest		$ 191,269.51		
Total Capital Gain Tax Paid		$ 28,690.43		
Net Gain		$ 162,579.08		

Final Closing Balance	$ 182,579.08
Total Invested	$ 20,000.00
Net Gain	$ 162,579.08
Average Annual Rate of Return	7.65%

Special Name Boxes

D3 = principle
D4 = interest_rate
D5 = capital_gain_tax_portion
D6 = income_tax_rate
D8 = income_tax_saving
D9 = additional_investment
D10 = net_tax_saving

4-2: Comparing RRSP's with Non-Registered Investments

	A	B	C	D	E	F	G	H	I	J	K
1	**4-2: Comparing RRSP's with Non-Registered Investments**										
2							Capital Gain Tax				
3			Principal 20000				Non-Registered Investment Earnings & Capital Gain Tax Portion & Marginal Tax Rate				
4			Interest Rate 0.09								
5			Capital Gain Tax Portion 0.5								
6			Average Income Tax & Marginal Tax Rate 0.3								
7											
8			Saving on Income Tax Return	principle*income_tax_rate			Saving on Income Tax Return	=principle*0.3			
9			Amount from Income Tax Saving into initial RRSP	6000							
10			Net Present Tax Saving	=income_tax_saving+additional_investment			Note: Assuming Capital Gain Tax is taken out each period.				
11											
12			Registered Plan				Non-Registered Plan				
13	Year	Opening Balance	Interest	Closing Balance		Year	Opening Balance	Interest	Capital Gain Tax	Closing Balance	
14	1	=principle+additional_investment	=B14*interest_rate	=B14+C14		1	=principle	=G14*interest_rate	=H14*capital_gain_tax_portion*income_tax_rate	=G14+H14+I14	
15	=A14+1	=D14	=B15*interest_rate	=B15+C15		=F14+1	=J14	=G15*interest_rate	=H15*capital_gain_tax_portion*income_tax_rate	=G15+H15+I15	
16	=A15+1	=D15	=B16*interest_rate	=B16+C16		=F15+1	=J15	=G16*interest_rate	=H16*capital_gain_tax_portion*income_tax_rate	=G16+H16+I16	
17	=A16+1	=D16	=B17*interest_rate	=B17+C17		=F16+1	=J16	=G17*interest_rate	=H17*capital_gain_tax_portion*income_tax_rate	=G17+H17+I17	
18	=A17+1	=D17	=B18*interest_rate	=B18+C18		=F17+1	=J17	=G18*interest_rate	=H18*capital_gain_tax_portion*income_tax_rate	=G18+H18+I18	
19	=A18+1	=D18	=B19*interest_rate	=B19+C19		=F18+1	=J18	=G19*interest_rate	=H19*capital_gain_tax_portion*income_tax_rate	=G19+H19+I19	
20	=A19+1	=D19	=B20*interest_rate	=B20+C20		=F19+1	=J19	=G20*interest_rate	=H20*capital_gain_tax_portion*income_tax_rate	=G20+H20+I20	
21	=A20+1	=D20	=B21*interest_rate	=B21+C21		=F20+1	=J20	=G21*interest_rate	=H21*capital_gain_tax_portion*income_tax_rate	=G21+H21+I21	
22	=A21+1	=D21	=B22*interest_rate	=B22+C22		=F21+1	=J21	=G22*interest_rate	=H22*capital_gain_tax_portion*income_tax_rate	=G22+H22+I22	
23	=A22+1	=D22	=B23*interest_rate	=B23+C23		=F22+1	=J22	=G23*interest_rate	=H23*capital_gain_tax_portion*income_tax_rate	=G23+H23+I23	
24	=A23+1	=D23	=B24*interest_rate	=B24+C24		=F23+1	=J23	=G24*interest_rate	=H24*capital_gain_tax_portion*income_tax_rate	=G24+H24+I24	
25	=A24+1	=D24	=B25*interest_rate	=B25+C25		=F24+1	=J24	=G25*interest_rate	=H25*capital_gain_tax_portion*income_tax_rate	=G25+H25+I25	
26	=A25+1	=D25	=B26*interest_rate	=B26+C26		=F25+1	=J25	=G26*interest_rate	=H26*capital_gain_tax_portion*income_tax_rate	=G26+H26+I26	
27	=A26+1	=D26	=B27*interest_rate	=B27+C27		=F26+1	=J26	=G27*interest_rate	=H27*capital_gain_tax_portion*income_tax_rate	=G27+H27+I27	
28	=A27+1	=D27	=B28*interest_rate	=B28+C28		=F27+1	=J27	=G28*interest_rate	=H28*capital_gain_tax_portion*income_tax_rate	=G28+H28+I28	
29	=A28+1	=D28	=B29*interest_rate	=B29+C29		=F28+1	=J28	=G29*interest_rate	=H29*capital_gain_tax_portion*income_tax_rate	=G29+H29+I29	
30	=A29+1	=D29	=B30*interest_rate	=B30+C30		=F29+1	=J29	=G30*interest_rate	=H30*capital_gain_tax_portion*income_tax_rate	=G30+H30+I30	
31	=A30+1	=D30	=B31*interest_rate	=B31+C31		=F30+1	=J30	=G31*interest_rate	=H31*capital_gain_tax_portion*income_tax_rate	=G31+H31+I31	
32	=A31+1	=D31	=B32*interest_rate	=B32+C32		=F31+1	=J31	=G32*interest_rate	=H32*capital_gain_tax_portion*income_tax_rate	=G32+H32+I32	
33	=A32+1	=D32	=B33*interest_rate	=B33+C33		=F32+1	=J32	=G33*interest_rate	=H33*capital_gain_tax_portion*income_tax_rate	=G33+H33+I33	
34	=A33+1	=D33	=B34*interest_rate	=B34+C34		=F33+1	=J33	=G34*interest_rate	=H34*capital_gain_tax_portion*income_tax_rate	=G34+H34+I34	
35	=A34+1	=D34	=B35*interest_rate	=B35+C35		=F34+1	=J34	=G35*interest_rate	=H35*capital_gain_tax_portion*income_tax_rate	=G35+H35+I35	
36	=A35+1	=D35	=B36*interest_rate	=B36+C36		=F35+1	=J35	=G36*interest_rate	=H36*capital_gain_tax_portion*income_tax_rate	=G36+H36+I36	
37	=A36+1	=D36	=B37*interest_rate	=B37+C37		=F36+1	=J36	=G37*interest_rate	=H37*capital_gain_tax_portion*income_tax_rate	=G37+H37+I37	
38	=A37+1	=D37	=B38*interest_rate	=B38+C38		=F37+1	=J37	=G38*interest_rate	=H38*capital_gain_tax_portion*income_tax_rate	=G38+H38+I38	
39	=A38+1	=D38	=B39*interest_rate	=B39+C39		=F38+1	=J38	=G39*interest_rate	=H39*capital_gain_tax_portion*income_tax_rate	=G39+H39+I39	
40	=A39+1	=D39	=B40*interest_rate	=B40+C40		=F39+1	=J39	=G40*interest_rate	=H40*capital_gain_tax_portion*income_tax_rate	=G40+H40+I40	
41	=A40+1	=D40	=B41*interest_rate	=B41+C41		=F40+1	=J40	=G41*interest_rate	=H41*capital_gain_tax_portion*income_tax_rate	=G41+H41+I41	
42	=A41+1	=D41	=B42*interest_rate	=B42+C42		=F41+1	=J41	=G42*interest_rate	=H42*capital_gain_tax_portion*income_tax_rate	=G42+H42+I42	
43	=A42+1	=D42	=B43*interest_rate	=B43+C43		=F42+1	=J42	=G43*interest_rate	=H43*capital_gain_tax_portion*income_tax_rate	=G43+H43+I43	
44											
45		Total Interest	=SUM(C14:C43)				Total Interest	=SUM(H14:H43)		Result of Some Sums	
46							Total Capital Gain Tax Paid	=SUM(I14:I43)		I29 = principle	
47							Net Gain	=H45+H6		J34 = interest_rate	
48		Final Closing Balance	=D43							I35 = capital_gain_tax_portion	
49		Net Present Tax Saving	=net_tax_saving							I36 = income_tax_rate	
50		Total Invested	=principle+additional_investment				Final ClosingBalance	=J43		I38 = income_tax_saving	
51	Average Income Tax (Marginal Tax) at Withdrawal	=D45*income_tax_rate					Total Invested	=principle		I39 = additional_investment	
52		Net Gain	=D45+net_tax_saving-D51-principle-additional_investment				Net Gain	=H50+principle		I310 = net_tax_saving	
53											
54		Average Annual Rate of Return	=(D46/B14)^(1/A43)-1				Average Annual Rate of Return	=(H50/G14)^(1/F43)-1			
55											

4-3: Investment Portfolios

Investment Portfolios: - the **different amounts** and **categories** of your total investment.

Investment Vehicles: - different ways of investing.

a) **Term Deposit:** (Lump Sum) – interest is paid if and only if **NO amount is withdrawn before each term had ended**. Interest is forfeited or reduced if any amount is withdrawn. Investment has a set compounded interest rate.

b) **Guaranteed Investment Certificate (GIC):** (Lump Sum) – interest is guaranteed for the amount invested. Money is **LOCKED IN** until the end of the entire investment period.

c) **Bonds:** (Lump Sum or Annuity) – when a government tries to raise money, they sell bonds certificates of different values for different terms of time. Interest rates are guaranteed initially. **The account is secured**.

d) **Debentures:** (Lump Sum or Annuity) – a type of bonds offer by governments of second or third world countries. Interest rates are not guaranteed but they are usually higher than bonds offer by governments of first world countries. **The account is NOT secured**.

e) **Stocks:** (Lump Sum or Annuity) – money is used to buy **shares (units of public company ownership)**. Each share has a specific value. It can **rise or fall** each minute of the working day depending on the economy and the performance of the company.

f) **Mutual Fund:** (Lump Sum or Annuity) – money is used to buy **fund units**. Each mutual fund consists of many shares from different companies. They do not fluctuate as much as one particular stock. "The eggs are NOT put into one basket."

Three Categories of Investment

1. Cash Investments

- include bank accounts, term deposits, money market, and mutual funds (non-registered).
- **easily accessible (liquidate)**: - convert back to cash.
- mostly **low risk but very little return** (except high risk type mutual funds and money market).

2. Fixed Income Investments

- include GIC, Bonds, Debentures
- can **liquidate only at certain time of a period**; otherwise, there is a penalty or no interest is paid.
- provide a **source of regular income with limited cash**.

3. Equity Investments

- include stocks and medium to high risk mutual funds (registered and non-registered).
- generally, they are **medium to high risk (NO guarantee rates but potentially high return)**.
- can be used as **Long Term Growth Investments**. The average annual rate of return will still be higher than rates offer in cash and fixed income investment. This is due to the fact that longer time of investment would likely rides out any downturn in the economy.

Recommended Asset Allocation in Retirement Investment Portfolio

	Early Years (Age 18 to 35)	Established Years (Age 35 to 50)	Pre-Retired Years (Age 50 to 65)	Retired Years (Age 65 and Over)
Cash	10%	10%	10%	20%
Fixed-Income	40%	15%	40%	50%
Equities	50%	70%	50%	30%

Nominal Interest Rate: - the stated interest rate that was advertised.

Average Annual Rate of Return (Effective Rate): - the actual interest rate if the interest were to calculate compound annually into of other type of compounding periods.

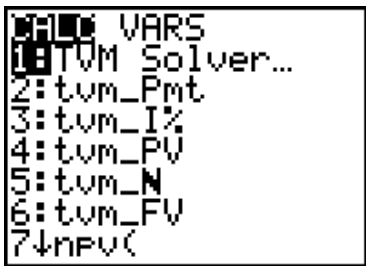
▸ **Eff (Effective Rate)**: - returns Effective Interest Rate given the Nominal Interest Rate in Percentage and Number of Compounding Terms Per Year.

▸ **Eff (Nominal Rate in %, Number of Compound Terms Per Year)**

To access ▸ Eff:

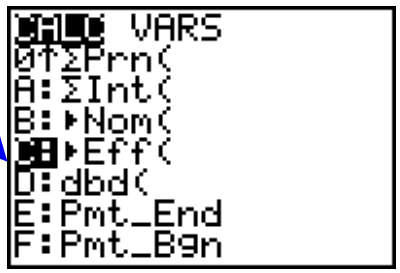
1. Press APPS

2. Select Option 1



3. Press ▲

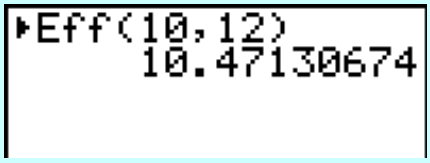
4. Select Option C



Example 1: Find the effective interest rate if an investment firm advertised at 10%/a compounded monthly.

Method 1: Using ▸ Eff

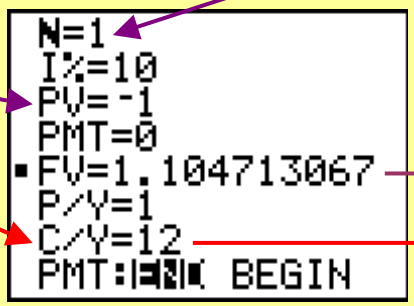
Nominal Rate = 10%
12 Compound Terms / Year



Effective Rate = 10.47%

Method 2: Using the TVM Solver

We can first calculate how much an investment of **$1** will be worth after **1 year** at 10%/a **compound monthly**.

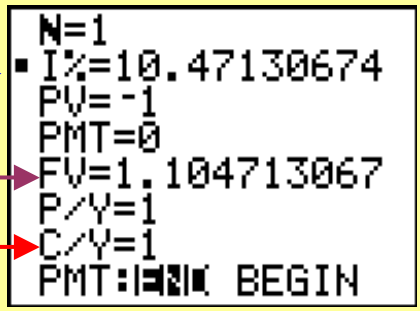


Solve for Effective Rate

SAME Return

Change to compound annually

Then, assuming the **same return** for an investment **compound annually**, the rate will be the **effective interest rate**.



Effective Rate 10.47%

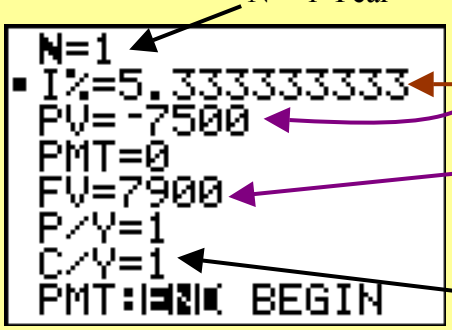
Example 2: Complete the table below. Find the average annual rate of return for the following portfolio. If this portfolio belongs to a person 25 years of age and she would like an additional investment of $3000 next year, how should she allocate her new asset?

2001 Investment Portfolio			
Investment	**Amount Invested**	**Average Rate of Return**	**Amount of Return**
Saving Account	$1500	2%	**$30**
GIC	$2000	7%	**$140**
Mutual Funds	$3000	9%	**$270**
Stocks	$1000	(4%)	**–$40**
Total	**$7500**	**Brackets mean a LOSS**	**$400**

TVM Solver

N = 1 Year

(**Cannot use ▸ Eff** because we do not know the nominal rate.)



Effective Rate = 5.333%

Future Value = $7500 invested + $400 return

Assume Compound Annually for Effective Rate

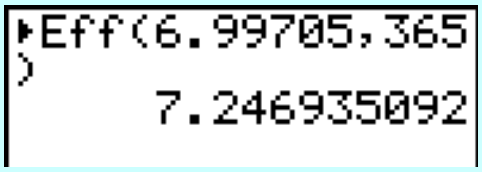
Due to her age she should put most of the additional $3000 into her equities and fixed income investments (Mutual Funds and GIC).

Example 3: A line of credit offers by a major bank advertised the annual interest rate as 7% and the daily interest rate as 0.01917%. What is the effective rate?

The daily interest rate does NOT equal to 7% when it is multiplied by 365 days.
(0.01917% × 365 = 6.99705%)
It is so closed to 7% that the bank rounded it up and advertised it as the nominal rate.
The actual nominal rate is 6.99705%

Using ▸Eff

Nominal Rate = 6.99705%
365 Compound Terms / Year



Effective Rate = 7.247%

Example 4: Mary invested $5000 in a particular natural resource mutual fund at the beginning of 1997. It has the following performance over the next 5 years.

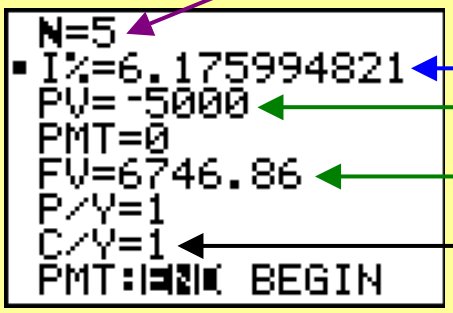
Year	**1997**	**1998**	**1999**	**2000**	**2001**
Average Rate of Return	5.2%	6.4%	(3.1%)	7.9%	15.3%

a. Complete the following table.

Year	Opening Balance	Average Rate of Return	Calculations	Closing Balance
1997	$5000.00	5.2%	$5000 (1 + 0.052)	$5260.00
1998	$5260.00	6.4%	$5260 (1 + 0.064)	$5596.64
1999	$5596.64	(3.1%)	$5596.64 (1 – 0.031)	$5423.14
2000	$5423.14	7.9%	$5423.14 (1 + 0.079)	$5851.57
2001	$5851.57	15.3%	$5851.57 (1 + 0.153)	$6746.86

b. What is the average annual interest rate for the period from 1997 to 2001?

TVM Solver N = 5 Years (2001 to 1997 inclusive)



Average Annual Interest Rate (Effective Rate) 6.176%

Initial Opening Balance

Final Closing Balance

Assume Compound Annually for Effective Rate

4-3 Assignment: pg. 171-172 # 1 to 7

Finance Project: Retirement Planning

Purpose: To calculate the amount of contribution to make **every month** for your retirement.

Procedure: (Use the TVM Solver for all your calculations)

1. **Basic Information (Copy this table into your final write up)**

At what age do you want to start saving?	
At what age do you want to retire?	
At what age do you think you will die?	
What is the average annual inflation rate? (choose between 2%/a to 5%/a compound annually)	
What is the average interest rate on your savings? (choose between 6%/a to 15%/a) Compound Period Per Year? (Annually, Semi-annually, Quarterly, or Monthly)	
What is the annual gross income you would like to retire with in today's value? (must be above $25000 per year)	

2. **Total Withdrawal Amount of your Retirement**
   - **a.** Using the average annual inflation rate, calculate how much you will have to earn a year when you **start retirement**.
   - **b.** Using the average annual inflation rate, calculate how much you will have to earn a year **just before you die**.
   - **c.** Average the two results above. **This is your average annual income during your retirement.**
   - **d.** Use the result in part **c.** above, multiply by the number of years you will spend retiring. **This is your Total Withdrawal Amount of your Retirement.**

3. **Retirement Goal (amount needed at the beginning of the retirement)**
   - Using the above result in **2c.**, and the average interest rate for your savings, calculate how much you will need at the beginning of your retirement. **This is your Retirement Goal.**

4. **Monthly Contribution for your Retirement**
   - Using the average interest rate on your savings, the number of months you have to save for your retirement goal and the retirement goal itself, calculate the **Monthly Contribution for your retirement.**

5. **Final Analysis**
   - **a.** Determine the **total interest earned** from the time you started saving until the time you die.
   - **b.** Find the **average annual rate of return** each year from the time you start saving until the time you die.

Note:

- Students can work together to discuss the project, but they should each have their own numbers to work with and do their own calculations. **Students who copied from each other will end up sharing the mark.** Let's say the mark was 70% and two students were involved in copying each other's work. They each get 35%.
- Final write up **MUST include all TVM Solver layouts** (numbers used), and all work must be shown.
- Project handed in **one day late** has a penalty of **30% off the total mark**. **Project handed in two days after the due date and later will not be marked.**

Due Date: _____

4-4: Leasing

Taxes and Extra Charges

When purchasing a new vehicle, there are many other charges to consider along with the sales tax.

1. **GST**: - 7% of the final price of the vehicle
2. **Shipping & Freights**: - the cost of shipping the vehicle to Alberta from Ontario or BC (about $850.00).
3. **Tires & Air Conditioning Tax**: - government charges for environmental purposes (about $100).
4. **Document & Process Fee**: - the cost of processing documents related to the vehicle sale (about $125).
5. **Additional Accessories**: - these are extra options that do not come with the standard features of the vehicle (example: underside coating and fabric protections, rear spoiler, hood deflector, running board, head lights protector).

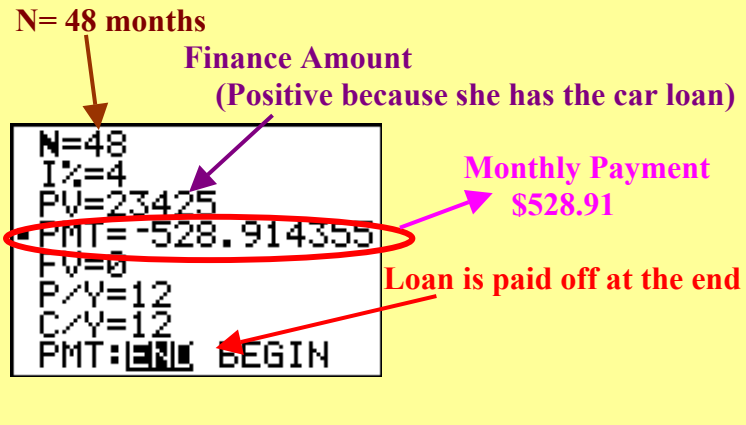
Financing

- the buyer agreed to **pay it off** with equal payments per month over a number of years.
- **balance owing at the end** of the finance term is **$0**.
- **all taxes and extra charges are added up into the final price of the vehicle before monthly payment is calculated**. Therefore, there is NO GST on the monthly payment.
- at the end of the finance term, **the car belongs to the buyer**.
- when we bargain with the dealer, we **bargain on the final price of the vehicle**.
- the **finance rate (compounds monthly) is advertised**, and monthly payment is then calculated after the bargaining is over.
- **payment is always made at the END of the month**.

Example 1: Mary wants to finance a new vehicle that has a final price of $20000. She went with a finance term of 48 months at the interest rate of 4%/a. Suppose her extra charges are the same as outlined above with the additional accessories of $950 that covers the underside coating and fabric protection.

a. Calculate her monthly payment with no down payment.

Final Price	$20000
GST (7% × $20000)	$1400
Shipping & Freights	$850
Tire & AC Tax	$100
Document & Process Fee	$125
Additional Accessories	$950
Finance Amount	$23425



b. Calculate the total cost of the vehicle.

Total Cost = Number of Months × Monthly Payment
Total Cost = 48 × $528.91

Total Cost = $25387.68

c. What is the total interest paid over 4 years?

Interest Paid = Total Cost – Finance Amount
Interest Paid = $25387.68 – $23425.00

Interest Paid = $1962.68

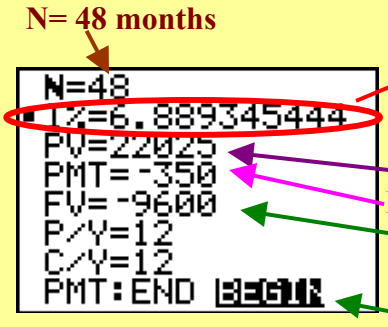
Leasing

- the buyer agreed to **pay off a portion of the final price** with equal month payments over time.
- the **balance owing** at the end of the finance term called the **residual** (usually 45% to 55% of the final vehicle price).
- **only extra charges are added up into the final price of the vehicle**. GST is added on the monthly payment and residual at the end of the term.
- at the end of the lease, the buyer has **two options**. He or she can pay the residual with GST and keep the car. He or she can walk away and the dealer will take the vehicle back.
- the monthly payment is like the monthly rental cost of the vehicle
- when we bargain with the dealer, we **bargain on the monthly lease payment** of the vehicle.
- the **lease rate (compounds monthly) is NEVER advertised**, and it is then calculated after the bargaining is over.
- payment is always made at the **beginning** of the month. **(On the TVM Solver, PMT is set to BEGIN)**.

Example 2: John wants the same new vehicle as Mary's, which has a final price of $20000. He went with a lease term of 48 months with a monthly payment of $350 plus tax. At the end of the lease, the residual amount is 48% of the final price of the vehicle. Suppose his extra charges are the same as outlined above with the additional accessories of $950 that covers the underside coating and fabric protection.

a. Calculate his interest rate for the lease with no down payment.

Final Price	**$20000**
Shipping and Freights	**$850**
Tire and AC Tax	**$100**
Document and Process Fee	**$125**
Additional Accessories	**$950**
Lease Amount	**$22025**
Monthly Payment (before GST)	**$350**
Residual before GST (48% × $20000)	**$9600**



Leasing Payment is at the BEGINNING of the month.

b. Calculate the total cost of the vehicle including GST.

Total Cost = (Number of Payments × Monthly Payment with GST) + (Residual with GST)
Total Cost = (48 × $350 × 1.07) + ($9600 × 1.07)

Total Cost = $28248

c. What is the total interest paid over 4 years?

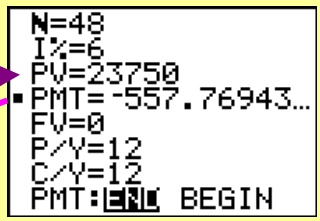
Interest Paid = Total Cost – Lease Amount with GST on the Final Price
Interest Paid = $28248 – ($20000 × 1.07 + $850 + $100 + $125 + $950)
Interest Paid = $28248 – $23425

Interest Paid = $4823

Example 3: Mr. Tang bought a $25000 car. His down payment included the GST and all other extra costs.

a. Calculate his monthly payment if he goes with 6%/a for 48 months with $3000 down payment. Determine what he would pay in total and in interest.

Finance Amount = Final Price with GST – Down Payment
Finance Amount = $25000 × 1.07 – $3000
Finance Amount = $23750



Monthly Payment = $557.77

Total Cost = Total Payments + Down Payment
Total Cost = 48 × $557.77 + $3000

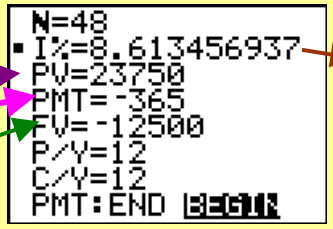
Total Cost = $29772.96

Interest = Total Cost – Final Price with GST
Interest = $29772.96 – ($25000 × 1.07)
Interest = $29772.96 – $26750

Interest Paid = $3022.96

b. Calculate his leasing rate if he pays $1250 for down payment (which covers all other extra costs), and $365 + GST for 48 months with a 50% residual value. Determine what he would pay in total and in interest.

Lease Amount = Final Price – Down Payment
Lease Amount = $25000 – $1250
Lease Amount = $23750
Monthly Lease Payment = $365
Residual = 50% × $25000 = **$12500**



Leasing Rate = 8.613%

Total Cost = Total Payments with GST + Down Payment + Residual with GST
Total Cost = 48 × $365 × 1.07 + $1250 + $12500 × 1.07

Total Cost = $33371.40

Interest Paid = Total Cost – (Final Price with GST)
Interest Paid = $33371.40 – ($25000 × 1.07)
Interest Paid = $33371.40 – $26750

Interest Paid = $6621.40

4-4 Assignment: Worksheet 4-4: Leasing and Financing

Applied Math 30 Worksheets: 4-4 Leasing and Financing a Vehicle

For all the questions below, assume the price of the vehicle includes the costs of freights / shipping, tire & A/C tax, documentation / administration fee, and any additional accessories. The price of the vehicle does NOT include GST (7%). There is no PST, as we live in the wonderful province of Alberta. All interest rates are compounded monthly; all payments are paid monthly.

As you work through these problems with the TVM Solver, be sure to copy down all the values you have entered and circle the final answer.

1. A Toyota 4-Runner is sold for $42000 plus GST. The dealership advertises the finance rate at 1.9% over 3 years and 3.9% over 4 years. The 4 years leasing program has a 45% residual amount with a monthly payment of $750 plus GST. All three programs have zero down payments.
- **a)** Calculate the final purchase price of the vehicle (with GST).
- **b)** Calculate the monthly payment of the 3 years finance program and the total cost with GST.
- **c)** Calculate the monthly payment of the 4 years finance program and the total cost with GST.
- **d)** For the 4 years leasing program, calculate the leasing rate, which is hidden from the advertisement.
- **e)** Calculate the total cost of the vehicle with GST.
- **f)** Compare the total interest paid for all 3 programs.
- **g)** Using residual amount of the leasing program, find the annual depreciation rate of the vehicle.

2. Mr. Tang wants to buy an Acura TL. The price of the vehicle is $38500 plus GST. He found out that the finance rate is 7.5% over 60 months, and the 48 months leasing program has a rate of 9% with a 50% residual amount. From the sale of his last vehicle, Mr. Tang can put down $11214.95 plus GST as down payment.
- **a)** Find the monthly payment of the 60 months finance program.
- **b)** Calculate the total cost with GST of the finance program.
- **c)** Find the residual payment with GST of the leasing program.
- **d)** Find the monthly payment with GST for the leasing program.
- **e)** Calculate the total cost with GST of the leasing program.
- **f)** Find the purchase price of the vehicle (buying it out right) with GST.
- **g)** Compare the total interest paid for the two programs.
- **h)** Using the residual amount of the leasing program, find the annual depreciation rate of the vehicle. Does this accurately reflect the actual resell value of the car? Explain.

3. In most newspaper advertisements, a lot of car dealerships do not state the retail price of their vehicles. Ford recently advertised "the Focus can be bought with a finance rate of 0.9% for 36 months at an incredible monthly payment of $450 with zero down payment!"
- **a)** Calculate the retail price of the Focus with GST.
- **b)** What is the MSRP (Manufacturer Suggested Retail Price) of the Focus (without GST)?
- **c)** How much interest is paid over the 36 months?
- **d)** If you decide to put $5000 down payment, how much lower will your monthly payment be?
- **e)** How much interest is paid over the 36 months with $5000 down payment? How much interest would have been saved compared to no down payment?
- **f)** If you are buying this Focus, and your $5000 is in a mutual fund that is earning 8%/a compound quarterly, is it worth it to withdraw the amount and use it as the down payment of the new vehicle? Explain.

4. In a TV advertisement, Chrysler announced, "Our Intrepid will only cost you $380 a month for 48 months with no money down!" As you read the fine prints at the bottom carefully, you found that GST on the monthly lease payment is extra and the residual amount is $13500 plus GST.
- **a)** If the leasing rate is 6.2%, what is the actual retail price of the Intrepid before GST?
- **b)** Find the purchase cost of the Intrepid with GST.
- **c)** Calculate the total cost of the lease with GST.
- **d)** Find the total interest paid over the 48 months.
- **e)** What is the residual amount of the Intrepid in percentage?

5. Jimmy leased a $15750 Pontiac Firefly for 48 months. His monthly payment was $250 plus GST. Since Jimmy did not listen to Mr. Tang when he took Applied Math 30, he had completely forgotten about the 43% residual amount at the end of the lease. The dealership sent him a letter that he has the option of buying the car back or giving it up. Jimmy really needs his wheels, because the public transit workers are on strike. He negotiated with the dealership to refinance the residual payment over the next 36 months at 4.25%
- **a)** What is the residual payment (without GST) for Jimmy at the end of the lease?
- **b)** What is the residual payment (with GST) for Jimmy at the end of the lease?
- **c)** What was the leasing rate?
- **d)** How much money did Jimmy pay (with GST) at the end of the 48 months lease?
- **e)** What is Jimmy monthly payment for the 36 months refinancing program?
- **f)** How much money in total did Jimmy pay for the refinancing period of 36 months?
- **g)** What is the total amount of money Jimmy paid for the Firefly?
- **h)** Find the total interest Jimmy paid over the 7 years.
- **i)** What is his average interest paid per year for the 7 years?
- **j)** What is his average annual interest rate for the 7 years?
- **k)** Using residual amount of the leasing program, find the annual depreciation rate of the vehicle.
- **l)** Using the answer you obtained in **5k)**, what would be the value of the car when he finally paid it off?

Answers

1a. $44940 **1b.** $1285.24, $46268.64 **1c.** $1012.69, $48609.12 **1d.** 10.49% **1e.** $58743
1f. 3 years finance program: $1328.64; 4 years finance program: $3669.12; leasing program: $13803
1g. 13.75%

2a. $585.01 **2b.** $47100.60 **2c.** $20597.50 **2d.** $341.76; with GST: $365.68
2e. $50150.29 **2f.** $41195 **2g.** Finance: $5905.6; Lease: $8955.29 **2h.** 12.5%; No, because the car is probably going to worth a lot more than the residual amount of $19250, Mr. Tang will likely buy back the car at the end of the lease.

3a. $15977.34 **3b.** $14932.09 **3c.** $222.66 **3d.** $309.18 **3e.** $153.14; Saved $69.52
3f. No, a saving of $69.52 over 3 years is chicken feed. If you would have put the $5000 in the mutual fund account that earns 8%/a compound quarterly over 3 years, you would have made $630.81. This is more than 9 times as much as the money you saved.

4a. $26742.38 **4b.** $28614.35 **4c.** $33961.80 **4d.** $5347.45 **4e.** 50.48%
5a. $6772.50 **5b.** $7246.58 **5c.** 6.68% **5d.** $12840 **5e.** $214.75 **5f.** $7731
5g. $20571 **5h.** $3718.50 **5i.** $531.21 **5j.** 2.89% **5k.** 14.25% **5l.** $39.38

4-4: Leasing and Financing a Vehicle

Purchasing

MSRP (Manufacturer Suggested Retail Price)	$ 20,000.00
GST	$ 1,400.00
Freights and Shippings	$ 850.00
Documentation and Administration Fee	$ 100.00
Tire & Air Conditioning Tax	$ 125.00
Additional Accessories	$ 950.00
Total Purchase Price	$ 23,425.00

Financing

Total Purchase Price	$ 23,425.00
Down Payment	$ -
Finance Amount	$ 23,425.00
Number of Months	48
Finance Rate	4.00%
Monthly Payment	$528.91

Leasing

MSRP (Manufacturer Suggested Retail Price)	$ 20,000.00
Freights and Shippings	$ 850.00
Documentation and Administration Fee	$ 100.00
Tire & Air Conditioning Tax	$ 125.00
Additional Accessories	$ 950.00
Total Price Before Tax	$ 22,025.00
Down Payment Before Tax	$ -
Leasing Amount	$ 22,025.00
Number of Months	48
Residual Rate	48.00%
Residual Amount	$ 9,600.00

To Solve for Leasing Rate		To Solve for Leasing Payment	
Lease Payment (Enter Value)	$ 350.00	Lease Rate (Enter Value)	6.889%
Lease Rate (Calculated)	6.889%	Lease Payment (Calculated)	$350.00
Lease Payment (with GST)	$ 374.50	Lease Payment (with GST)	$ 374.50

Special Name Boxes

E4 = MSRP	J9 = finance_months
E5 = GST	J10 = finance_rate
E7 = Shipping	J12 = finance_payment
E8 = Fee	J17 = finance_cost
E9 = Tire_AC_Tax	J18 = purchase_cost
E10 = Accessories	J20 = finance_interest
E12 = purchase_price	J22 = annual_finance_interest
E23 = lease_amount	J26 = lease_payment
E27 = lease_months	J27 = leasing_cost
E29 = residual_rate	J28 = final_lease_cost
E30 = residual	J31 = lease_interest
J5 = down_payment	J33 = annual_lease_interest
J7 = finance_amount	J39 = avg_annual_depreciation rat

Final Analysis

FINANCING	
Total Finance Cost	$ 25,387.89
Total Purchase Cost	$ 23,425.00
Total Finance Interest Paid	$ 1,962.89
Average Annual Interest	$ 490.72
Average Annual Interest Rate	2.03%

LEASING	
Lease Payment before GST (Enter Value)	$ 350.00
Total Leasing Cost (before GST)	$ 26,400.00
Total Leasing Cost (with GST)	$ 28,248.00
Total Purchase Cost	$ 23,425.00
Total Lease Interest Paid	$ 4,823.00
Average Annual Interest	$ 1,205.75
Average Annual Interest Rate	4.79%

DEPRECIATION (based on residual)	
Residual Rate	48.00%
Number of Leased Months	48
Average Annual Depreciation Rate	13.00%
Age of the Vehicle in Years (Enter Value)	4
Future Value of the Vehicle (Calculated)	$ 9,600.00

4-4: Leasing and Financing a Vehicle

Purchasing

MSRP (Manufacturer Suggested Retail Price) 21000

GST =E4*0.07

Freights and Shippings 850
Documentation and Administration Fee 100
Tire & Air Conditioning Tax 125
Additional Accessories 850

Total Purchase Price =SUM(E4:E10)

Leasing

MSRP (Manufacturer Suggested Retail Price) =MSRP
Freights and Shippings =Shipping
Documentation and Administration Fee =Fee
Tire & Air Conditioning Tax =Tire_AC_Tax
Additional Accessories =Accessories

Total Price Before Tax =SUM(E16:E20)
Down Payment Before Tax =down_payment*1.07

Leasing Amount =E27-E23

Number of Months 48

Residual Rate 0.48

Residual Amount =MSRP*residual_rate

To Solve for Leasing Rate

Lease Payment (Enter Value)	350	Lease Rate (Enter Value)	0.0889
Lease Rate (Calculated)	=RATE(lease_months,-C33,lease_amount,-residual)*12	Lease Payment (Calculated)	=PMT(E33/12,lease_months,-lease_amount,residual,1)*1.07
Lease Payment with GST	=C33*1.07	Lease Payment with GST	=E34*1.07

To Solve for Leasing Payment

Special Name	
E4 = MSRP	
E5 = GST	
E7 = Shipping	
E8 = Fee	
E9 = Tire_AC_Tax	
E10 = Accessories	
E12 = purchase_price	
E23 = lease_amount	
E27 = lease_months	
E28 = residual_rate	
E30 = residual	
E5 = down_payment	
J7 = finance_amount	

J9 = finance_months	
J10 = finance_rate	
J12 = finance_payment	
J17 = finance_cost	
J18 = purchase_cost	
J20 = finance_interest	
J22 = annual_finance_interest	
J26 = lease_payment	
J27 = leasing_cost	
J28 = final_lease_cost	
J31 = lease_interest	
J33 = annual_lease_interest	
J39 = avg_annual_depreciation_rate	

Financing

Total Purchase Price =purchase_price
Down Payment 0

Finance Amount=purchase_price-down_payment

Number of Months 48
Finance Rate 0.04

Monthly Payment =PMT(finance_rate/12,finance_months,-finance_amount,0,0)

Final Analysis

FINANCING

Total Finance Cost=finance_months*finance_payment+down_payment

Total Purchase Cost=purchase_price

Total Finance Interest Paid=finance_cost-purchase_cost

Average Annual Interest=J20/finance_months*12

Average Annual Interest Rate=(finance_cost-purchase_cost)*(1/(finance_months/12))-1

LEASING

Lease Payment before GST (Enter Value) 350

Total Leasing Cost (before GST)=lease_months*lease_payment+residual+E23

Total Leasing Cost (with GST)=leasing_cost*1.07

Total Purchase Cost=purchase_price

Total Lease Interest Paid=final_lease_cost-purchase_cost

Average Annual Interest=lease_interest/finance_months*12

Average Annual Interest Rate=(final_lease_cost/purchase_cost)^(1/(lease_months/12))-1

DEPRECIATION (based on residual)

Residual Rate=residual_rate

Number of Leased Months=lease_months

Average Annual Depreciation Rate=(100%-residual_rate)/lease_months*12

Age of the Vehicle in Years (Enter Value) 4

Future Value of the Vehicle (Calculated)=MSRP*(100%-J41*avg_annual_depreciate_rate)

4-5: Mortgage Calculations

Mortgage: - the amount of money borrowed from a financial institution for the purchase of a home.

Maximum Mortgage Allowed: - the **maximum amount of money a person can borrow** from a financial institution based on the borrower(s)' incomes, expenses, and credit history. **This does NOT mean the borrower has to borrow the maximum amount**.

Liabilities: - **fixed expenses** like credit card bills, student loans, personal credit line payments, car payments, insurance …etc.

Amortization: - the number of years needed to pay off the mortgage. The most common amortization periods are 5, 10, 15, 20, 25 and 30 years.

Mortgage Rate: - the interest rate of the mortgage.

- **Fixed Rate**: - also called a **CLOSED Mortgage**, the **Rate is FIXED** for the **duration of the term**. A Mortgage term can be 6 months, 1, 2, 3, 4, 5, 7, 10, or 18 years. **When a mortgage term has passed, it needs to be re-negotiated**. **(In Canada, a Close Mortgage is always calculated compounded semi-annually)**.
- **Variable Rate**: - also called an **OPEN Mortgage**, the **Rate can FLUCTUATE**. The borrower can convert to a fixed rate when interest rates start to climb due to a better economy. However, the Variable Rate is always LOWER than any kind of Fixed Rate. **(In Canada, an Open Mortgage is always calculated compounded monthly)**.

Mortgage Payments: - the **amount** a borrower **pays** to the financial institution **every period** until the mortgage is paid off.

Payment Arrangements	Monthly	Semi-monthly	Biweekly	Weekly	Daily
Number of Payments per Year	12	24	26	52	365

Down Payment: - the amount of cash you want to put up as the **initial deposit** on the house.

- it guarantees your place during the negotiation of the final price of the house.
- the home buyer(s) may borrow up to **$20,000** from the RRSP account(s) with NO penalty! **(Home Buyers Plan – HBP)**. It is a **single lifetime allowance**, and the home buyer(s) will have to pay back the borrowed amount in a form of decreasing the RRSP contribution limit for the next15 years.
- **Minimum 5% of the final price**.
- **if the down payment is between 5% to 25%**, it is **subjected to an insurance charge** by **CMHC (Canada Mortgage and Housing Corporation** http://www.cmhc-schl.gc.ca). The reason is that most banks only accept mortgage application if the borrower puts down 25% down payment. Therefore, CMHC has to guarantee the bank that homebuyers with less than 25% down payment will not default on their mortgages. This insurance cost will then be added on to total mortgage amount.

Size of Down Payment of the House	5%	5% to 10%	10% to 15%	15% to 20%	20% to 25%
CMHC Insurance (expressed as % of the final house price)	3.75%	2.50%	2.00%	1.25%	0.75%

Calculating Maximum Mortgage Amount Allowed

Step 1: Calculate Gross Monthly Income

$$\text{Gross Monthly Income} = \frac{\text{All Gross Annual Incomes}}{12}$$

Step 2: Calculate Maximum Monthly Mortgage Payments Allowed (32% of Gross Monthly Income)

Maximum Monthly Mortgage Payments Allowed = Gross Monthly Income × 32%

Step 3: List ALL Liabilities

Liabilities	Monthly Amount
Car Payments	
Insurance (Life, Auto)	
Student Loans	
Personal Credit Line	
Credit Cards	
Total Liabilities	

NOTE: The Total Liability should not be over 40% of your gross monthly income.

Liabilities Difference = 40% of Gross Income – Total Liabilities

Mortgage Approved: when Liability Difference is POSITIVE.

Mortgage Denied: when Liability Difference is NEGATIVE.

40% of Gross Income – Total Liabilities	Liabilities Difference	Approved / Denied

Step 4: Find the ACTUAL Maximum Monthly Mortgage Payment Allowed

a. Compare the 32% of Gross Monthly Income (in step 2) with the Liabilities Differences (in step 3).
b. Take the smaller of the two amounts. This is the maximum amount you may spend on the monthly mortgage payment and other house related expenses.

Actual Maximum Monthly Mortgage Payment Allowed = The SMALLER Amount of Step 2 and Step 3 – Heat & Property Tax

Step 5: Calculate the Maximum Mortgage Amount Allowed (Use TVM Solver)

N= Amortization Period × 12

I%= Mortgage Rate

PV= Maximum Mortgage Amount Allowed (SOLVE)

PMT= – Actual Maximum Monthly Mortgage Payment Allowed (from step 4)

FV= 0 (Balance is $0 at the end of the Amortization Period)

P/Y= 12 (12 Monthly Payments per Year)

C/Y= 2 (assume fixed rate)

PMT: END BEGIN

Example 1: Jack and Jill have a combined annual gross income of $85,000. They have an average credit card bill of $250 per month, and a monthly car payment of $300. Suppose they predict that heat and property tax will cost them on average $100 and $180 a month respectively, what is their maximum mortgage amount allowed at 7.750%/a over 25 years?

Step 1: Gross Monthly Income

$$\text{Gross Monthly Income} = \frac{\text{All Gross Annual Incomes}}{12} = \frac{\$85000}{12}$$

Gross Monthly Income = $7083.33

Step 2: Maximum Monthly Mortgage Payments Allowed (32% of Gross Monthly Income)

Maximum Monthly Mortgage Payments Allowed = Gross Monthly Income × 32%
$= \$7083.33 \times 0.32$

Maximum Monthly Mortgage Payments Allowed = $2266.67

Step 3: List ALL Liabilities

Liabilities	Monthly Amount
Car Payments	$300
Credit Cards	$250
Total Liabilities	**$550**

Since the Liabilities Difference is Positive, the Mortgage is APRROVED.

40% of Gross Income – Total Liabilities	Liabilities Difference
0.4 × $7083.33 – $550	**$2283.33**

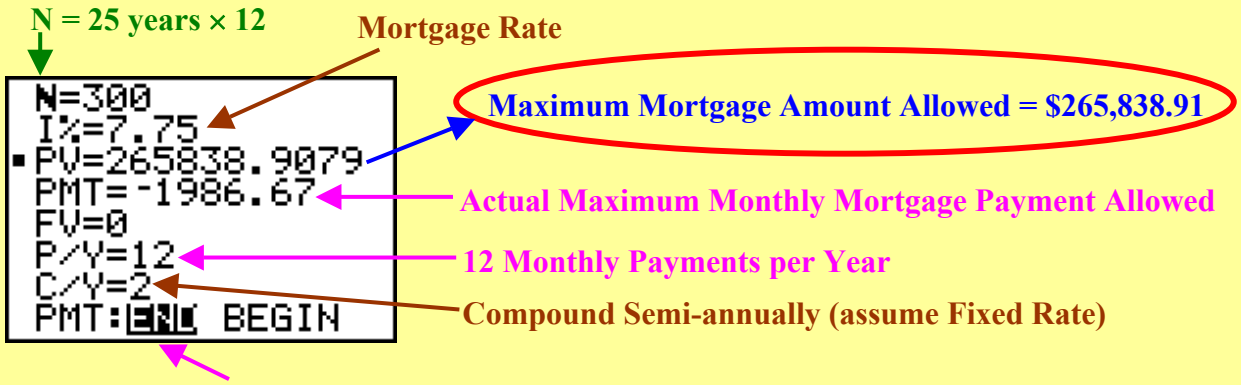
Step 4: ACTUAL Maximum Monthly Mortgage Payment Allowed

Actual Maximum Monthly Mortgage Payment Allowed = **The SMALLER Amount of Step 2 and Step 3 – Heat & Property Tax**

= $2266.67 (**SMALLER** Amount from Step 2) – ($100 + $180)

Actual Maximum Monthly Mortgage Payment Allowed = $1986.67

Step 5: Maximum Mortgage Amount Allowed (Use TVM Solver)

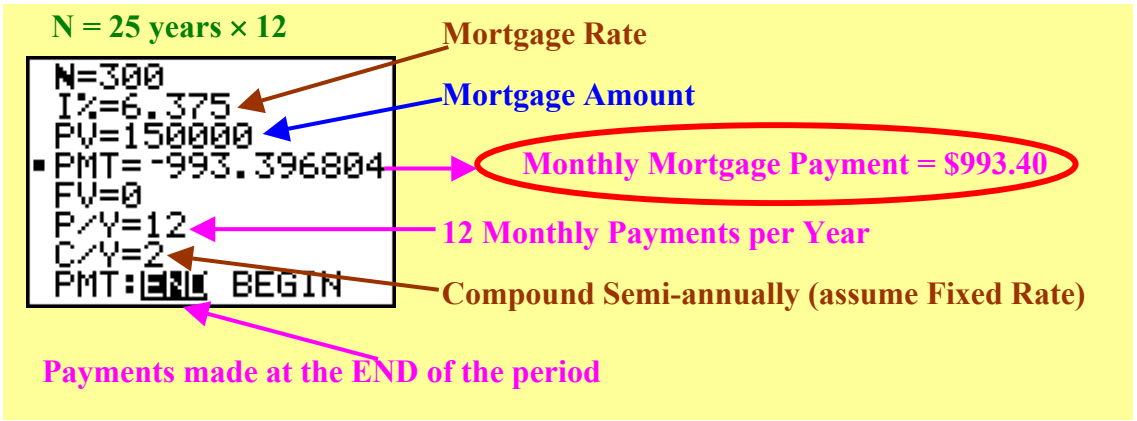


Calculating Mortgage Payments and Other Variables

- payment is always made at the **END** of a period.
- in Canada, only **Fixed Rate** is **compounded Semi-annually**, **Variable Rate** is **compounded Monthly**. **(**When the question did **NOT SPECIFIED** the **rate type, ALWAYS ASSUME** it is a **FIXED rate)**.
- if the borrower decides to make **Semi-monthly or Biweekly Mortgage Payments** instead, the payment is **calculated by Dividing the Monthly Payment by 2**.

Example 2: A $150,000 mortgage has a fixed mortgage rate of 6.375%/a over 25 years.

a. Calculate the monthly payment.



b. Determine the total amount and the interest paid over 25 years.

Total Cost = Total Payments
Total Cost = 25 × 12 × \$993.40

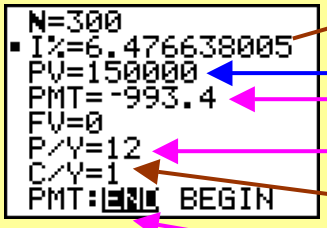
Total Cost = \$298,020

Interest = Total Cost – Mortgage Amount
Interest = \$298,020 – \$150,000

Interest Paid = \$148,020

c. What is the effective rate?

N = 25 years × 12



Effective Rate = 6.477%

Mortgage Amount

Monthly Mortgage Payment

12 Monthly Payments per Year

Effective Rate (Compound ANNUALLY)

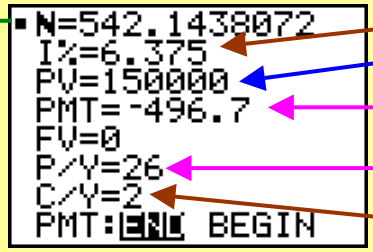
Payments made at the END of the period

d. If the payment is made biweekly, calculate the biweekly payment?

$$\text{Biweekly Payment} = \frac{\text{Monthly Payment}}{2} = \frac{\$993.40}{2}$$

Biweekly Payment = \$496.70

e. What is the new amortization period?



Mortgage Rate

Mortgage Amount

Biweekly Mortgage Payment

26 BIWEEKLY Payments per Year

Fixed Rate (Compound Semi-annually)

N = 542.1438072 Payments

$$\text{Amortization Period} = \frac{542.1438072 \text{ Payments}}{26 \text{ Payments per Year}}$$

Amortization Period = 20.852 Years

f. What is the total interest paid over this new amortization period?

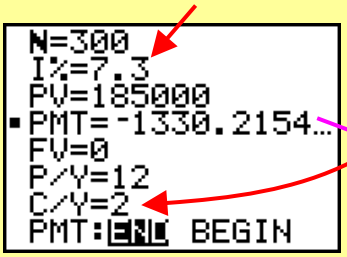
Interest = (Number of Payments × Biweekly Payments) – Mortgage Amount
Interest = 542.1438072 Payments × \$496.70 – \$150,000
Interest = \$269282.83 – \$150,000

Interest Paid = \$119,282.83

Example 3: A major bank advertises its 5-years fixed mortgage rate as 7.300% and its variable mortgage rate as 3.625%. Assuming the rates do not change over the entire amortization period of 25 years. Consider an $185,000 mortgage with monthly payments

a. Using the TVM Solver and showing your work, which mortgage rate is a better option? How much interest would be saved when the better option is chosen?

Fixed Rate (7.300%/a Compound Semi-annualy)

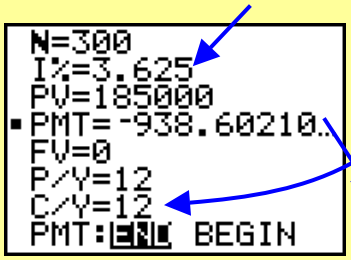


Monthly Payment $1330.22

Total Cost = Total Payments
Total Cost = 25 × 12 × $1330.22

Total Cost = $399,066

Variable Rate (3.625%/a compound monthly)



Monthly Payment $938.60

Total Cost = Total Payments
Total Cost = 25 × 12 × $938.60

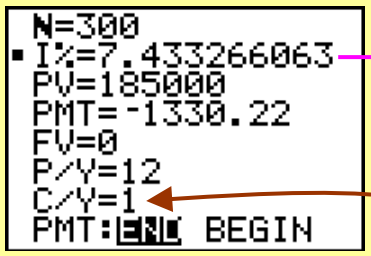
Total Cost = $281,580

Difference = $399,066 – $281,580

The VARIABLE Rate is Better. Overall Saving = $117,486

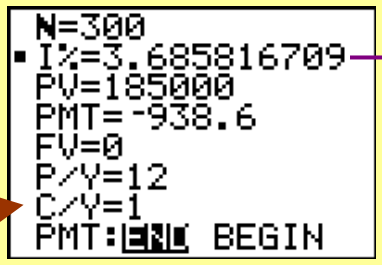
b. Determine the effective rate for both options.

Fixed Rate



Effective Rate 7.433%

Variable Rate



Effective Rate 3.686%

Effective Rate (assume compound ANNUALLY)

c. Under what circumstances will a borrower consider "locking" the interest rate (converting from a variable rate to a fixed rate)?

When the economy becomes more active, the central banks (both Canada and the US) will raise interest rate (prime lending rate) to prevent or slow down inflation. **If the trend is that the Interest Rate will RISE SHARPLY**, then it make sense to convert the variable rate to a fixed rate. Even though, the fixed rate might be presently double the variable rate, **in the future** the **PRESENT Fixed Rate** will be **LOWER** than the **FUTURE Variable Rate**.

4-5 Assignment: pg 187 – 189 # 1 to 7 and Worksheet 4-5: Mortgages

APPLIED MATH 30 WORKSHEET: 4-5 MORTGAGES

For all the questions below, assume that all mortgages are paid at the end of the payment period, and that the interest is calculated semi-annually. As you work through these problems with the TVM solver, be sure to copy down all the values you have entered and circle the final answer.

1. Carol and John have been approved for of $90,000.00 mortgage.
- **a.** Calculate the monthly payment for 25 years for the interest rate of 8.5%
- **b.** What is the total amount paid at the end of the 25 years? How much of it is interest?
- **c.** Calculate the monthly payment if they decide to go with a 20 years amortization at the same interest rate of 8.5%.
- **d.** What is the total amount paid after 20 years? How much of it is interest?
- **e.** How much did they save by changing the mortgage to 20 years from 25 year?

2. A $40,000.00 mortgage is to be repaid over a 10-year period by monthly payments.
- **a.** What is the monthly payment if the interest is 7%?
- **b.** If the mortgage were to be changed from 10 to 15 years at the same interest rate of 7%, what would be the new monthly payment?
- **c.** What is the difference on the monthly payments by changing it to 15 years?
- **d.** What is the difference on the amount of interest paid by changing it to 15 years?
- **e.** Is the 15-year better then the 10-year mortgage? Explain your answer.

3. Andrea and Anthony want to get approved for a mortgage. Their combined annual income is $130,000.00.
- **a.** Using the 32% of gross monthly income as their maximum monthly mortgage payment, determine the maximum monthly mortgage payment they can afford.
- **b.** Anthony has a student loan for $150 per month and Andrea has a monthly car payment of $275. Both of them have credit card expenses of $250.00 per month. Calculate their liabilities difference using 40% of gross monthly income as their total liability allowed.
- **c.** Suppose that the heating cost is $100 per month and the monthly property tax is $150. Calculate the maximum amount they can spend on the actual mortgage payment.
- **d.** Suppose that the current interest rate is 7.5% and Andrea and Anthony decide on a 20-year amortization. Determine the maximum mortgage amount that the bank will lend them.
- **e.** Using the answer of **3d** as the price of the house, if Anthony and Andrea get a present from their parents of a 15% down payment to put toward their house, find the maximum price of a house that they can afford if the bank offers a 7.5% with a 20 years amortization.

4. A contractor's price for a new building was $96,000.00 with Slade Inc., the buyers of the building, paid $12,000.00 down and financed the balance by making equal payments at the end of every month for 10 years. Interest is at 6.4%.
- **a.** What is the size of the monthly payment?
- **b.** What is the total cost of the building for Slade Inc.?
- **c.** How much of the total is interest?

5. A mortgage of $80,000.00 is amortized over 15 years with monthly payments of $826.58.
- **a.** What is the annual interest rate?
- **b.** What would the monthly payments be if the interest were at 4.5% over 20 years?
- **c.** Which option would have the smaller interest paid?

6. Angela and Marco decide that they want to buy a house and they are not sure which option to choose.

a. Copy and complete the following table.

Variables	Option A	Option B
Amortization	25	25
Mortgage Amounts	$150,000	$150,000
Interest Rates	7.2%	7.2%
Payment Arrangements	Monthly	Biweekly
Payment per Period ($)		
Actual Number of Years to pay off Mortgage		
Total Amount Paid		
Total Interest Paid		

b. Which one is the better option? Explain.

7. What is annual interest rate for a $195,000.00 mortgage if it has a monthly payment of $1606.87 over 20 years?

8. Sylvie Cardinal bought a house for $145,000.00. She made a down payment of $20,000.00 and agreed to repay the balance by monthly payments for 15 years at the interest rate of 8%.
a. What is her monthly payment?
b. What is the total paid? What is the total interest paid for the house?
c. If she paid biweekly instead of monthly, what is her biweekly payment?
d. How many years would it actually take to pay off the mortgage with biweekly payments?
e. Using **8c** and **8d,** what is the total paid? What is the total interest paid for the house?

9. A mortgage of $95,000 is repaid by payments of $900.00 monthly. The interest rate is 10%.
a. How many payments are required to repay the mortgage? How many years would it take?
b. What is the total paid? What is the total interest paid?
c. If the monthly payment were $1000.00, how long would it take to pay off the loan? How much would the saving be, compare to **9b**?

Answers

1a. $715.83 **1b.** $214749; $124749 **1c.** $772.70 **1d.** $185448; $95448 **1e.** $29301
2a. $462.38 **2b.** $357.30 **2c.** $105.08 **2d.** $8828.40 more
2e. Yes and No. Payment is less per month, but pay more interest at the end.
3a. $3466.67 **3b.** $3658.33 **3c.** $3216.67 **3d. $**402787.57 **3e.** $463205.71
4a. $945.97 **4b.** $125516.40 **4c.** $29516.40 **5a.** 9.50% **5b.** $504.32 **5c.** Second Option
6a. $1069.21; $534.61 25 years; 20.44 years $320763; $284048.99 $170763; $134048.99
6b. Option B; less interest paid out and less time to repaid the mortgage. **7.** 7.93% **8a.** $1185.19
8b. $213334.20; $88334.20 **8c.** $592.60 **8d.** 12.93 years **8e.** $199220.27; $74220.27
9a. 243.42 or 244 payments; 20.28 years **9b.** $219078; $124078 **9c.** 15.32 years; saved $35278

4-5A: Calculating Mortgage Amount

	A	B	C	D	E	F	G	H	I	J
1	4-5A: Calculating Mortgage Amount									
2										
3	Basic Information						Montly Liabilities List			
4	Annual Family Gross Income				$ 85,000.00		Car Payments		$ 300.00	
5	Monthly Family Gross Income				$ 7,083.33		Credit Cards		$ 250.00	
6							Personal Line of Credit		$ -	
7	Maximum Rate House Related Expenses				32.00%		Student Loan		$ -	
8	Maximum Monthly House Related Expenses				$ 2,266.67		Others		$ -	
9										
10	Maximum Rate of Liability Allowed				40.00%		Total Monthly Liabilities		$ 550.00	
11	Maximum Monthly Liability Amount Allowed				$ 2,833.33					
12	Liabilities Difference				$ 2,283.33					
13							Final Calculation			
14	Maximum Monthly House Related Expenses Allowed				$ 2,266.67		Amortization Period		25	
15							Mortgage Rate		7.750%	
16	Property Tax Mill Rate				6.791		Effective Rate		0.00635646161629	
17	Approximate Monthly Property Tax				$ 180.00		Pre-Approved Mortgage Amount		$265,838.46	
18	Approxiamte Monthly Heating Cost				$ 100.00					
19	ACTUAL Maximum Monthly Mortagage Payment Allowed				$ 1,986.67		Actual Annual Property Tax		$ 1,805.31	
20							Actual Monthly Property Tax		$ 150.44	
21										
22	Special Name Boxes									
23	E4 = annual_gross_income			E12 = liability_difference			I4 = car_payments	I14 = amortization		
24	E5 = monthly_gross_income			E16 = mill_rate			I5 = credit_cards	I15 = mortgage_rate		
25	E7 = house_expenses_rate			E17 = approx_prop_tax			I6 = personal_loans	I16 = Eff_rate		
26	E8 = max_house_expenses			E18 = heating			I7 = student_loans	I17 = max_mortgage		
27	E10 = max_liability_rate			E19 = max_mortgage_Pmt			I8 = other_liability	I19 = annual_prop_tax		
28	E11 = max_liability						I10 = total_liability	I20 = monthly_prop_tax		
29										

4-5A: Calculating Mortgage Amount

	A	B	C	D	E	F	G	H	I	J
1	4-5A: Calculating Mortgage Amount									
2										
3	Basic Information						Montly Liabilities List			
4	Annual Family Gross Income				85000		Car Payments		300	
5	Monthly Family Gross Income				=annual_gross_income/12		Credit Cards		250	
6							Personal Line of Credit		0	
7	Maximum Rate House Related Expenses				0.32		Student Loan		0	
8	Maximum Monthly House Related Expenses				=monthly_gross_income*house_expenses_rate		Others		0	
9										
10	Maximum Rate of Liability Allowed				0.4		Total Monthly Liabilities		=car_payments+credit_cards+personal_loans+student_loans+other_liability	
11	Maximum Monthly Liability Amount Allowed				=monthly_gross_income*max_liability_rate					
12	Liabilities Difference				=max_liability-total_liability					
13							Final Calculation			
14	Maximum Monthly House Related Expenses Allowed				=MIN(max_house_expenses,liability_difference)		Amortization Period		25	
15							Mortgage Rate		0.0775	
16	Property Tax Mill Rate				6.791		Effective Rate		=EXP(2/12*LN(mortgage_rate/2+1))-1	
17	Approximate Monthly Property Tax				180		Pre-Approved Mortgage Amount		=(-max_mortgage_Pmt/Eff_rate)*((1/(1+Eff_rate)^(amortization*12))-1)	
18	Approxiamte Monthly Heating Cost				100					
19	ACTUAL Maximum Monthly Mortagage Payment Allowed				=E14-E17-heating		Actual Annual Property Tax		=max_mortgage*mill_rate/1000	
20							Actual Monthly Property Tax		=annual_prop_tax/12	
21										
22	Special Name Boxes									
23	E4 = annual_gross_income			E12 = liability_difference			I4 = car_payments	I14 = amortization		
24	E5 = monthly_gross_income			E16 = mill_rate			I5 = credit_cards	I15 = mortgage_rate		
25	E7 = house_expenses_rate			E17 = approx_prop_tax			I6 = personal_loans	I16 = Eff_rate		
26	E8 = max_house_expenses			E18 = heating			I7 = student_loans	I17 = max_mortgage		
27	E10 = max_liability_rate			E19 = max_mortgage_Pmt			I8 = other_liability	I19 = annual_prop_tax		
28	E11 = max_liability						I10 = total_liability	I20 = monthly_prop_tax		
29										

4-5B: Calculating Mortgage Payments

Basic Information							
Final House Price	$ 200,000.00						
Down Payment	$ 50,000.00						
Mortgage Amount	$ 150,000.00						
Mortgage Rate	6.375%	(Compound Semi-annually)					
Amortization (years)	25						
Monthly		Biweekly					
Effective Rate	0.005243287	Effective Rate	0.002418572				
Number of Payments	300	Number of Payments	542.1474407				
Actual Amortization (years)	25	Actual Amortization (years)	20.852				
Monthly Payment	$ 993.40	Biweekly Payment	$ 496.70				
Total House Cost	$ 298,019.01	Total House Cost	$ 269,283.75		Total Saving	$ 28,735.29	
Total Interest Paid	$ 148,019.01	Total Interest Paid	$ 119,283.75		Total Saving	$ 28,735.29	
Special Name Boxes		B11 = monthly_Eff_rate					
B4 = house_price		B12 = monthly_N					
B5 = down_payment		B14 = monthly_payment					
B6 = mortgage		E11 = biweekly_Eff_rate					
B7 = mortgage_r		E12 = biweekly_N					
B8 = amortization		E14 = biweekly_payment					

4-5B: Calculating Mortgage Payments

Basic Information							
Final House Price	200000						
Down Payment	50000						
Mortgage Amount	=house_price-B5						
Mortgage Rate	0.06375	(Compound Semi-annually)					
Amortization (years)	25						
Monthly		Biweekly					
Effective Rate	=EXP(12/2*LN(mortgage_r/2+1))-1	Effective Rate	=EXP(12/26*LN(mortgage_r/2+1))-1				
Number of Payments	=amortization*12	Number of Payments	=LN(biweekly_payment/(biweekly_payment-mortgage*biweekly_Eff_rate))/LN(1+biweekly_Eff_rate)				
Actual Amortization (years)	=amortization	Actual Amortization (years)	=biweekly_N/26				
Monthly Payment	=monthly_Eff_rate*mortgage/(mortgage*(1+monthly_Eff_rate)^B12-1))	Biweekly Payment	=monthly_payment/2				
Total House Cost	=monthly_payment*monthly_N+down_payment	Total House Cost	=biweekly_payment*biweekly_N+down_payment		Total Saving	=B16-E16	
Total Interest Paid	=monthly_payment*monthly_N-mortgage	Total Interest Paid	=biweekly_payment*biweekly_N-mortgage		Total Saving	=B17-E17	
Special Name Boxes		B11 = monthly_Eff_rate					
B4 = house_price		B12 = monthly_N					
B5 = down_payment		B14 = monthly_payment					
B6 = mortgage		E11 = biweekly_Eff_rate					
B7 = mortgage_r		E12 = biweekly_N					
B8 = amortization		E14 = biweekly_payment					

4-6: Renting versus Buying

BUYING A HOUSE	
BENEFITS	**COSTS / DISADVANTAGES**
You own the place after paying off the mortgage. Your house usually appreciates in value through the years.	Mortgage Payment is usually higher than Rent at the beginning.
It can be liquidated fairly easily.	Have to pay for Property Tax.
It can be turned into a Rental Property to offset any Mortgage Payments.	Have to buy Insurance. (Fire, Theft, Water, and Hail Damages)
Mortgage Payment will be equal or less than the rent payment towards the end of the amortization.	Have to pay for other maintenance. (Lawn Care, Interior and Exterior Fix-ups)
Mortgage Payments are usually the same every month (assuming no change in mortgage rate).	Have to pay for Utilities. (Gas, Electricity, Water, and Sewage)
Pride of being a Home Owner.	Have to buy Mortgage Life Insurance.

RENTING A PROPERTY	
BENEFITS	**COSTS / DISADVANTAGES**
Usually, the landlord pays for most of the Utilities for attached dwellings. (The renter pays for all utilities for single housing unit.)	Rent amount would be the same or more than the Mortgage Payment at the end if you were to buy a house 15 to 20 years earlier.
No commitment. You can move out anytime.	Your rent pays someone else's mortgage.
You do not pay Property Tax	You don't get to own the place.
Monthly Rent is usually less than the Mortgage Payment initially.	No guarantee against rent increases. They happen when utility costs or property tax has gone up.
No Maintenance to worry about. Call the Superintendent to fix things if they are broken.	You have to pay rent as long as you live there.

Calculating Annual and Monthly Property Tax

Mill Rate: - the property tax expressed in every one thousand dollars.

$$\textbf{Annual Property Tax} = \frac{\textbf{Assessed Value of Property} \times \textbf{Mill Rate}}{\textbf{1000}}$$

Example 1: Laura owns a $130,000.00 home. The city of Calgary has a mill rate is 10.1. Find her monthly property tax payment.

$$\text{Annual Property Tax} = \frac{\text{Assessed Value of Property} \times \text{Mill Rate}}{1000} = \frac{\$130{,}000 \times 10.1}{1000} = \textbf{\$1,313}$$

$$\text{Monthly Property Tax} = \frac{\$1313}{12}$$

Monthly Property Tax = $109.42

Comparing Cost of Renting and Buying a Property

Step 1. Fill in the known information and calculate all others using the table below.

BASIC INFORMATION			
Home Purchase Price	Enter Value	Average Property Appreciation Rate	Enter Value
Down Payment Amount	Enter Value	Home Value at the **End** of Amortization Period	TVM
Amortization Period	Enter Value	Average Annual Inflation Rate (Rent, Utilities, Insurance, Taxes & Maintenance)	Enter Value
Mortgage Rate	Enter Value		
Payment Arrangement	Enter Value	Property Tax Mill Rate	Enter Value
Amount of Payment per Period	TVM Solver	Annual Property Tax	Calculate
Amount of Payment per Month	Calculate	Monthly Property Tax	Calculate

Mortgage Payment Calculation

N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN

Home Value at the END of Amortization Period

N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN

Property Tax Calculation

a. Annual Property Tax Payment

b. Monthly Property Tax Payment

Step 2. Calculate those **Monthly Expenses that are subjected to INFLATION for Renting and Buying.**

Buying a Property		Renting a Property	
Expenses	**Monthly Cost**	**Expenses**	**Monthly Cost**
Electricity	Enter Value	Electricity	Enter Value
Gas	Enter Value	Gas	Enter Value
Water & Sewage	Enter Value	Water & Sewage	Enter Value
Property Tax	Calculated	Rent	Enter Value
Maintenance	Enter Value		
Mortgage & Home Insurance	Enter Value		
Total Monthly Expense	Calculate	**Total Monthly Expense**	Calculate
Total Annual Expense	Calculate	**Total Annual Expense**	Calculate

Step 3. Using the formula below, find the **CUMULATIVE EXPENSES** for the entire amortization period **for both Buying and Renting. (Fill in the table in Step 4.)**

$$A = \frac{P\left[(1+r)^n - 1\right]}{r}$$

A = **Cumulative Expenses**
P = **Initial Annual Expenses**
r = **Average Annual Inflation Rate**
n = **Amortization Period (years)**

Step 4. Find the **Total and Net Cost** of **Renting** and **Buying.**

a. **Total Cost** of **Renting** = **Cumulative Expenses**

b. **Net Cost** of **Renting** = **Total Cost** of **Renting** – **Asset ($0)**

c. **Total Cost** of **Buying** = **Cumulative Expenses** + **Down Payment** + **Total Mortgage Payments**

d. **Net Cost** of **Buying** = **Total Cost** of **Buying** – **Home Value at the END of the Amortization Period**

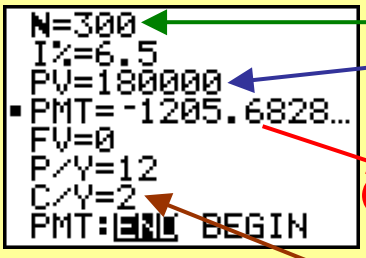
Buying a Property		**Renting a Property**	
Cumulative Expenses		Cumulative Expenses	
Down Payment			
Total Mortgage Payments			
Total Cost of Buying		**Total Cost of Renting**	
Home Value at the End of the Amortization Period		Property Asset	$0.00
Net Cost of Buying		**Net Cost of Renting**	

Example 2: John and Mary plan to buy a house. Use the information below to complete the analysis of renting versus buying.

Step 1. Fill in the known information and calculate all others using the table below.

BASIC INFORMATION			
Home Purchase Price	$220,000	Average Property Appreciation Rate	2.500%
Down Payment Amount	$40,000	Home Value at the **End** of Amortization Period	**$407,867.70**
Amortization Period	25 years	Average Annual Inflation Rate (Rent, Utilities, Insurance, Taxes & Maintenance)	3.000%
Mortgage Rate	6.500%		
Payment Arrangement	Monthly	Property Tax Mill Rate	10.5
Amount of Payment per Period	**$1205.68**	Annual Property Tax	**$2310.00**
Amount of Payment per Month	**$1205.68**	Monthly Property Tax	**$192.50**

Mortgage Payment Calculation



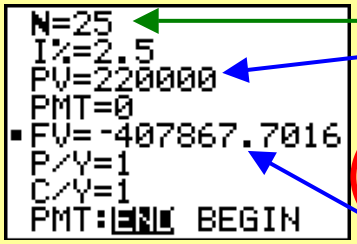
N = 25 × 12

Mortgage Amount
$220,000 – $40,000

Monthly Payment $1205.68

Fixed Rate (Compound Semi-annually)

Home Value at the END of Amortization Period



N = 25 years
Present Home Value

Home Value at the END of the Amortization Period $407,867.70

Property Tax Calculation

a. Annual Property Tax Payment

$$\text{Annual Property Tax} = \frac{\$220{,}000 \times 10.5}{1000}$$

Annual Property Tax = $2310

b. Monthly Property Tax Payment

$$\text{Monthly Property Tax} = \frac{\$2310}{12}$$

Monthly Property Tax = $192.50

Step 2. Calculate those **Monthly Expenses that are subjected to INFLATION for Renting and Buying.**

Buying a Property		Renting a Property	
Expenses	**Monthly Cost**	**Expenses**	**Monthly Cost**
Electricity	$60.00	Electricity	$60.00
Gas	$100.00	Gas (included in rent)	None
Water & Sewage	$30.00	Water & Sewage (included)	None
Property Tax	**$192.50**	Rent (2 bedrooms)	$850.00
Maintenance	$75.00		
Mortgage & Home Insurance	$45.00		
Total Monthly Expense	**$502.50**	**Total Monthly Expense**	**$910.00**
Total Annual Expense	**$6,030.00**	**Total Annual Expense**	**$10,920.00**

Step 3. Using the formula below, find the **CUMULATIVE EXPENSES** for the entire amortization period for both **Buying** and **Renting**. **(Fill in the table in Step 4.)**

$$A = \frac{P\left[(1+r)^n - 1\right]}{r}$$

***A* = Cumulative Expenses**
***P* = Initial Annual Expenses**
***r* = Average Annual Inflation Rate**
***n* = Amortization Period (years)**

a. Cumulative Expenses of Buying

$$A = \frac{\$6{,}030\left[(1+0.03)^{25} - 1\right]}{0.03}$$

Cumulative Expenses of Buying = $219,849.36

b. Cumulative Expenses of Renting

$$A = \frac{\$10{,}920\left[(1+0.03)^{25} - 1\right]}{0.03}$$

Cumulative Expenses of Renting = $398,135.17

Step 4. Find the **Total** and **Net Cost** of **Renting** and **Buying**.

a. **Total Cost of Renting = Cumulative Expenses**

b. **Net Cost of Renting = Total Cost of Renting – Asset ($0)**

c. **Total Cost of Buying = Cumulative Expenses + Down Payment + Total Mortgage Payments**

d. **Net Cost of Buying = Total Cost of Buying – Home Value at the END of the Amortization Period**

Buying a Property		Renting a Property	
Cumulative Expenses	**$219,849.36**	Cumulative Expenses	**$398,135.17**
Down Payment	**$40,000.00**		
Total Mortgage Payments **(25 × 12 × $1205.68)**	**$361,704.00**		
Total Cost of Buying	**$621,553.36**	**Total Cost of Renting**	**$398,135.17**
Home Value at the End of the Amortization Period	**$407,867.70**	Property Asset	$0.00
Net Cost of Buying	**$213,685.66**	**Net Cost of Renting**	**$398,135.17**

It is true that the Total Cost of Buying a property is MORE than Renting over the amortization period. But because the BUYER eventually OWNS the property, it would cost him / her LESS.

Difference between Net Cost of Renting and Buying = $398,135.17 – $213,685.66

Buying a Property would have actually SAVED $184,449.51 over 25 years compared to Renting.

4-6 Assignment: pg 194 # 1 and Worksheet 4-6: Renting versus Buying

Applied Math 30 Worksheet: 4-6 Renting versus Buying

1. Jack and Jill plan to buy a house. Use the information below to complete the renting versus buying analysis.

BASIC INFORMATION			
Home Purchase Price	$260,000	Average Property Appreciation Rate	2.250%
Down Payment Amount	$50,000	Home Value at the **End** of Amortization Period	
Amortization Period	20 years	Average Annual Inflation Rate (Rent, Utilities, Insurance, Taxes & Maintenance)	2.750%
Mortgage Rate	6.500%		
Payment Arrangement	Monthly	Property Tax Mill Rate	12.1
Amount of Payment per Period		Annual Property Tax	
Amount of Payment per Month		Monthly Property Tax	

Mortgage Payment Calculation

N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN

Home Value at the END of Amortization Period

N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT: END BEGIN

Property Tax Calculation

Annual Property Tax Payment

Monthly Property Tax Payment

Buying a Property	
Expenses	**Monthly Cost**
Electricity	$90.00
Gas	$120.00
Water & Sewage	$45.00
Property Tax	
Maintenance	$125.00
Mortgage & Home Insurance	$65.00
Total Monthly Expense	
Total Annual Expense	

Renting a Property	
Expenses	**Monthly Cost**
Electricity	$90.00
Gas (included in rent)	None
Water & Sewage (included)	None
Rent (2 bedrooms)	$975.00
Total Monthly Expense	
Total Annual Expense	

Cumulative Expenses** of **Buying

Cumulative Expenses** of **Renting

Buying a Property	
Cumulative Expenses	
Down Payment	
Total Mortgage Payments	
Total Cost of Buying	
Home Value at the End of the Amortization Period	
Net Cost of Buying	

Renting a Property	
Cumulative Expenses	
Total Cost of Renting	
Property Asset	$0.00
Net Cost of Renting	

Answers: Net Cost of Buying = $239,791.77 Net Cost of Renting = $334,802.74

