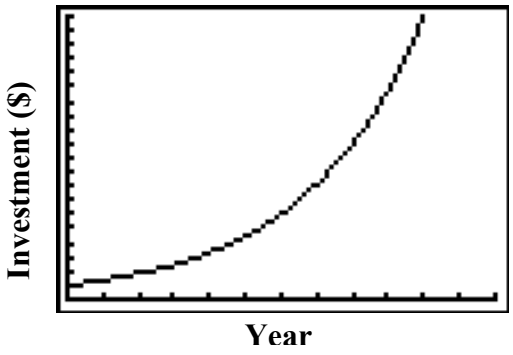
# Unit 6: Sinusoidal Data

## 5-1: Collecting and Plotting Periodic Data

**Periodic Data**: - data that contains cycles which repeat at a regular interval.

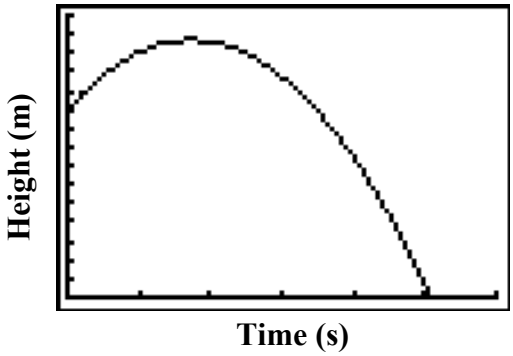
Example 1: Classify and explain whether the graphs below are periodic.

a.



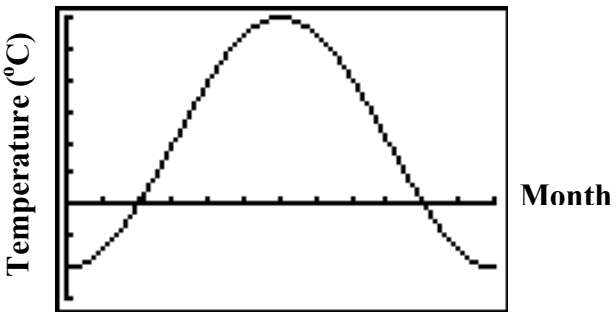

**NOT Periodic. The Pattern does NOT repeat. The graph keeps going up.**

b.



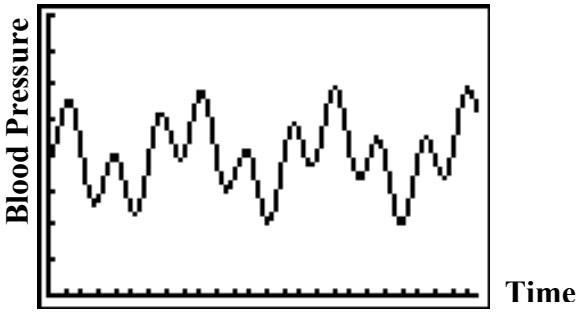

**NOT Periodic. The Pattern does NOT repeat. The graph goes up and down, and then stops.**

c.



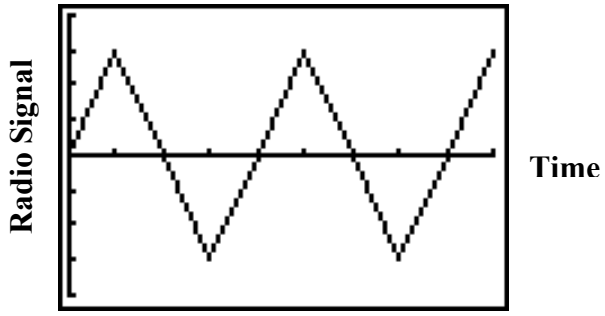

**PERIODIC. The Pattern REPEATS. The graph goes up and down and continues to be the same way if the *x*-axis is extended.**

d.



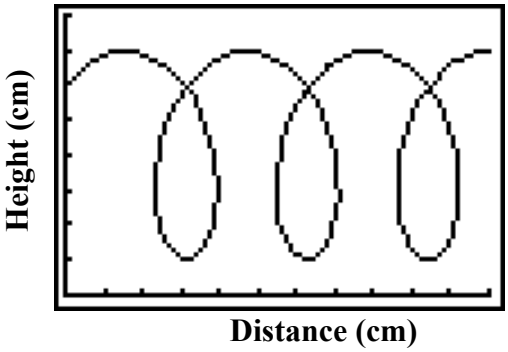

**PERIODIC. The Pattern REPEATS. The graph goes up and down and continues to be the same way if the *x*-axis is extended.**

e.




**PERIODIC. The Pattern REPEATS. The graph goes up and down and continues to be the same way if the *x*-axis is extended.**

f.



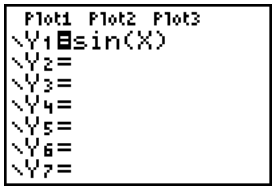

**PERIODIC. The Pattern REPEATS. The graph goes up and down and continues to be the same way if the *x*-axis is extended.**

**Sinusoidal Data**: - periodic data, when graphed, looks like a Sine Wave (the graph of an equation using the Sine function).

Example 2: Using the graphing calculator in **DEGREE Mode**, complete the following table for the equation $y = \sin\theta$. Grpah the equation between $-90^o$ to $720^o$.

**Step 1: Enter the Equation.**

**Y=**



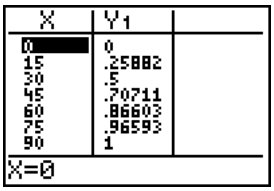

**Step 2: Enter Table Setting**

**2nd** **TBLSET**


**Step 3: Display Table**

**2nd** **TABLE**

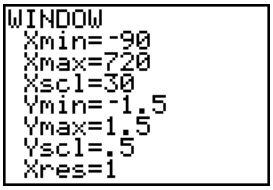
**WINDOW** **GRAPH**




| θ | *y* | θ | *y* | θ | *y* | θ | *y* |
|---|---|---|---|---|---|---|---|
| $0^o$ | 0 | $105^o$ | 0.96593 | $195^o$ | –0.25882 | $285^o$ | –0.96593 |
| $15^o$ | 0.25882 | $120^o$ | 0.86603 | $210^o$ | –0.5 | $300^o$ | –0.86603 |
| $30^o$ | 0.5 | $135^o$ | 0.70711 | $225^o$ | –0.70711 | $315^o$ | –0.70711 |
| $45^o$ | 0.70711 | $150^o$ | 0.5 | $240^o$ | –0.86603 | $330^o$ | –0.5 |
| $60^o$ | 0.86603 | $165^o$ | 0.25882 | $255^o$ | –0.70711 | $345^o$ | –0.25882 |
| $75^o$ | 0.96593 | $180^o$ | 0 | $270^o$ | –1 | $360^o$ | 0 |
| $90^o$ | 1 | | | | | | |

**Step 4: Set the Proper Windows.**

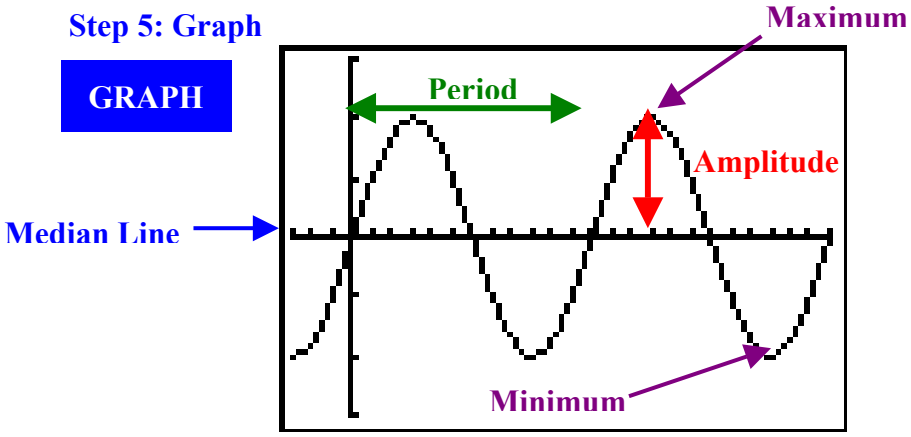
**WINDOW**




***x*: [–90, 720, 30]**
***y*: [ –1.5, 1.5, 0.5]**

**Step 5: Graph**

**GRAPH**




**Period**: - the length or the time it takes to complete one cycle of the wave

**Amplitude**: - the **height** of the wave **from the median line (middle line)** of the wave.

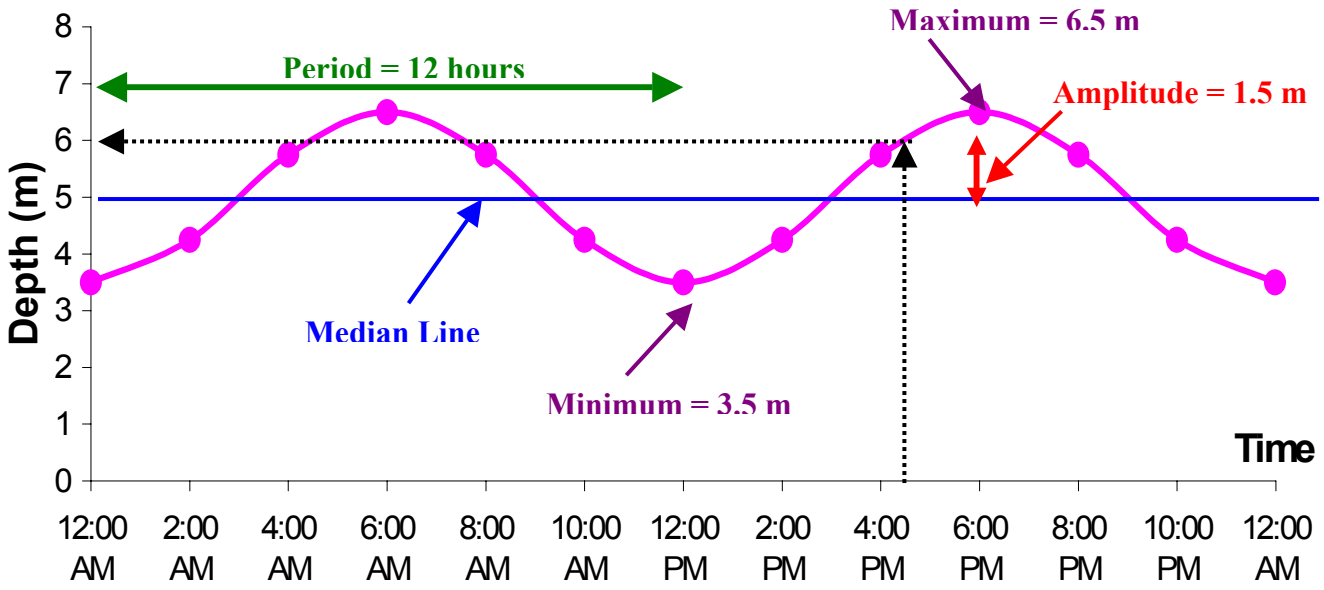
 

Example 3: A particular beach has various depth of water because of high and low tides. The depths of the water in metres and the times of the day is recorded.

| **Time** | Midnight | 2:00 AM | 4:00 AM | 6:00 AM | 8:00 AM | 10:00 AM |
|---|---|---|---|---|---|---|
| **Depth (m)** | 3.5 | 4.25 | 5.75 | 6.5 | 5.75 | 4.25 |
| **Time** | 12:00 PM | 2:00 PM | 4:00 PM | 6:00 PM | 8:00 PM | 10:00 PM |
| **Depth (m)** | 3.5 | 4.25 | 5.75 | 6.5 | 5.75 | 4.25 |

a. Graph the data using a graphing calculator. Label all scales and axis.
b. Describe the pattern of the graph by stating its period, amplitude, maximum and minimum.
c. What is the depth of the water at 4:30 PM? How many cycles has it been since midnight of the same day?

## Water Depth versus Time of Day




**a.** The graph is **SINUSOIDAL**.
**Period = 12 hours**
**Maximum = 6.5 m** **Minimum = 3.5 m**
**Median Line** $= \frac{3.5\text{ m} + 6.5\text{ m}}{2} =$ **5 m**
**Amplitude = 6.5 m – 5 m = 1.5 m**

**c.** **At 4:30 PM, the depth of the water is at around 6 m.**

**There had been 1 cycle of high tide and low tide since midnight.**

**5-1 Assignment**: **pg. 216 – 217 #1 to 7**

## 5-2: Radian Measures and Sine Curves

**Radians**: - a unit **(rad)** to measure the size of an angle.

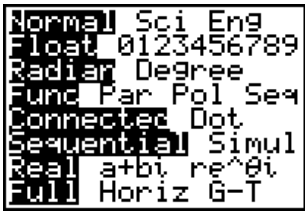
$$\pi \text{ rad} = 180^o \quad \text{OR} \quad \frac{\pi}{180} \text{ rad} = 1^o$$

**Divide both Sides by 180**

**Converting Degree to Radian Using Graphing Calculator.**

**Step 1: Set Mode to RADIAN.**

MODE



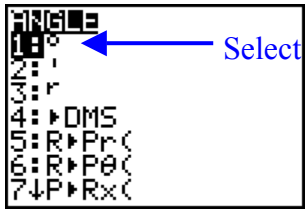

**Step 2: Enter Degree**


**Step 3: Specify Degree Unit and Convert**

2nd

ANGLE

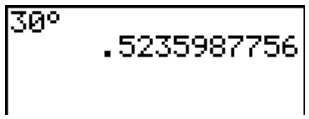
APPS




Select Option 1

ENTER

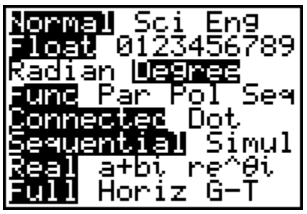
ENTER



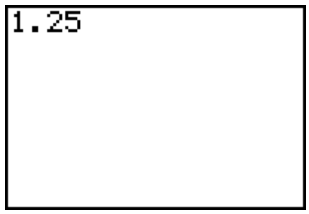

**Converting Radian to Degree Using Graphing Calculator.**

**Step 1: Set Mode to Degree.**

MODE




**Step 2: Enter Radian**

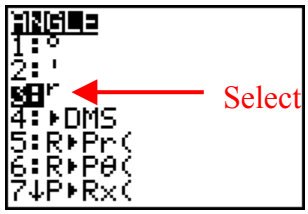



**Step 3: Specify Radian Unit and Convert**

2nd

ANGLE

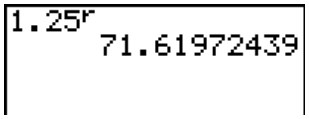
APPS




Select Option 3

ENTER

ENTER



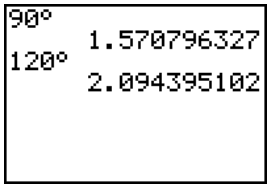

Example 1: Convert the following into radian.

a. $90^o$

$$1^o = \frac{\pi}{180} \text{ rad}$$

$$90^o = 90 \times \frac{\pi}{180} \text{ rad} = \frac{90\pi}{180} \text{ rad}$$

$$90^o = \frac{\pi}{2} \text{ rad} \approx 1.57 \text{ rad}$$




b. $120^o$

$$1^o = \frac{\pi}{180} \text{ rad}$$

$$120^o = 120 \times \frac{\pi}{180} \text{ rad} = \frac{120\pi}{180} \text{ rad}$$

$$120^o = \frac{2\pi}{3} \text{ rad} \approx 2.09 \text{ rad}$$

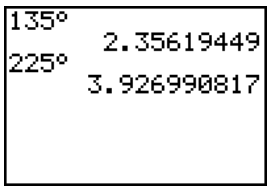
 

c. $135^o$

$1^o = \frac{\pi}{180}$ rad

$135^o = 135 \times \frac{\pi}{180}$ rad $= \frac{135\pi}{180}$ rad

**$135^o = \frac{3\pi}{4}$ rad $\approx 2.36$ rad**




d. $225^o$

$1^o = \frac{\pi}{180}$ rad

$225^o = 225 \times \frac{\pi}{180}$ rad $= \frac{\mathbf{225\pi}}{\mathbf{180}}$ rad

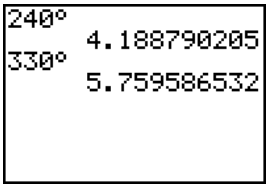
**$225^o = \frac{5\pi}{4}$ rad $\approx 3.93$ rad**

e. $240^o$

$1^o = \frac{\pi}{180}$ rad

$240^o = 240 \times \frac{\pi}{180}$ rad $= \frac{240\pi}{180}$ rad

**$240^o = \frac{4\pi}{3}$ rad $\approx 4.19$ rad**




f. $330^o$

$1^o = \frac{\pi}{180}$ rad

$330^o = 330 \times \frac{\pi}{180}$ rad $= \frac{330\pi}{180}$ rad

**$330^o = \frac{11\pi}{6}$ rad $\approx 5.76$ rad**

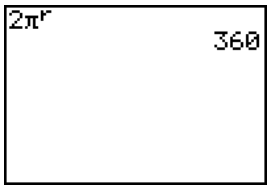
Example 2: Convert the following into degree.

a. $2\pi$ rad

$\pi$ rad $= 180^o$

$2\pi$ rad $= 2(180^o)$

**$2\pi$ rad $= 360^o$**

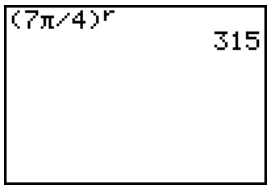



b. $\frac{7\pi}{4}$ rad

$\pi$ rad $= 180^o$

$\frac{7\pi}{4}$ rad $= \frac{7(180^o)}{4}$

**$\frac{7\pi}{4}$ rad $= 315^o$**

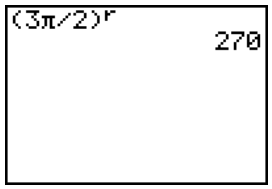



c. $\frac{3\pi}{2}$ rad

$\pi$ rad $= 180^o$

$\frac{3\pi}{2}$ rad $= \frac{3(180^o)}{2}$

**$\frac{3\pi}{2}$ rad $= 270^o$**




d. 3.42 rad

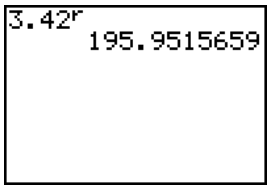
$\pi$ rad $= 180^o$

3.42 rad $= x$ degrees

$\frac{\pi}{3.42} = \frac{180^o}{x}$

$x = \frac{3.42 \times 180}{\pi}$

**3.42 rad $= 195.95^o$**



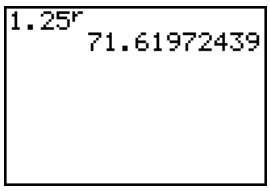
e. 1.25 rad

$\pi$ rad = 180°
1.25 rad = $x$ degrees

$$\frac{\pi}{1.25} = \frac{180^\circ}{x}$$

$$x = \frac{1.25 \times 180}{\pi}$$

**1.25 rad = 71.62°**



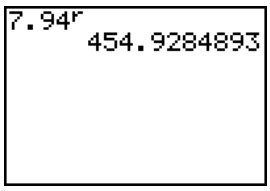

f. 7.94 rad

$\pi$ rad = 180°
7.94 rad = $x$ degrees

$$\frac{\pi}{7.94} = \frac{180^\circ}{x}$$

$$x = \frac{7.94 \times 180}{\pi}$$

**7.94 rad = 454.93°**




Example 3: Draw $y = \sin\theta$ for $-360^\circ \le \theta \le 360^\circ$ in Degree mode and $-2\pi \le \theta \le 2\pi$ in Radian mode. What are the maximum and minimum coordinates for both graphs?

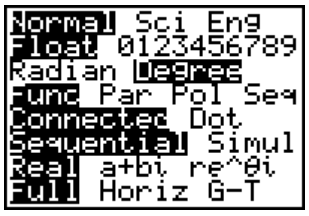
a. Find the value of sin (120°) and other values of θ that produce the same value of sin (120°).

b. Find the value of $\sin\left(\frac{7\pi}{4}\right)$ and other values of θ that produce the same value of $\sin\left(\frac{7\pi}{4}\right)$.

**Graph 1: $y = \sin\theta$ for $-360^\circ \le \theta \le 360^\circ$**

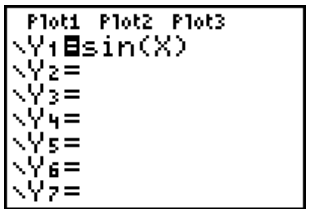
**Step 1: Set Mode to Degree**

**MODE**




**Step 2: Enter the Equation**

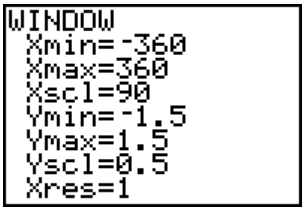
**Y=**




**Step 3: Set the Proper Window**

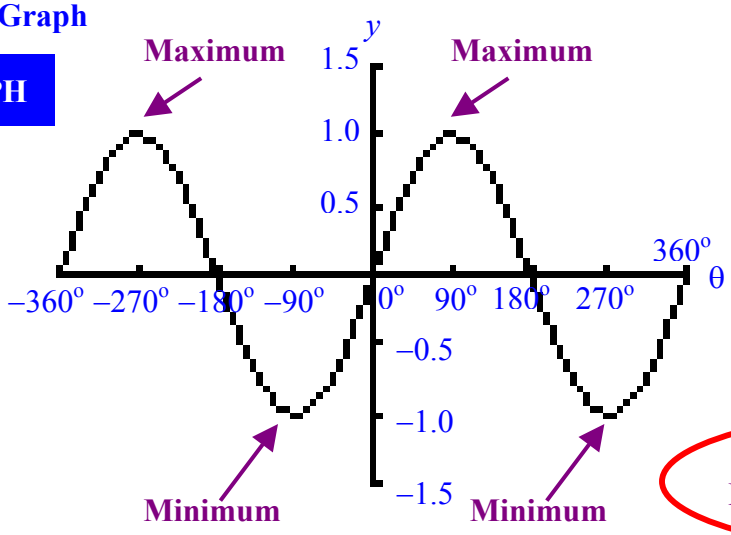
**WINDOW**

**$x$: [–360, 360, 90]**
**$y$: [ –1.5, 1.5, 0.5]**



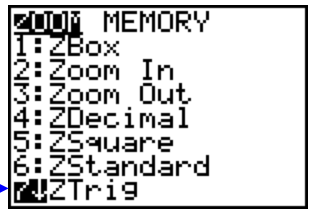

**Step 4: Graph**

**GRAPH**




**OR Using Zoom Trig (automatically select the above Window Setting)**

**ZOOM**



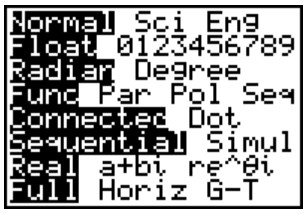

Select Option 7

**Maximums at (–270°, 1) and (90°, 1)**
**Minimums at (–90°, –1) and (270°, –1)**

**Graph 2: $y = \sin\theta$ for $-2\pi \le \theta \le 2\pi$**

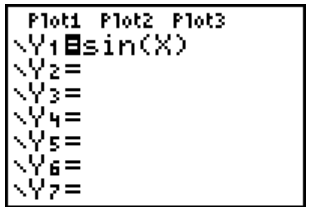
**Step 1: Set Mode to Radian**

**MODE**




**Step 2: Enter the Equation**

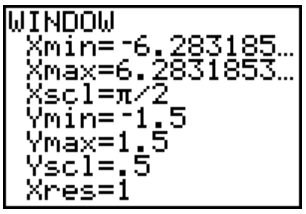
**Y=**




**Step 3: Set the Proper Window**

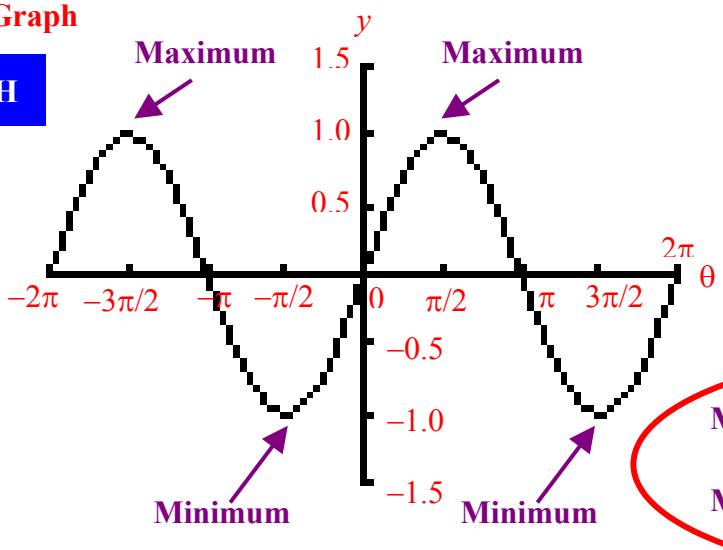
**WINDOW**

*x*: [–2π, 2π, π/2]
*y*: [ –1.5, 1.5, 0.5]



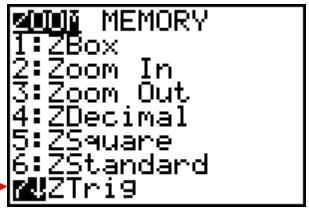

**Step 4: Graph**

**GRAPH**




**OR Using Zoom Trig (automatically select the above Window Setting)**

**ZOOM**




Select Option 7

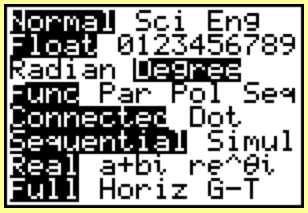
**Maximums at $(\frac{-3\pi}{2}, 1)$ and $(\frac{\pi}{2}, 1)$**

**Minimums at $(\frac{-\pi}{2}, -1)$ and $(\frac{3\pi}{2}, -1)$**

**a.** Since we are given *y*-value to find other angles (θ), we have to run the INTERSECT function with the graphing calculator after entering $Y_2 = \sin(120)$.

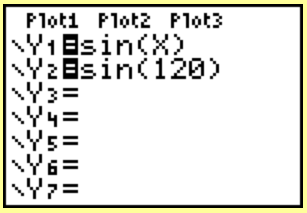
**Step 1: Set Mode to Degree**

**MODE**



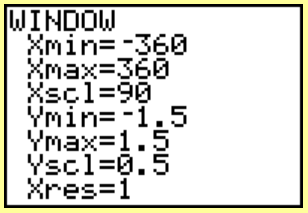

**Step 2: Enter the Equation**

**Y=**




**Step 3: Set the Proper Window**

**WINDOW**




**Step 4: Run INTRESECT at all Intersecting Points**

*x*: [–360, 360, 90]
*y*: [ –1.5, 1.5, 0.5]

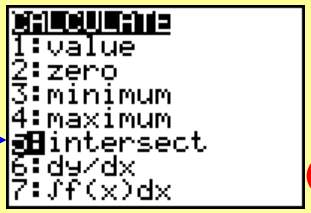
**2nd** **CALC** **TRACE**

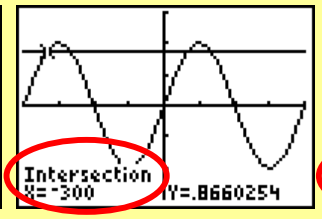
i. Take Cursor close to each intersecting point.

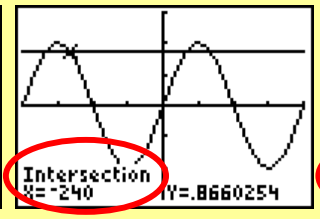
ii. Press **ENTER** 3 Times.

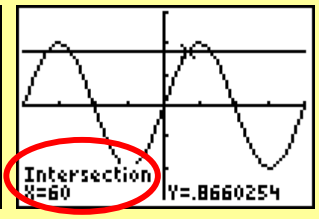
**sin 120° = 0.8660254**

Select Option 5



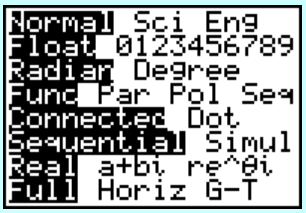



**b.** Since we are given *y*-value to find other angles (θ), we have to run the INTERSECT function with the graphing calculator after entering $Y_2 = \sin\left(\frac{7\pi}{4}\right)$.

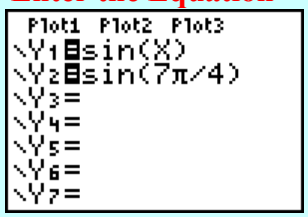
**Step 1: Set Mode to Radian**

**MODE**



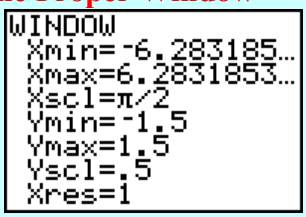

**Step 2: Enter the Equation**

**Y=**




**Step 3: Set the Proper Window**

**WINDOW**




***x*: [–2π, 2π, π/2]**
***y*: [ –1.5, 1.5, 0.5]**

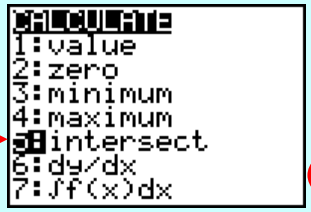
**Step 4: Run INTRESECT at all Intersecting Points**

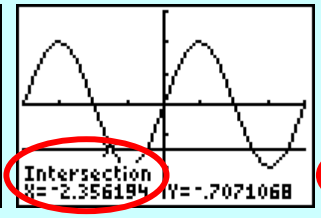
**2nd** **CALC** **TRACE**

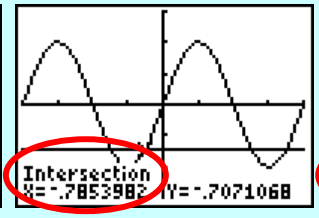
i. **Take Cursor close to each intersecting point.**

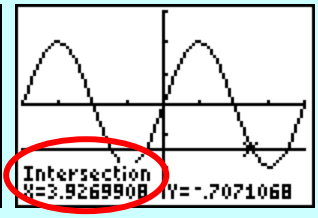
ii. **Press ENTER 3 Times.**

**Select Option 5**




$\sin\left(\frac{7\pi}{4}\right) = -0.7071068$

$-0.7853982 \text{ rad} = -45^\circ$

$-45^\circ = -45 \times \frac{\pi}{180} \text{ rad} = \frac{-45\pi}{180} \text{ rad}$

$-0.7853982 \text{ rad} = \frac{-\pi}{4}$

$-2.356194 \text{ rad} = -135^\circ$

$-135^\circ = -135 \times \frac{\pi}{180} \text{ rad} = \frac{-135\pi}{180} \text{ rad}$

$-2.356194 \text{ rad} = \frac{-3\pi}{4}$

$3.9269908 \text{ rad} = 225^\circ$

$225^\circ = 225 \times \frac{\pi}{180} \text{ rad} = \frac{225\pi}{180} \text{ rad}$

$3.9269908 \text{ rad} = \frac{5\pi}{4}$

**5-2 Assignment**: **5-2 Worksheet: Radian Measures and Sine Curves**

## 5-2 Worksheet: Radian Measures and Sine Curve

1. Convert the following from degrees to radians. Express the answer in terms of π.
   a. $30^o$ b. $45^o$ c. $60^o$ d. $150^o$ e. $180^o$ f. $210^o$
   g. $270^o$ h. $300^o$ i. $315^o$ j. $360^o$ k. $390^o$ l. $405^o$

2. Convert the following from degrees to radians. Express answers in two decimal places.
   a. $100^o$ b. $225^o$ c. $57.3^o$ d. $-125^o$ e. $-65^o$ f. $150^o$ g. $30^o$

3. Convert the following from radians to degrees.
   a. $\frac{\pi}{2}$ rad b. $\frac{3\pi}{4}$ rad c. $-\frac{2\pi}{3}$ rad d. $\frac{7\pi}{6}$ rad e. $\frac{\pi}{4}$ rad f. $-\frac{3\pi}{2}$ rad
   g. $2\pi$ rad h. $-\frac{5\pi}{3}$ rad i. $\frac{5\pi}{4}$ rad j. $\frac{\pi}{6}$ rad k. $-\frac{11\pi}{6}$ rad

4. Convert the following from radians to degrees. Express answers to 1 decimal place.
   a. 2 rad b. −5 rad c. 3.2 rad d. 1.8 rad e. −0.7 rad f. 6.7 rad

5. Find the value of $\sin 30^o$ and other values of θ that give the same value of $\sin 30^o$ for $-360^o \le \theta \le 360^o$.

6. Find the value of $\sin\left(\frac{\pi}{3}\right)$ and other values of θ that give the same value of $\sin\left(\frac{\pi}{3}\right)$ for $-2\pi \le \theta \le 2\pi$.

7. Find the value of $\sin 240^o$ and other values of θ that give the same value of $\sin 240^o$ for $-360^o \le \theta \le 360^o$.

8. Find the value of $\sin\left(\frac{11\pi}{6}\right)$ and other values of θ that give the same value of $\sin\left(\frac{11\pi}{6}\right)$ for $-2\pi \le \theta \le 2\pi$.

**Answers**

1 a. $\frac{\pi}{6}$ rad b. $\frac{\pi}{4}$ rad c. $\frac{\pi}{3}$ rad d. $\frac{5\pi}{6}$ rad e. $\pi$ rad f. $\frac{7\pi}{6}$ rad
g. $\frac{3\pi}{2}$ rad h. $\frac{5\pi}{3}$ rad i. $\frac{7\pi}{4}$ rad j. $2\pi$ rad k. $\frac{13\pi}{6}$ rad l. $\frac{9\pi}{4}$ rad

2 a. 1.75 rad b. 3.93 rad c. 1.00 rad d. −2.18 rad e. −1.13 rad f. 2.62 rad g. 0.52 rad

3 a. $90^o$ b. $135^o$ c. $-120^o$ d. $210^o$ e. $45^o$ f. $-270^o$
g. $360^o$ h. $-300^o$ i. $225^o$ j. $30^o$ k. $-330^o$

4 a. $114.6^o$ b. $-286.5^o$ c. $183.3^o$ d. $103.1^o$ e. $-40.1^o$ f. $383.9^o$

5. $\sin 30^o = 0.5$, at $-330^o, -210^o, 150^o$

6. $\sin\left(\frac{\pi}{3}\right) = 0.86603$, at $-\frac{5\pi}{3}, -\frac{4\pi}{3}, \frac{2\pi}{3}$

7. $\sin 240^o = -0.86603$, at $-120^o, -60^o, 300^o$

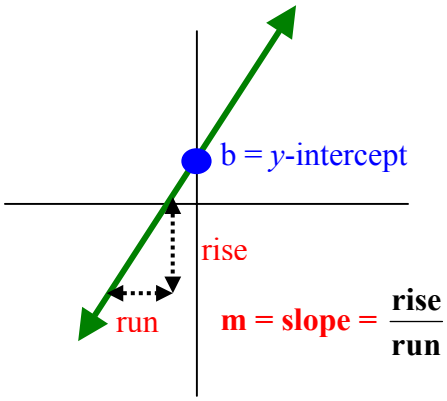
8. $\sin\left(\frac{11\pi}{6}\right) = -0.5$, at $-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}$

## 5-3: Fitting Sine Curves to Data

Recall finding an equation from a list of data, we have to use the **REGRESSION** function of the graphing calculator.

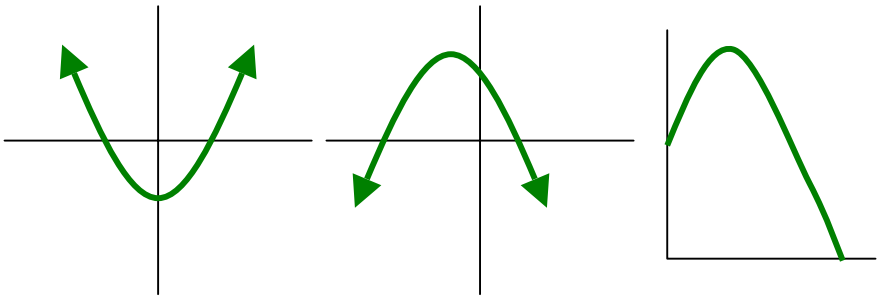
### Linear Regression

$y = ax + b$ (the same as $y = mx + b$)




### Quadratic Regression

$y = ax^2 + bx + c$

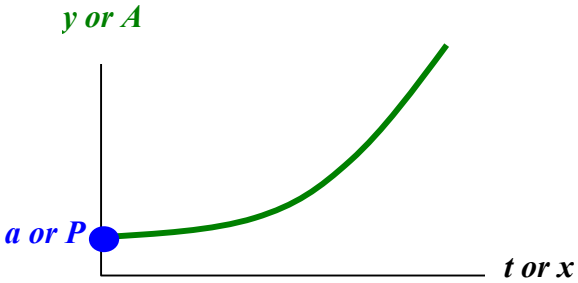


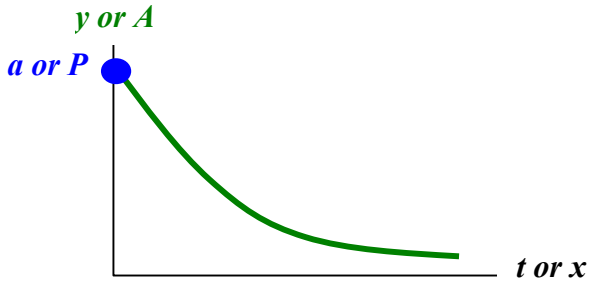
### Exponential Regression

$y = ab^x$ OR $A = P(M)^{\frac{t}{T}}$

$A$ = Final Amount at time $t$ $P$ = Initial Amount at time 0

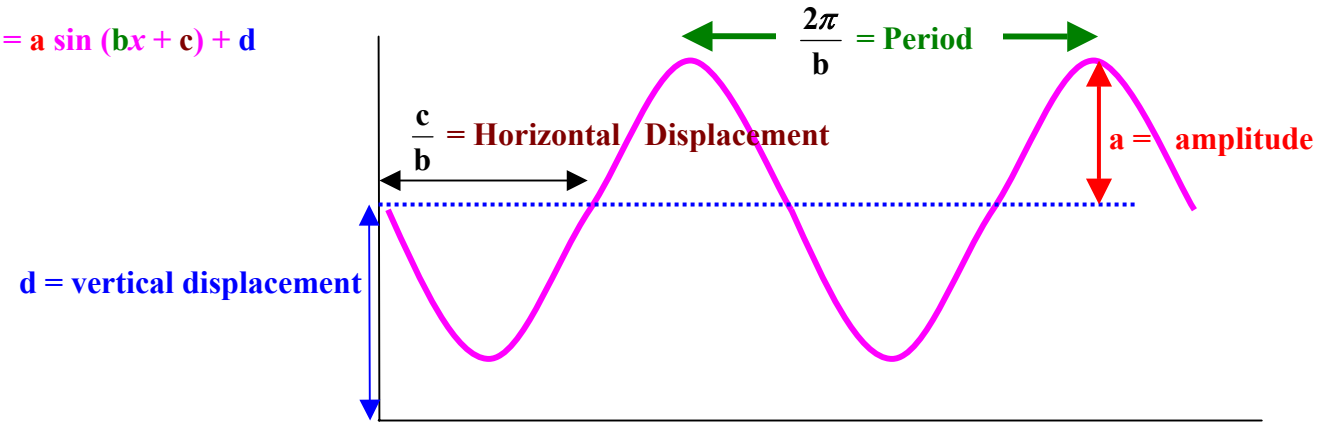
$M$ = Multiplying Factor $T$ = time per multiplying period




### Sinusoidal Regression (Calculator automatically generate equation in RADIAN measure.)

$y = a \sin(bx + c) + d$



Example 1: The following is the hour of sunrise for Calgary at the beginning of each month in 2001.

| 2001 | Day Number | Hour of Sunrise | 2001 | Day Number | Hour of Sunrise |
|---|---|---|---|---|---|
| Jan 1 | 1 | 8.77 | July 1 | 183 | 4.20 |
| Feb 1 | 32 | 8.20 | Aug 1 | 214 | 4.63 |
| Mar 1 | 60 | 7.30 | Sept 1 | 245 | 5.54 |
| Apr 1 | 91 | 6.13 | Oct 1 | 275 | 6.66 |
| May 1 | 121 | 5.08 | Nov 1 | 306 | 7.78 |
| June 1 | 153 | 4.35 | Dec 1 | 335 | 8.54 |

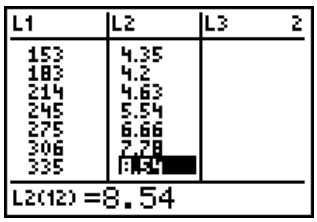
**Conversion of 12-hour Clock to Decimal Time:**

6:26 AM $= 6\frac{26}{60} = 6.43$ hour  8:41 PM = 20:41 (24 hour time) $= 20\frac{41}{60} = 20.683$ hour

a. Graph the data above and obtain an equation from your graphing calculator. Label your scales and your mode used.
b. Explain parameters a, b, c, d in terms of the data above.
c. Use your equation to calculate the hour of sunrise for May 14, 2001.
d. On what date did the sunrise the earliest?
e. How many days out of the year 2001 did the sun rise before 7:30 AM?

**1. Enter Table in Stat Editor**

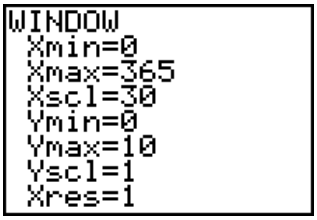
STAT ENTER




**2. Set WINDOW**

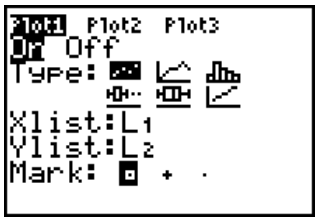
WINDOW

*x*: [0, 365, 30]
*y*: [ 0, 10, 1]



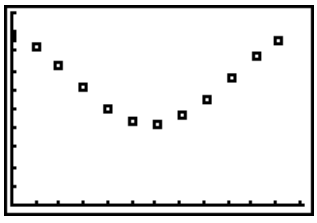

**3. Turn Stat Plot ON**

2nd STAT PLOT Y=




**4. Graph Scatter Plot**

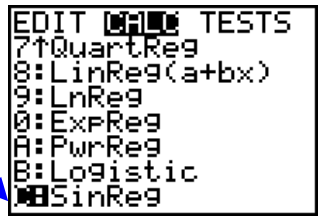
GRAPH

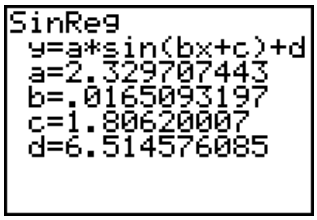


**5. Run Sinusoidal Regression**

STAT ENTER

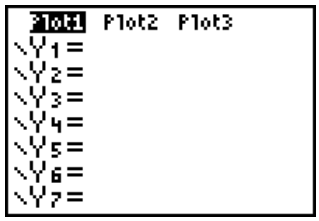
Select Option C



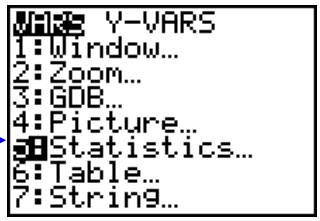

**6. Copy Regression Equation onto Y= Editor**

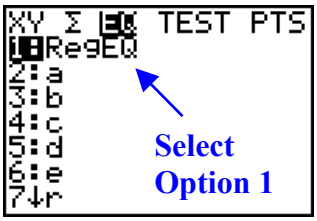
Y=




VARS

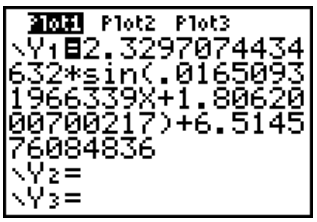
Select Option 5



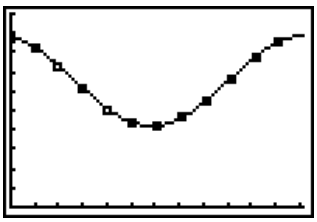



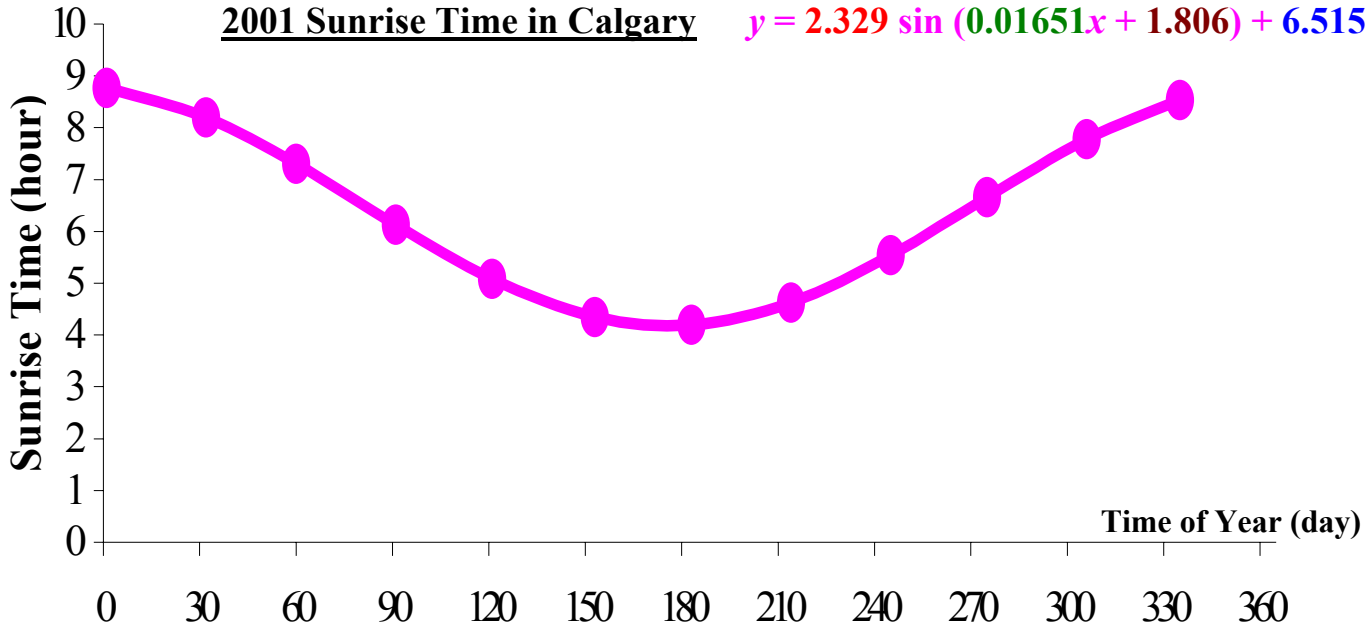

Select Option 1

**7. Graph Regression Equation along with Scatter Plot (A Good Fit!)**




GRAPH






**b. Explanations of Parameters**

**a = amplitude (amount of variance from the median sunrise time) = 2.329 hours (2 hrs 20 mins)**

**d = vertical displacement (median sunrise time) = 6.515 hours (6:31 AM)**

**b = parameter needed to find the Period (b is NOT the PERIOD)**

$$\text{Period} = \frac{2\pi}{b} = \frac{2\pi}{0.01650931966339}$$

**Period = 380.58 days (should be 365 days = 1 year)**

**c = parameter needed to find the Horizontal Displacement (c is NOT the Horizontal Displacement)**

$$\text{Horizontal Displacement} = \frac{c}{b} = \frac{1.8062000700217}{0.01650931966339}$$

**Horizontal Displacement = 109.4 days (April 19)**
**(should be 80.9 days = March 22 = Spring Equinox)**

**Spring Equinox**: - when the sun rise at the median sunrise time, $d$ = 6.515 hrs, for the first time of the year)

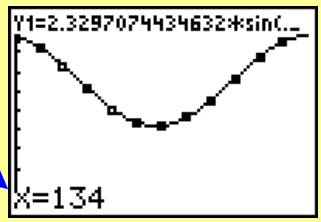
**c. Sunrise Time for May 14, 2001**

May 14th = 31 days (Jan) + 28 days (Feb) +31 days (Mar) + 30 Days (Apr) + 14 days (part of May)

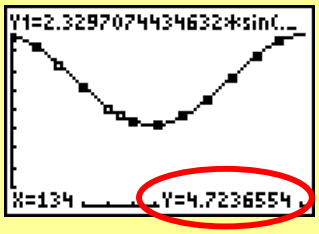
**May 14th = 134 days**

**Use TRACE Function (Be sure Y1 is selected – Use ▼ to switch equation)**

**TRACE**

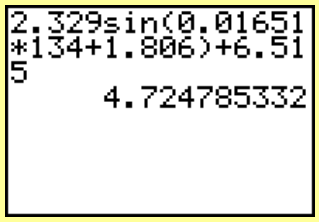



**ENTER**




**Type 134**

**OR Substitute $x$ = 134 into Equation (in RADIAN Mode)**




**Sunrise Time on May 14th = 4.72 hours = 4:43 AM**

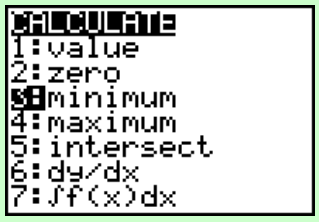
**d. Earliest Sunrise Time**

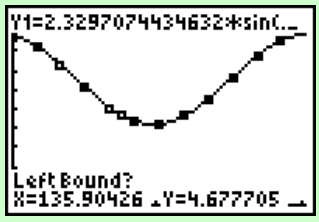
**1. Use MINIMUM Function** **2. Set Left and Right Boundaries by using ◀ or ▶**

**2nd** **CALC**

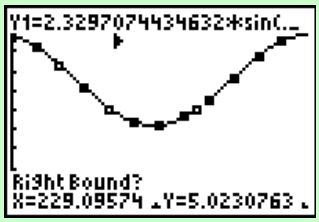
**TRACE**

**(Take cursor left of the minimum, then right of the minimum point)**



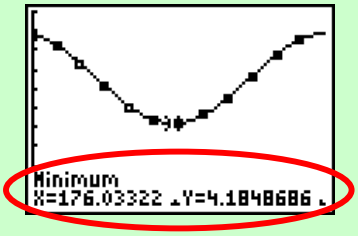




**ENTER**




**ENTER**

**3. Press ENTER again for Minimum Point**




**Earliest Sunrise Time = 4.185 hours on 176th day**
**Earliest Sunrise Time = 4:11 AM on June 25th**

**Actual Date for Earliest Sunrise Time = June 21st (Summer Solstice)**

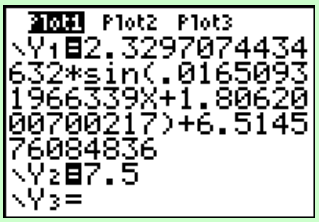
**e. Number of Days Sun Rose before 7:30 AM (7.5 hrs)**

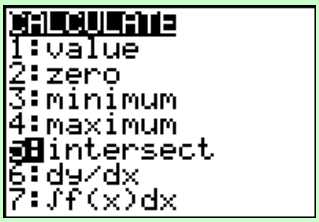
**1. Enter $y = 7.5$**

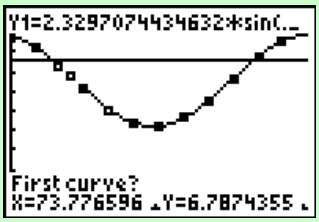
**2. Find the First Intersecting Point**

**2nd** **CALC** **TRACE**

**(Take cursor near first intersecting point by using ◀ )**

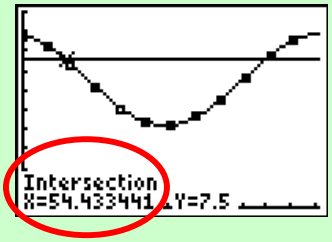


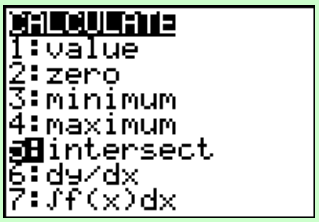



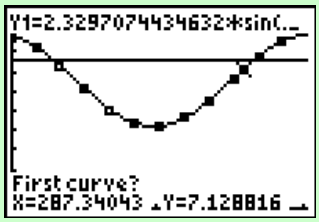




**ENTER** **ENTER** **ENTER**

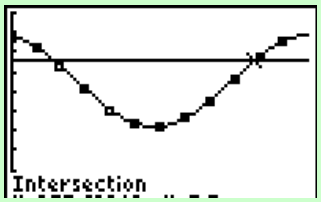
**3. Find the Second Intersecting Point**
**(Take cursor near the second intersecting point by using ▶ )**




**ENTER** **ENTER** **ENTER**



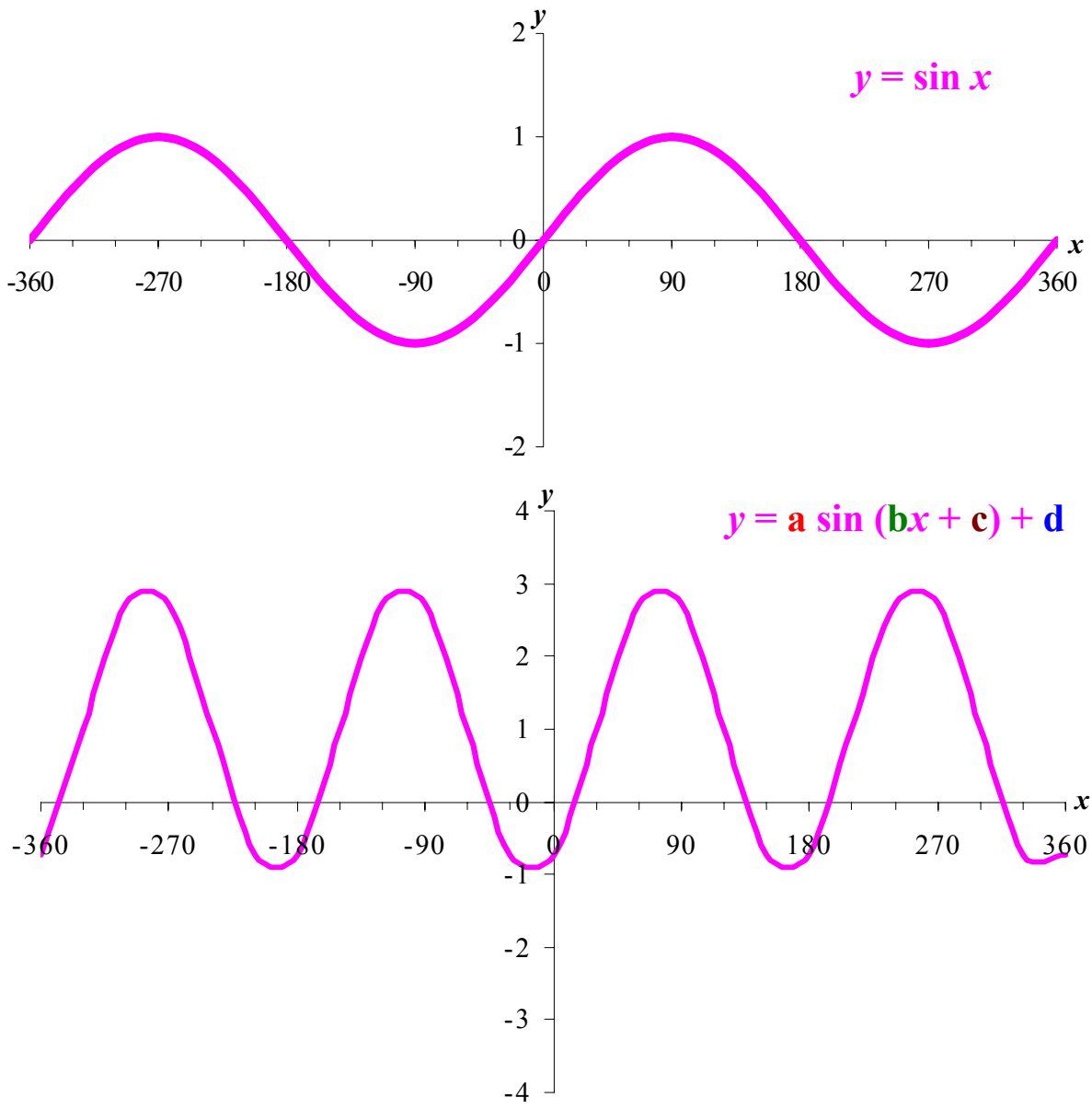

Number of Days Sun Rose Before 7:30 AM (7.5 hrs) = 298$^{th}$ day – 54$^{th}$ day

**Number of Days Sun Rose Before 7:30 AM = 244 days**

**5-3 Assignment**: pg. 229 – 232 #1 to 6

## 5-4: The Characteristics of $y = a \sin (bx + c) + d$







$$y = a \sin (bx + c) + d$$

$a$ = Amplitude

$d$ = Vertical Displacement (how far away from the $x$–axis)

Period = $\frac{2\pi}{b} = \frac{360^\circ}{b}$

$\frac{c}{b}$ = Horizontal Translation / Displacement

$\frac{c}{b} > 0$ (shifted left) $\frac{c}{b} < 0$ (shifted right)

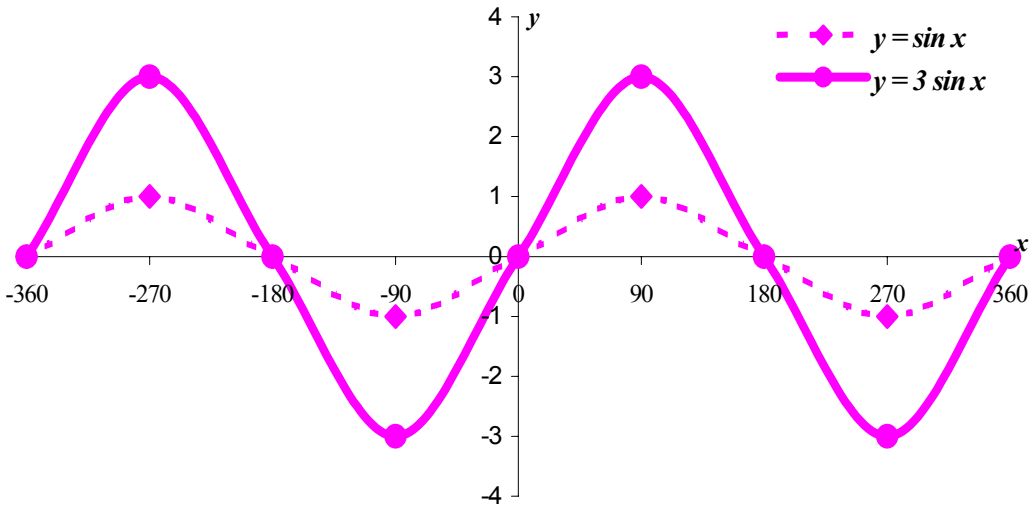
Range = Minimum $\leq y \leq$ Maximum

Example 1: Graph $y = \sin x$ in DEGREE mode along with each of the following equations. Use the specified WINDOW setting.

**a.** $y = 3 \sin x$

$x$: $[-360^o, 360^o, 90^o]$
$y$: $[-4, 4, 1]$

| | |
|---|---|
| **a** | **3** |
| **b** | **1** |
| **c** | **0** |
| **d** | **0** |
| **Maximum** | **3** |
| **Minimum** | **–3** |

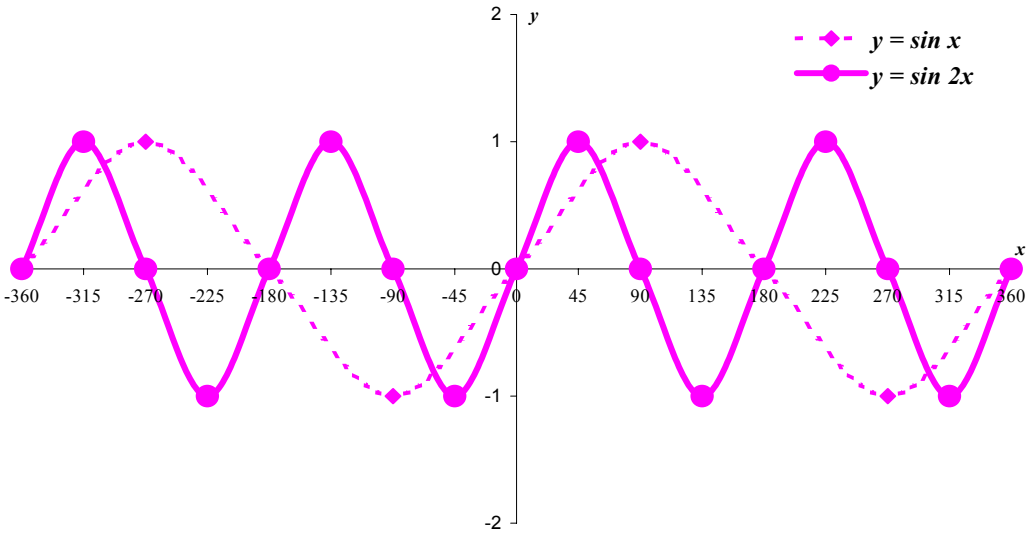



| | | |
|---|---|---|
| **Amplitude = 3** | **Vertical Displacement = 0** | **Range: $-3 \le y \le 3$** |
| Period $= \frac{360^o}{b} = \frac{360^o}{1}$ **Period = 360°** | Horizontal Displacement $= \frac{c}{b} = \frac{0}{1}$ **Horizontal Displacement = 0** | |

**b.** $y = \sin 2x$

$x$: $[-360^o, 360^o, 45^o]$
$y$: $[-2, 2, 1]$

| | |
|---|---|
| **a** | **1** |
| **b** | **2** |
| **c** | **0** |
| **d** | **0** |
| **Maximum** | **1** |
| **Minimum** | **–1** |

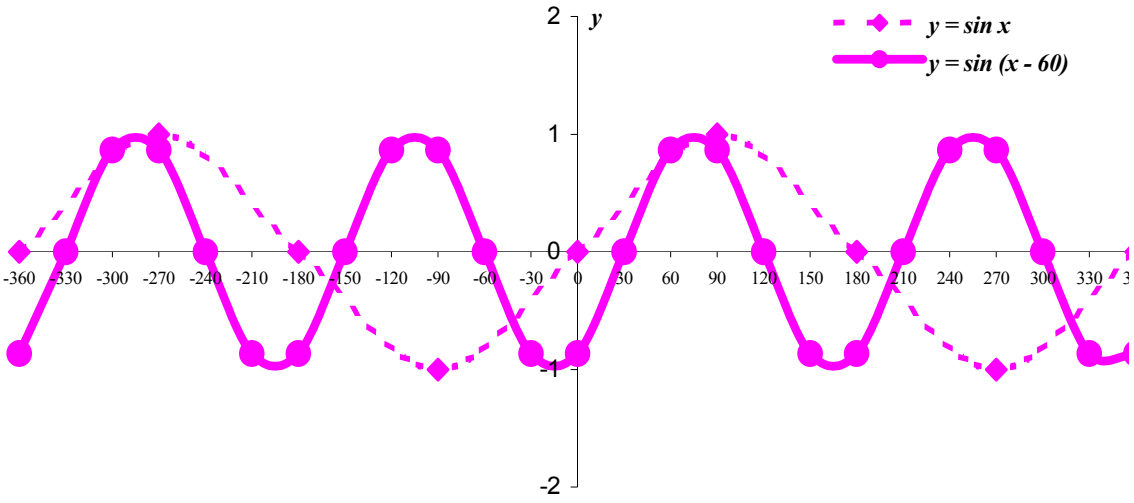



| | | |
|---|---|---|
| **Amplitude = 1** | **Vertical Displacement = 0** | **Range: $-1 \le y \le 1$** |
| Period $= \frac{360^o}{b} = \frac{360^o}{2}$ **Period = 180°** | Horizontal Displacement $= \frac{c}{b} = \frac{0}{2}$ **Horizontal Displacement = 0** | |

**c.** $y = \sin(x - 60^o)$

$x$: $[-360^o, 360^o, 30^o]$
$y$: $[-2, 2, 1]$

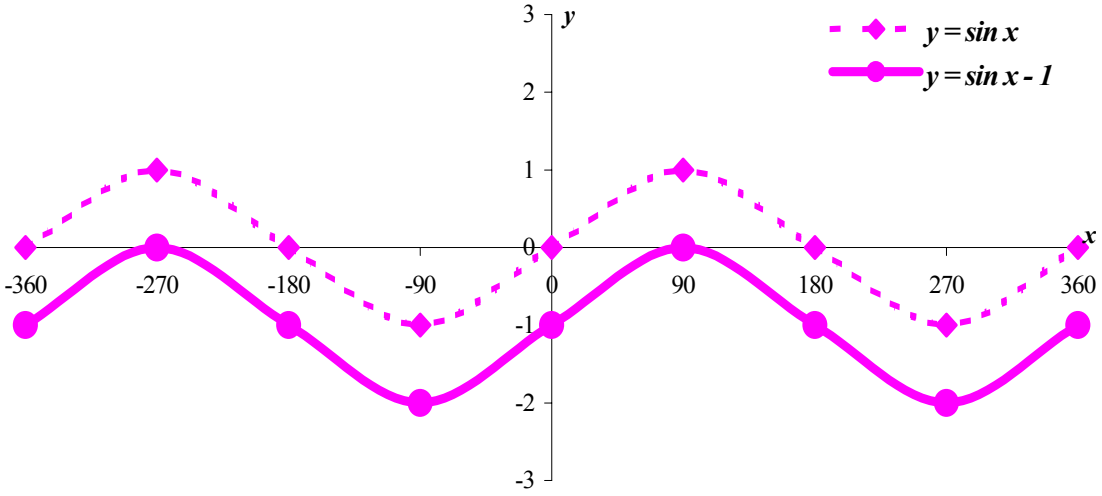



| | |
|---|---|
| a | 1 |
| b | 1 |
| c | $-60^o$ |
| d | 0 |
| Maximum | 1 |
| Minimum | –1 |

| | | |
|---|---|---|
| **Amplitude = 1** | **Vertical Displacement = 0** | **Range: $-1 \le y \le 1$** |
| Period $= \frac{360^o}{b} = \frac{360^o}{1}$ **Period = $360^o$** | Horizontal Displacement $= \frac{c}{b} = \frac{-60^o}{1} = -60^o$ **Horizontal Displacement = $60^o$ to the right** | |

**d.** $y = \sin x - 1$

$x$: $[-360^o, 360^o, 90^o]$
$y$: $[-3, 3, 1]$



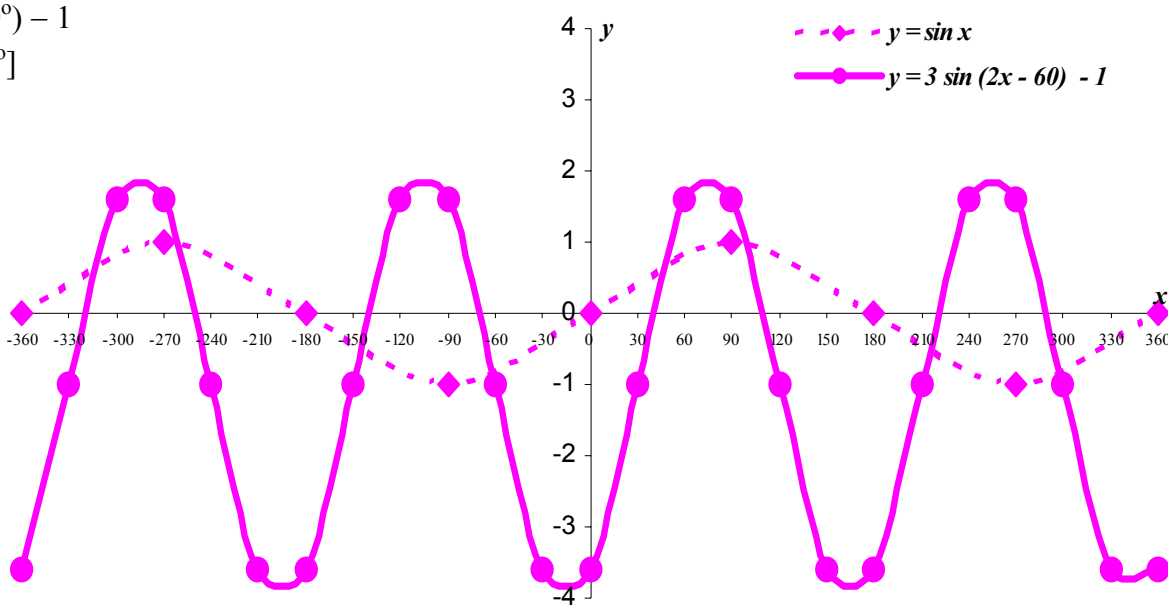

| | |
|---|---|
| a | 1 |
| b | 1 |
| c | 0 |
| d | –1 |
| Maximum | 1 |
| Minimum | –1 |

| | | |
|---|---|---|
| **Amplitude = 1** | **Vertical Displacement = 1 down** | **Range: $-2 \le y \le 0$** |
| Period $= \frac{360^o}{b} = \frac{360^o}{1}$ **Period = $360^o$** | Horizontal Displacement $= \frac{c}{b} = \frac{0}{1}$ **Horizontal Displacement = 0** | |

**e.** $y = 3 \sin (2x - 60^o) - 1$

$x$: [ $-360^o$, $360^o$, $30^o$]

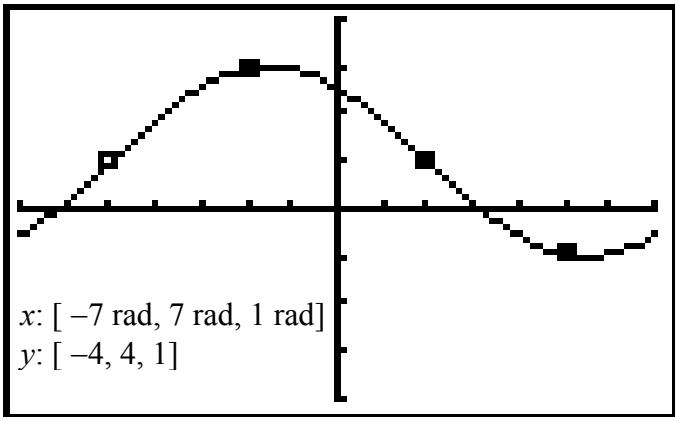
$y$: [ $-4$, $4$, $1$]




| | |
|---|---|
| a | 3 |
| b | 2 |
| c | $-60^o$ |
| d | $-1$ |
| Maximum | 2 |
| Minimum | $-4$ |

| | | |
|---|---|---|
| **Amplitude = 3** | **Vertical Displacement = 1 down** | **Range: $-4 \le y \le 2$** |
| Period $= \frac{360^o}{b} = \frac{360^o}{2}$ **Period = $180^o$** | Horizontal Displacement $= \frac{c}{b} = \frac{-60^o}{2} = -30^o$ **Horizontal Displacement = $30^o$ to the right** | |

Example 2: Find the equation of the following graph in radians.



$x$: [ $-7$ rad, $7$ rad, $1$ rad]

$y$: [ $-4$, $4$, $1$]

**Amplitude = 2**

**Vertical Displacement = 1 up**

**Range: $-1 \le y \le 3$**

**Period = 14 rad**

Period $= \frac{2\pi}{b}$ $b = \frac{2\pi \text{ rad}}{\text{Period}} = \frac{2\pi \text{ rad}}{14 \text{ rad}}$ $b = \frac{\pi}{7}$

Horizontal Translation $= \frac{c}{b}$

$c$ = Horizontal Displacement $\times$ $b$ $= 5 \text{ rad} \times \frac{\pi}{7}$ $c = \frac{5\pi}{7}$

| a | b | c | d | Maximum | Minimum |
|---|---|---|---|---|---|
| 2 | $\frac{\pi}{7}$ | $\frac{5\pi}{7}$ | 1 | 3 | $-1$ |

$$y = 2 \sin \left(\frac{\pi}{7}x + \frac{5\pi}{7}\right) + 1$$

Example 3: Graph $y = 3 \sin (3x + \frac{\pi}{2}) + 2$ for $-2\pi \le x \le 2\pi$. Select and write down the proper scales for your WINDOW settings. What are the values of a, b, c and d? How are they related to the amplitude, horizontal translation, vertical displacement and the range?

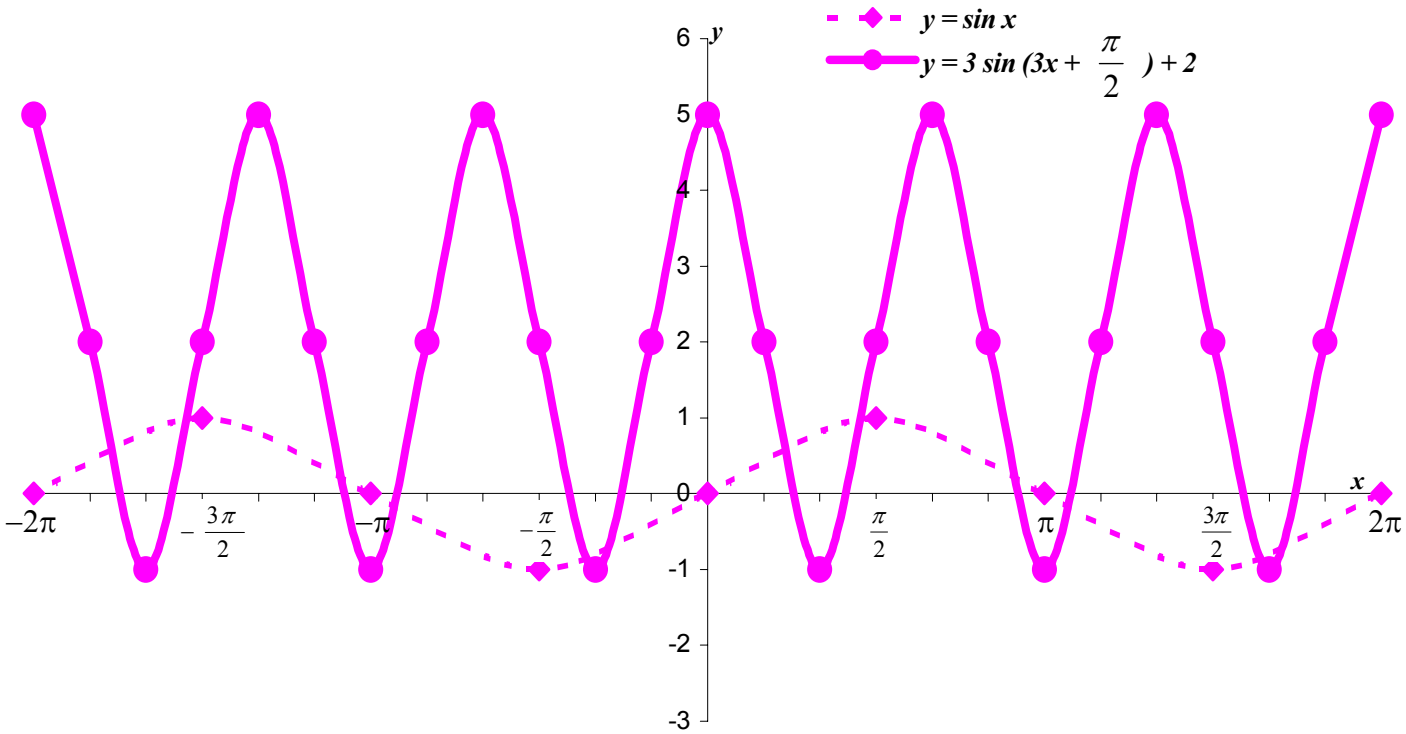
| a | b | c | d | Maximum | Minimum |
|---|---|---|---|---|---|
| 3 | 3 | $\frac{\pi}{2}$ | 2 | 5 | –1 |

| Amplitude = 3 | Vertical Displacement = 2 up | Range: $-1 \le y \le 5$ |
|---|---|---|

Period = $\frac{2\pi}{b} = \frac{2\pi}{3}$ **Period = $\frac{2\pi}{3}$ rad**

Horizontal Displacement = $\frac{c}{b} = \frac{\left(\frac{\pi}{2}\right)}{3} = \frac{\pi}{6}$

**Horizontal Displacement = $\frac{\pi}{6}$ rad to the left**




**5-4 Assignment: pg. 238 – 240 #1 to 8**

## 5-5: Applications of Sinusoidal Data

Sometimes, a data table is not provided for the sketch of a scatter plot. But instead, a description of the periodic pattern is given instead. In these cases, it is very important to **determine the features of the graph (amplitude, period, horizontal displacement, and vertical displacement)**. They will be used to generate the parameters needed for the basic sinusoidal equation, $y = a \sin(bx + c) + d$.

Example 1: A mechanical pendulum has a height of 3 m off the ground. When it is swung to the highest point, its height is 7 m off the ground. It makes 15 complete swing per minute, and the starting point is on the right side of the rest position.

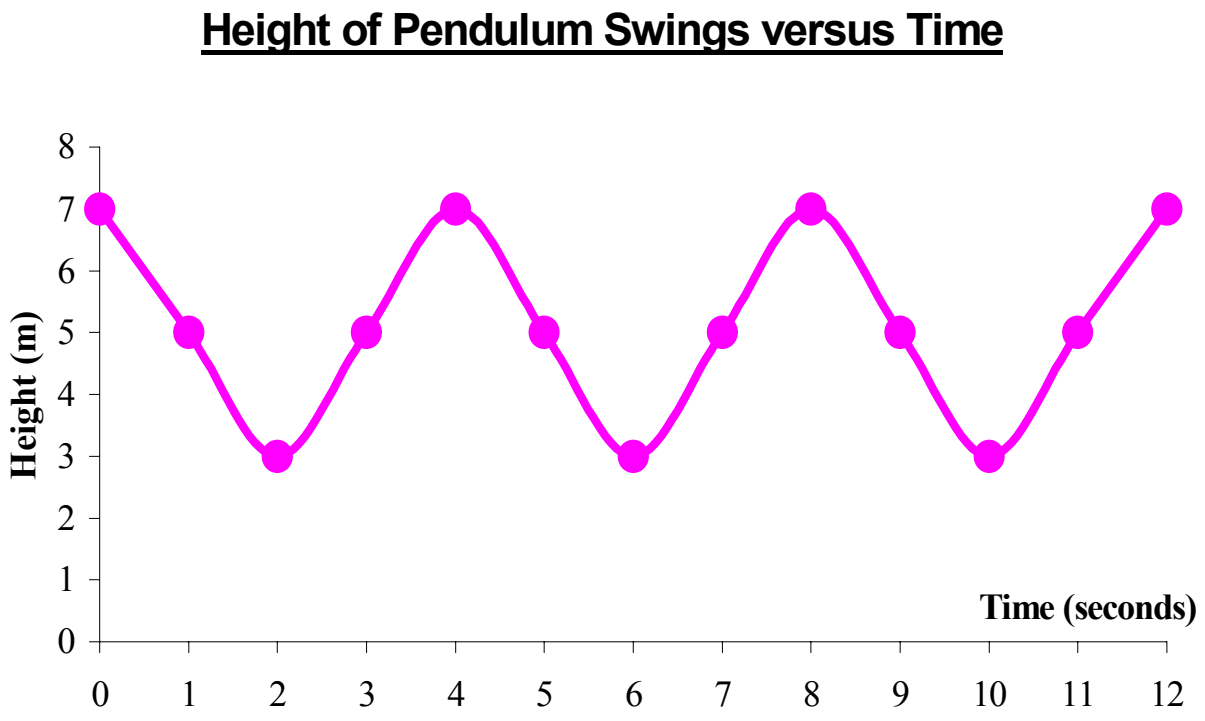
a. What is the period of the pendulum?
b. Draw a graph to describe the height of the pendulum versus time for 3 complete cycles.
c. Explain all the features of the graph and determine the equation of height in terms of time.
d. Find the height of the pendulum at 10.3 seconds.
e. At what time(s) will the height of the pendulum be at 5.5 m during the first complete cycle?

**a.** $\text{Frequency} = 15 \text{ swings / min} = \dfrac{15 \text{ swings}}{60 \text{ seconds}} = \dfrac{1 \text{ swing}}{4 \text{ seconds}}$

$\text{Period} = \dfrac{\text{time in seconds}}{1 \text{ cycle (or swing)}} = \dfrac{4 \text{ seconds}}{1 \text{ swing}}$

**Period = 4 sec / cycle**

**b.**



**c. Characteristics of the Graph**

**Amplitude = a = 2 m (how far the height is varied from one side of the swing to the rest position)**

**Vertical Displacement = d = 5 m (the height between the rest position and one side of the swing)**

**Range: 3 m $\le h \le$ 7 m (the minimum and maximum heights of the pendulum)**

**Period = 4 seconds (time to complete one full swing)** **Horizontal Translation = –3 second (right)** $= \frac{c}{b}$

Period $= \frac{2\pi}{b}$ $b = \frac{2\pi \text{ rad}}{\text{Period}} = \frac{2\pi}{4}$ $\boxed{b = \frac{\pi}{2}}$ $c$ = Horizontal Displacement $\times\ b = -3 \times \frac{\pi}{2}$ $\boxed{c = -\frac{3\pi}{2}}$

**OR**

Period $= \frac{360^\circ}{b}$ $b = \frac{360^\circ}{\text{Period}} = \frac{360^\circ}{4}$ $\boxed{b = 90^\circ}$ $c$ = Horizontal Displacement $\times\ b = -3 \times 90^\circ$ $\boxed{c = -270^\circ}$

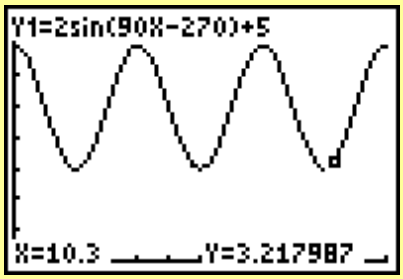
| a | b | c | d |
|---|---|---|---|
| 2 | $\frac{\pi}{2}$ or $90^\circ$ | $-\frac{3\pi}{2}$ or $-270^\circ$ | 5 |

**In Radian Mode:** $h = 2\sin\left(\frac{\pi}{2}t - \frac{3\pi}{2}\right) + 5$ **In Degree Mode:** $h = 2\sin(90t - 270) + 5$

**d. Height at 10.3 seconds**

1. **Enter equation in Degree Mode**
2. **Run TRACE.**

**TRACE**

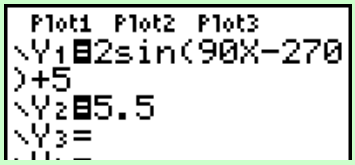



**x: [0, 12, 1]**
**y: [ 0, 8, 1]**

**Height = 3.218 m**

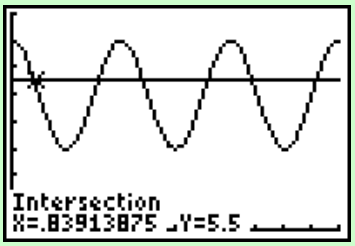
**e. When will the pendulum reach 5.5 m?**

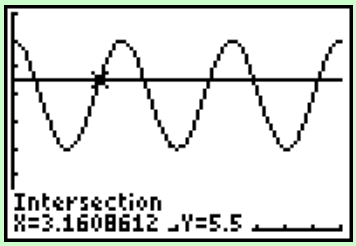
1. **Enter equation in Degree Mode**
2. **Enter $Y_2$ equation as 5.5**




**x: [0, 12, 1]**
**y: [ 0, 8, 1]**

3. **Run Intersect twice on the first cycle.**




**$t$ = 0.839 seconds and 3.161 seconds**

**5-5 Assignment**
**pg. 244 – 247 #1 to 7**

# Unit 6 Project: Sunrise and Sunset

**Purpose:** To analyze the patterns of sunrise and sunset times throughout a year, and to compare sunrise and sunset times at different locations around the world.

**Procedure:**

1. Select three cities from the list below, one from the Northern Hemisphere (above the Tropic of Cancer 23°N), another from the Southern Hemisphere (below the Tropic of Capricorn 23°S), and the last one from the equatorial region (between the Tropic of Cancer and Capricorn).

| Northern Hemisphere | | | |
|---|---|---|---|
| **City, Country** | **Longitude (Degree °, Minute ‘)** | **Latitude (Degree °, Minute ’)** | **Time Zone (Hours East/West of Greenwich)** |
| Calgary, Canada | W 114° 09’ | N 51° 10’ | 7 Hours West |
| San Francisco, USA | W 122° 18’ | N 37° 40’ | 8 Hours West |
| Houston, USA | W 95° 25’ | N 29° 45’ | 6 Hours West |
| Beijing, China | E 116° 26’ | N 39° 55’ | 8 Hours East |
| Berlin, Germany | E 13° 25’ | N 52° 32’ | 1 Hour East |
| Rome, Italy | E 12° 30’ | N 41° 53’ | 1 Hour East |
| Moscow, Russia | E 37° 45’ | N 55° 36’ | 3 Hours East |
| Yellowknife, Canada | W 114° 29’ | N 62° 30’ | 7 Hours West |
| Tokyo, Japan | E 139° 45’ | N 35° 40’ | 9 Hours East |
| Toronto, Canada | W 79° 46’ | N 43° 42’ | 5 Hours West |

| Southern Hemisphere | | | |
|---|---|---|---|
| **City, Country** | **Longitude (Degree °, Minute ‘)** | **Latitude (Degree °, Minute ’)** | **Time Zone (Hours East/West of Greenwich)** |
| Falkland Island, UK | W 60° 00’ | S 52° 30’ | 4 Hours West |
| Wellington, New Zealand | E 174° 47’ | S 41° 17’ | 12 Hours East |
| Santiago, Chile | W 70° 40’ | S 33° 30’ | 4 Hours West |
| Buenos Aires, Argentina | W 58° 30’ | S 34° 40’ | 3 Hours West |
| Cape Town, South Africa | E 18° 28’ | S 33° 56’ | 2 Hours East |
| Sydney, Australia | E 151° 10’ | S 33° 55’ | 10 Hours East |
| Melbourne, Australia | E 144° 58’ | S 37° 45’ | 10 Hours East |
| Faraday, Antarctica | W 64° 16’ | S 65° 15’ | 4 Hours West |
| Johannesburg, S. Africa | E 28° 02’ | S 26° 10’ | 2 Hours East |
| Perth, Australia | E 115° 49’ | S 31° 58’ | 8 Hours East |

| Equatorial Region | | | |
|---|---|---|---|
| **City, Country** | **Longitude (Degree °, Minute ‘)** | **Latitude (Degree °, Minute ’)** | **Time Zone (Hours East/West of Greenwich)** |
| Hong Kong, China | E 114° 10’ | N 22° 10’ | 8 Hours East |
| Singapore, Singapore | E 103° 40’ | N 1° 20’ | 7 Hours East |
| Honolulu, USA | W 157° 50’ | N 21° 19’ | 10 Hours West |
| Kingston, Jamaica | W 76° 48’ | N 17° 58’ | 5 Hours West |
| Manila, Philippines | E 120° 58’ | N 14° 37’ | 8 Hours East |
| Lima, Peru | W 78° 40’ | S 12° 06’ | 5 Hours West |
| Darwin, Australia | E 130° 44’ | S 12° 23’ | 8 Hours East |
| Mexico City, Mexico | W 99° 10’ | N 19° 25’ | 6 Hours West |
| Galapagos Isles, Ecuador | W 90° 00’ | S 0° 05’ | 6 Hours West |
| Colombo, Sri Lanka | E 79° 52’ | N 6° 55’ | 6 Hours East |

**2.** Go to the U.S. Naval Observatory Astronomical Applications Department Data Service's web site **http://aa.usno.navy.mil/data/docs/RS_OneYear.html** and print out the table for the sunrise and sunset times of the current year for your selected cities. Make sure you print out your tables in Landscape. If you have to save your files on your desktop first, open them with Microsoft WORDS, select the entire table, change the font size to 8, and change the Page Set-up to Landscape before printing. **Attach these printouts to your final report.**

3. Copy and complete three different tables as shown below. The first one is for sunrise times, the next one is for sunset times, and the last one is for amount of daylight. Convert all calendar dates (the 1st of each month) to day numbers, and all 24-hour times to decimal times. Sample conversions must be included. Explain and show how you would calculate the amount of daylight.

| Sunrise Times | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Date** | **Day Number** | **City 1:** | | **City 2:** | | **City 3:** | |
| | | **24-hour Time** | **Decimal Time** | **24-hour Time** | **Decimal Time** | **24-hour Time** | **Decimal Time** |
| Jan 1 | 1 | | | | | | |
| Feb 1 | 32 | | | | | | |
| Mar 1 | 60 | | | | | | |
| Apr 1 | | | | | | | |
| May 1 | | | | | | | |
| June 1 | | | | | | | |
| July 1 | | | | | | | |
| Aug 1 | | | | | | | |
| Sept 1 | | | | | | | |
| Oct 1 | | | | | | | |
| Nov 1 | | | | | | | |
| Dec 1 | | | | | | | |

| Sunset Times | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Date** | **Day Number** | **City 1:** | | **City 2:** | | **City 3:** | |
| | | **24-hour Time** | **Decimal Time** | **24-hour Time** | **Decimal Time** | **24-hour Time** | **Decimal Time** |
| Jan 1 | 1 | | | | | | |
| Feb 1 | 32 | | | | | | |
| Mar 1 | 60 | | | | | | |
| Apr 1 | | | | | | | |
| May 1 | | | | | | | |
| June 1 | | | | | | | |
| July 1 | | | | | | | |
| Aug 1 | | | | | | | |
| Sept 1 | | | | | | | |
| Oct 1 | | | | | | | |
| Nov 1 | | | | | | | |
| Dec 1 | | | | | | | |

| Amount of Sunlight (in Hours) | | | | |
|---|---|---|---|---|
| **Date** | **Day Number** | **City 1:** | **City 2:** | **City 3:** |
| Jan 1 | 1 | | | |
| Feb 1 | 32 | | | |
| Mar 1 | 60 | | | |
| Apr 1 | | | | |
| May 1 | | | | |
| June 1 | | | | |
| July 1 | | | | |
| Aug 1 | | | | |
| Sept 1 | | | | |
| Oct 1 | | | | |
| Nov 1 | | | | |
| Dec 1 | | | | |

4. Enter the sunrise data table into your graphing calculator. Select the appropriate WINDOW settings for your graph. Be sure to set the MODE to RADIAN. Draw out your graph on a fine graphing paper. Write out a title along with all the proper labeling on your axis and sine curves. Run Sinusoidal Regressions on all three curves. Include them on your graph paper. Select the appropriate variables.
5. Repeat Step 4 with the sunset, and the amount of sunlight tables.
6. What are the range, amplitude, phase shift, vertical displacement, and period of each of the sine curve in the amount of sunlight graph?
7. What do the parameters a, b, c, d represents for each of the three graphs?
8. Define and Explain Longitude, Latitude, Time Zone from Greenwich, Summer and Winter Solstices, Spring and Fall (Autumn) Equinoxes.
9. For each of the three cities, on which calendar dates has the most and least amount of sunlight? Locate and label them on your graph. How do these dates relate to some of the terms you define in the previous step?
10. When (calendar dates) did the spring and autumn equinoxes occur for each of the three cities? Locate and label them on your graph. Which feature(s) and/or parameter(s) from the Sinusoidal Regression Equation, $y = a \sin(bx + c) + d$ , corresponds with the definition of equinox? Explain.
11. Explain why the northern and southern hemispheric cities you have chosen have bigger ranges of sunlight amount annually compare to that of the equatorial city. Include any diagrams you may come across in your research to help illustrate your explanations.
12. **Bonus Marks:** Determine the longitudes, latitudes, and time zone for the Greenwich Meridian at the Equator, the North Pole, and the South Pole. Go to the same web site in Step 2. Print the tables. Determine the amount of sunlight for the 21$^{st}$ of each month. Explain your results. Again, you may use diagrams to enhance your explanations.

**Note:**

**1.** All graphs drawn must be properly labelled.

**2.** Please read carefully and do not omit any steps in the Procedure. All sample calculations, explanations, diagrams must be shown clearly.

**3.** Include all printouts from the Internet with your final report.

**4.** Students can work together to discuss the project, but they should each have their own numbers to work with and do their own calculations. Students who copied from each other will end up sharing the mark. Let's say the mark was 70% and two students were involved in copying each other's work. They each get 35%.

**5.** Late Project handed in one day after the due date is counted as 30% off the total mark. Project handed in two days and later will not be marked.

Due Date: _____

## Sample Tables of Calgary's 2001 Sunrise and Sunset Times

**(From the U.S. Naval Observatory Astronomical Applications Department Data Service's Web Site)**

**http://aa.usno.navy.mil/data/docs/RS_OneYear.html**

Location: W114 09, N51 10

CALGARY, CANADA
Rise and Set for the Sun for 2001

Zone: 7h West of Greenwich

Astronomical Applications Dept.
U. S. Naval Observatory
Washington, DC 20392-5420

| | Jan. | | Feb. | | Mar. | | Apr. | | May | | June | | July | | Aug. | | Sept. | | Oct. | | Nov. | | Dec. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Day | Rise | Set | Rise | Set | Rise | Set | Rise | Set | Rise | Set | Rise | Set | Rise | Set | Rise | Set | Rise | Set | Rise | Set | Rise | Set | Rise | Set |
| | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m | h m |
| 01 | 0841 | 1640 | 0813 | 1728 | 0721 | 1818 | 0612 | 1910 | 0509 | 1959 | 0427 | 2043 | 0426 | 2055 | 0502 | 2023 | 0550 | 1922 | 0638 | 1814 | 0730 | 1710 | 0819 | 1632 |
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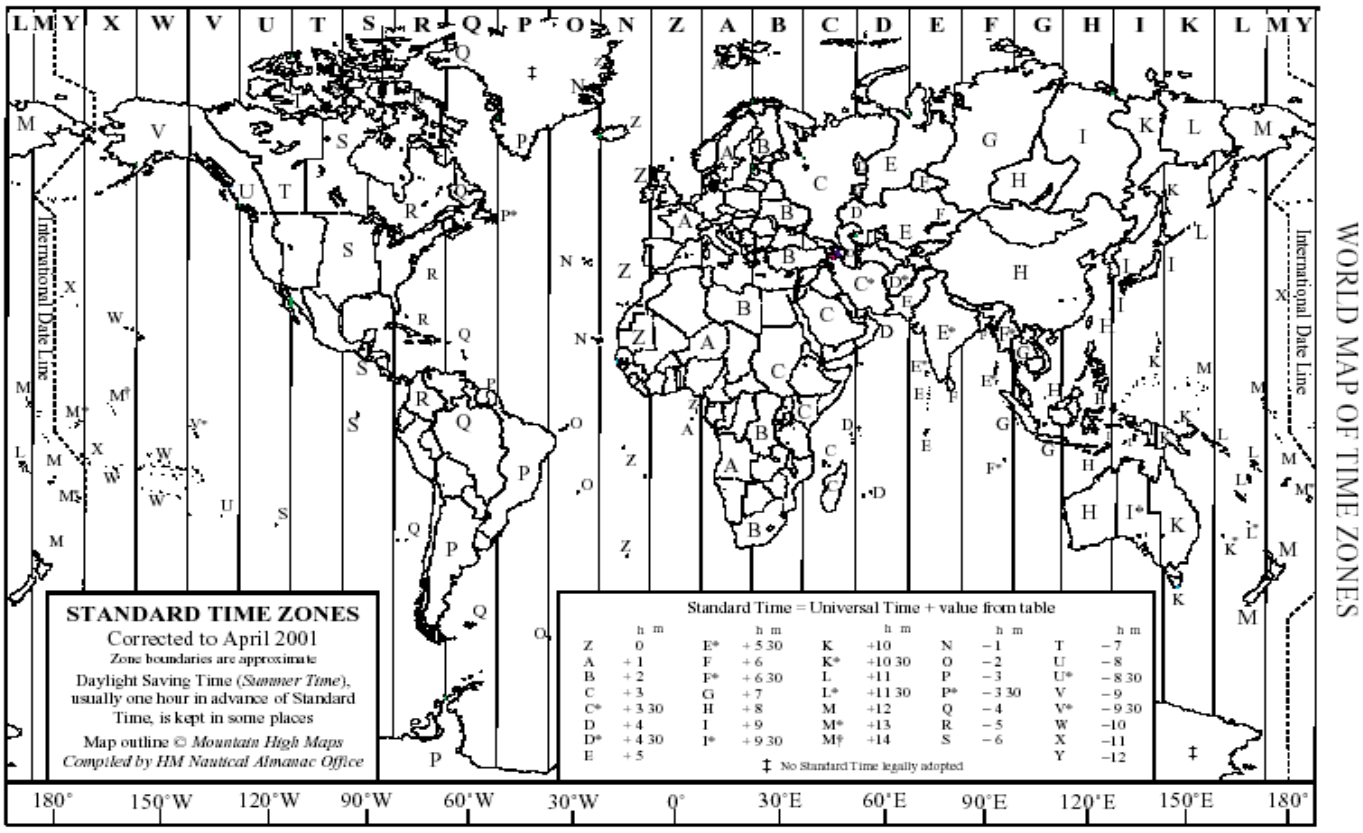


WORLD MAP OF TIME ZONES