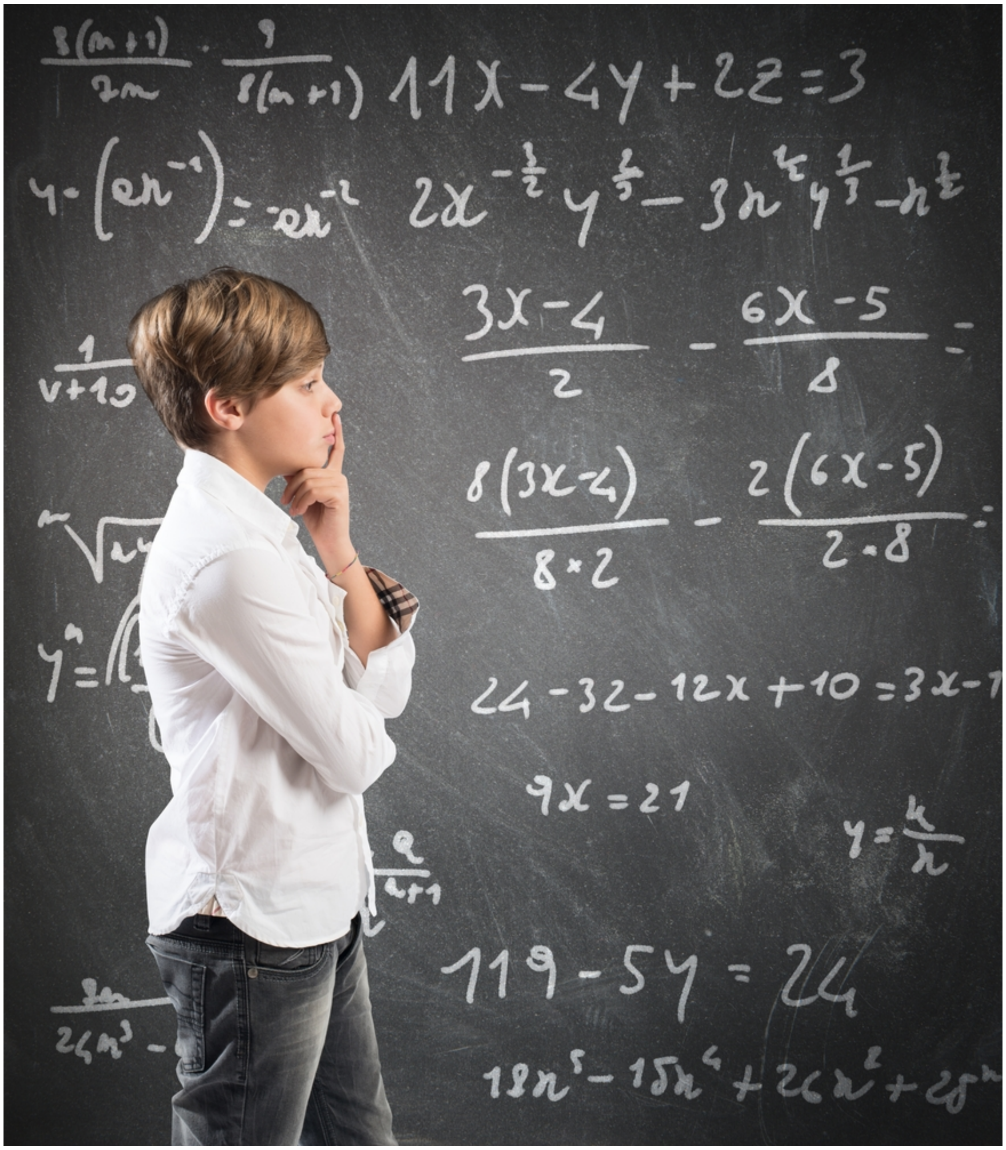
Woodside Priory School - Chemistry

Rearranging Formulae



This lesson covers rearranging formulae. The manipulation of algebraic expressions and equations is an important skill in Chemistry. Since we will encounter situations where you must solve for different variables in many formulae, you must complete this booklet at the beginning of this course.

Contents

Introduction

The ability to rearrange formulas or rewrite them in different ways is an important skill in mathematics. This course will explain how to rearrange some simple formulas.

The subject of a formula

You will be familiar with the formula for the area of a circle which states $A = \pi r^2$.
Here, A is the area and r is the radius.

In the form $A = \pi r^2$, we say that A is the **subject** of the formula. Usually the subject of a formula is on its own on the left-hand side. If you know the value of r then you can substitute directly to find A.

Changing the subject of a formula

Changing the subject of a formula is exactly the same as solving an equation. The key thing to remember is that ‘whatever you do to one side of the formula you must do to the other side’.

To rearrange a formula you may
- add or subtract the same quantity to or from both sides
- multiply or divide both sides by the same quantity

Example 1

Make x the subject of the formula in each of the following cases.

a) $a + x = y + z$

b) $a + 3x = y + z$

c) $ax = y + z$

Solutions:

a) $a + x = y + z$

$a + x - a = y + z - a$ *Subtract a from both sides*

$x = y + z - a$

b) $a + 3x = y + z$

$3x = y + z - a$ *Subtract a from both sides*

$x = \frac{y+z-a}{3}$ *Divide both sides by 3*

c) $ax = y + z$

$x = \frac{y+z}{a}$ *Simply divide both sides by a*

Exercise 1

1. Prepare yourself by making x the subject in each of the following cases:

a) $y = x + a$	e) $y = \frac{x}{5}$
b) $y = 2x - a$	f) $2w = 3x$
c) $y = 2x + 7$	g) $ax - y = 2y$
d) $y = 7 - 2x$	h) $ax - y + z = b$

2. In each case, make the letter at the end the subject of the formula.

a) $y = mx + c\,, (c)$	d) $2s = 2ut + at^2\,, (a)$
b) $y = mx + c\,, (m)$	e) $v^2 = u^2 + 2as\,, (a)$
c) $v^2 = u^2 + 2as\,, (s)$	f) $y = a^2x + b^2\,, (x)$

Formulae with brackets and fractions

If there are brackets included in the formula then it is easier if you expand them first. Look out for implied brackets in fractions, (part (c) below), and include them where necessary.

Example 2

Make x the subject of the formula in each of the following cases.

a) $a(x+b)=c$

b) $\frac{x}{a}=1+\frac{y}{b}$

c) $\frac{x+y}{y}=\frac{y}{a}+\frac{a}{y}$

Solutions:

a) $a(x+b)=c$

$ax+ab=c$	*Expand the brackets*
$ax=c-ab$	*Subtract ab from each side*
$x=\frac{c-ab}{a}$	*Divide both sides by a*

b) $\frac{x}{a}=1+\frac{y}{b}$

$bx=ab+a$	*Remove the fractions by multiplying both sides by ab*
$x=\frac{ab+a}{b}$	*Divide both sides by b*

c) $\frac{x+y}{y}=\frac{y}{a}+\frac{a}{y}$ *There is an implied bracket in the first term so put it in*

$\frac{(x+y)}{y}=\frac{y}{a}+\frac{a}{y}$	
$a(x+y)=y^2+a^2$	*Remove the fractions by multiplying both sides by* ay
$ax+ay=y^2+a^2$	*Expand the brackets*
$ax=y^2+a^2-ay$	*Subtract* ay *from both sides*
$x=\frac{y^2+a^2-ay}{a}$	*Divide both sides by* a

Exercise 2

1. In each of the following cases make x the subject:

a) $2(x+a)=y$	e) $\frac{x}{a}=\frac{y}{z}$
b) $\frac{x}{a}=\frac{y+z}{b}$	f) $\frac{1}{3}x+2y=3z$
c) $\frac{a(x+y)}{b}=c$	g) $a(x+y)=ay$
d) $a(x+y)=y(a+z)$	h) $\frac{x}{a}=\frac{a}{b}$

2. In each case, make the letter at the end the subject of the formula.

a) $\frac{v-u}{a} = t\,,(u)$	d) $s = ut + \frac{1}{2}at^2\,,(u)$
b) $s = ut + \frac{1}{2}at^2\,,(a)$	e) $\frac{v-u}{a} = t\,,(v)$
c) $s = \left(\frac{u+v}{2}\right)t\,,(u)$	f) $\frac{y-x^2}{x} = 3z\,,(y)$

Formulae which require factorising first

When changing the subject you start by collecting the term that you want on one side of the equation. If this term appears more than once then you will need to factorise before proceeding further.

Example 3

Make x the subject of the formula in each of the following cases.

a) $a(x+y) = b(x+y)$

b) $y = \frac{x+a}{x-a}$

c) $\frac{y}{x} + a = b$

Solutions:

a) $a(x+y) = b(x+y)$

$ax + ay = bx + by$	*Expand the brackets*
$ax - bx = by - ay$	*Collect all the terms in x on one side*
$x(a-b) = by - ay$	*Factorise*
$x = \frac{by-ay}{(a-b)}$	*Divide both sides by* $(a-b)$

b) $y = \frac{x+a}{x-a}$

$y(x-a) = x + a$	*Remove the fractions by multiplying both sides by* $(x-a)$
$yx - ya = x + a$	*Multiply out the bracket*
$yx - x = a + ya$	*Collect all the terms in x on one side*
$x(y-1) = a + ya$	*Factorise the left side*
$x = \frac{a+ay}{y-1}$	*Divide both sides by* $(a-b)$

c) $\frac{y}{x} + a = b$

$y + ax = bx$	*Remove the fractions by multiplying both sides by* x
$y = bx - ax$	*Collect the* x *terms on one side (in this case the right)*
$bx - ax = y$	*You can swap the sides over*
$x(b-a) = y$	*Factorise*
$x = \frac{y}{b-a}$	*Divide both sides by* $(b-a)$

Exercise 3

In each of the following cases make x the subject:

a) $x + xy = y$	e) $\frac{x}{a} = \frac{x}{b} - 1$
b) $x + y = xy$	f) $y(x + z) = 3z(x + y)$
c) $x = y + xy$	g) $ax + b(x - a) = ay$
d) $y = 2 - \frac{1}{x}$	h) $y = \frac{x+1}{x-1}$

Formulae with roots and powers

It is important to remember that square and square root are inverse functions, similarly cube and cube root. It is not always easy to spot when is the best time to square or square root when you are trying to change the subject; this comes with practice.

Example 4

Make x the subject of the formula in each of the following cases.

a) $\sqrt{x} - 3 = y$

b) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 4$

c) $\sqrt{x+3} = y$

Solutions:

a) $\sqrt{x} - 3 = y$

$\sqrt{x} = y + 3$ — *First get $\sqrt{x}$ on its own*

$x = (y+3)^2$ — *Square both sides -* ***remember the brackets***

b) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 4$

$b^2x^2 - a^2y^2 = 4a^2b^2$ — *Remove the fractions by multiplying both sides by* a^2b^2

$b^2x^2 = 4a^2b^2 + y^2a^2$ — *Get the x^2 term on one side*

$x^2 = \dfrac{4a^2b^2+y^2a^2}{b^2}$ — *Divide both sides by* b^2

$x = \pm\sqrt{\dfrac{4a^2b^2+y^2a^2}{b^2}}$ — *Finally square root both sides, remembering the $\pm$ sign*

$)\sqrt{x+3} = y$

$x + 3 = y^2$ — *Start by squaring both sides*

$x = y^2 - 3$

Exercise 4

1. In each of the following cases make x the subject:

a) $\sqrt{x} - 1 = y$	d) $\sqrt[3]{x} - y = 1$
b) $x^2 - y^2 = a^2$	e) $x^3 - y^3 = 1$
c) $\sqrt{x^2 + y^2} = y$	f) $x^2 - 4 = a^2$

2. In each case, make the letter at the end the subject of the formula.

a) $A = 4\pi r^2$, (r)	d) $E = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$, (u)
b)$V = \frac{4}{3}\pi r^3$, (r)	e) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, (y)
c) $V = \pi r^2 h$, (r)	f) $ay^2 = x^3$, (y)

Answers

Exercises 1	Exercises 2	Exercises 3	Exercises 4
1 a) $x = y - a$	1 a) $x = \frac{y-2a}{2}$	a) $x = \frac{y}{1+y}$	1 a) $x = (y+1)^2$
b) $x = \frac{y+a}{2}$	b) $x = \frac{ya+za}{b}$	b) $x = \frac{y}{y-1}$	b) $x = \pm\sqrt{a^2+y^2}$
c) $x = \frac{y-7}{2}$	c) $x = \frac{bc-ay}{a}$	c) $x = \frac{y}{1-y}$	c) $x = 0$
d) $x = \frac{7-y}{2}$	d) $x = \frac{yz}{a}$	d) $x = \frac{1}{2-y}$	d) $x = (1+y)^3$
e) $x = 5y$	e) $x = \frac{ay}{z}$	e) $x = \frac{ab}{a-b}$	e) $x = \sqrt[3]{1+y^3}$
f) $x = \frac{2w}{3}$	f) $x = 9z - 6y$	f) $x = \frac{2yz}{y-3z}$	f) $x = \pm\sqrt{a^2+4}$
g) $x = \frac{3y}{a}$	g) $x = 0$	g) $x = \frac{ay+ab}{a+b}$	2 a) $r = \pm\sqrt{\frac{A}{4\pi}}$
h) $x = \frac{b+y-z}{a}$	h) $x = \frac{a^2}{b}$	h) $x = \frac{1+y}{y-1}$	b) $r = \sqrt[3]{\frac{3V}{4\pi}}$
2 a) $c = y - mx$	2 a) $u = v - at$		c) $r = \pm\sqrt{\frac{V}{\pi h}}$
b) $m = \frac{y-c}{x}$	b) $a = \frac{2s-2ut}{t^2}$		d) $u = \pm\sqrt{\frac{mv^2-2E}{m}}$
c) $s = \frac{v^2-u^2}{2a}$	c) $u = \frac{2s-vt}{t}$		e) $y = \pm\sqrt{\frac{x^2b^2-a^2b^2}{a^2}}$
d) $a = \frac{2s-2ut}{t^2}$	d) $u = \frac{2s-at^2}{2t}$		f) $y = \pm\sqrt{\frac{x^3}{a}}$
e) $a = \frac{v^2-u^2}{2s}$	e) $v = u + at$		
f) $x = \frac{y^2-b^2}{a^2}$	f) $y = 3zx + x^2$		