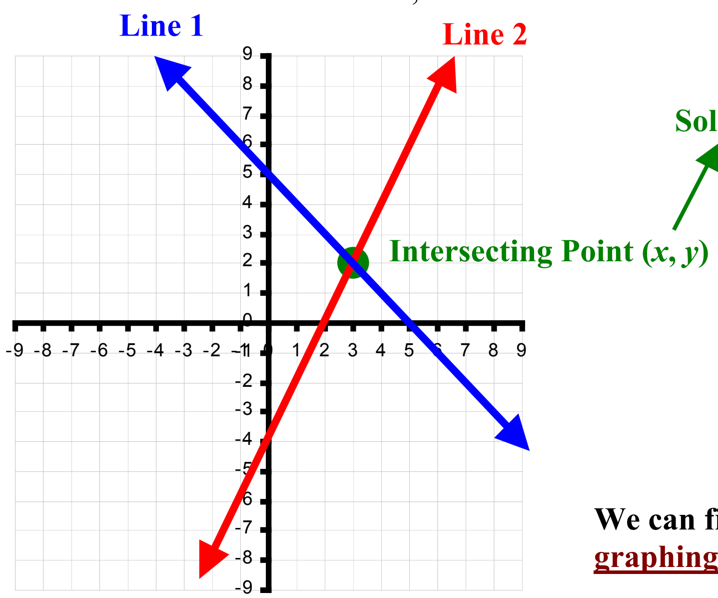
Chapter 1: Linear and Quadratic Functions

1-1: Points and Lines

System of Linear Equations: - two or more linear equations on the same coordinate grid.

Solution of a System of Linear Equations:

- the **intersecting point** of two or more linear equations
- on the Cartesian Coordinate Grid, the solution contains two parts: the ***x*-coordinate** and the ***y*-coordinate** (can be expressed as an **Ordered Pair**)



Solution of the System of Linear Equations

We can find the solution of a system of linear equation graphing manually or using a graphing calculator.

Example 1: Find the solution of the following system of equations by graphing manually using

$$x + 2y = 6$$
$$x - y = 3$$

a. the slope and y-intercept form

For Line 1:

$$x + 2y = 6$$
$$2y = -x + 6$$
$$y = \frac{-x+6}{2}$$
$$y = -\frac{1}{2}x + 3$$

y-int = (0, 3)

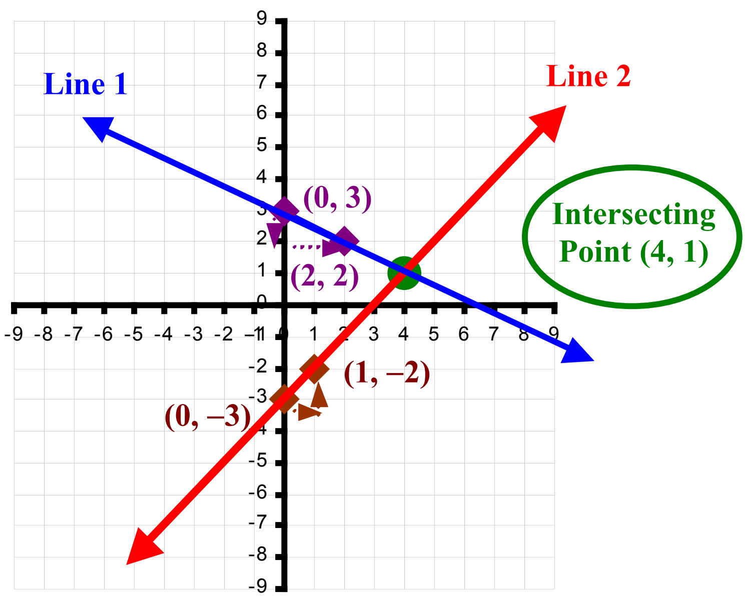
slope = $\frac{-1}{2} = \frac{\text{1 Down}}{\text{2 Right}}$

For Line 2:

$$x - y = 3$$
$$-y = -x + 3$$
$$y = x - 3$$

y-int = (0, −3)

slope = $\frac{1}{1} = \frac{\text{1 Up}}{\text{1 Right}}$



b. x and y-intercepts

Line 1:

For y-int, let $x = 0$

$x + 2y = 6$

$(0) + 2y = 6$

$y = 3$ $(0, 3)$

For x-int, let $y = 0$

$x + 2y = 6$

$x + 2(0) = 6$

$x = 6$ $(6, 0)$

Line 2:

For y-int, let $x = 0$

$x - y = 3$

$(0) - y = 3$

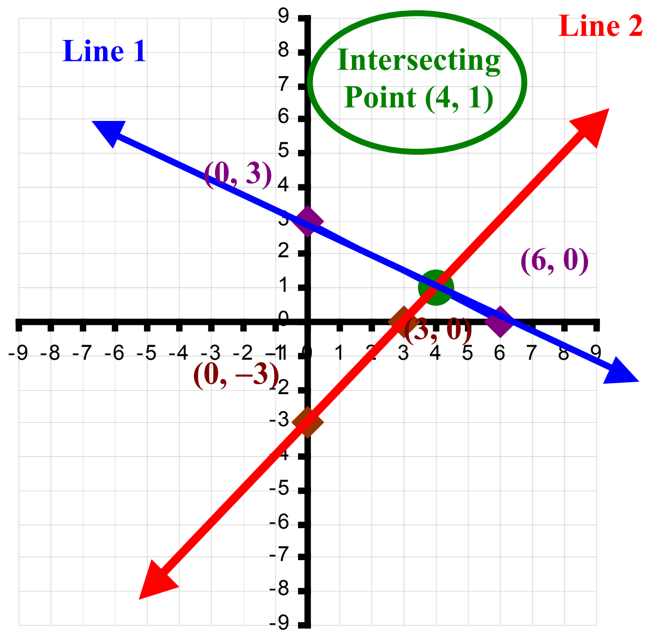
$y = -3$ $(0, -3)$

For x-int, let $y = 0$

$x - y = 3$

$x - (0) = 3$

$x = 3$ $(3, 0)$

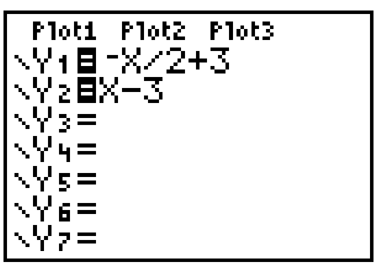


Example 2: Find the solution of the following system of equations by using the graphing calculator.

$$x + 2y = 6$$
$$x - y = 3$$

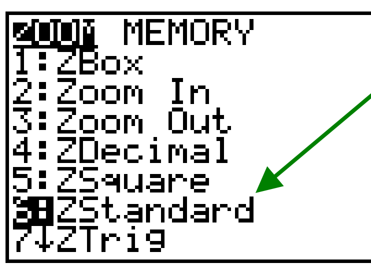
Step 1: Rearrange to Slope and y–intercept form

Y=



Step 2: Set WINDOWS to Standard Mode

ZOOM



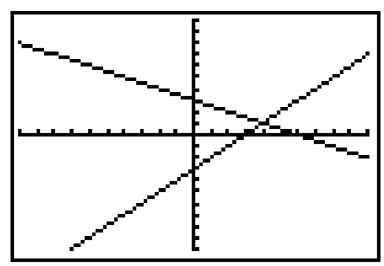
Choose Option 6 will set

x: [−10, 10, 1]

y: [−10, 10, 1]

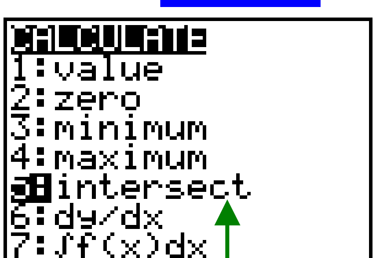
Step 3: Graph

GRAPH



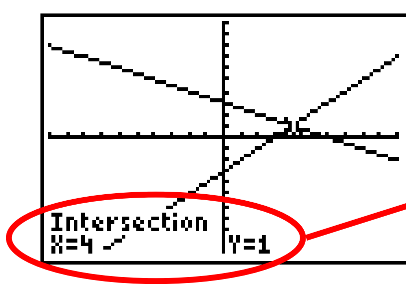
Step 4: Run INTERSECT

2nd **CALC** **TRACE**



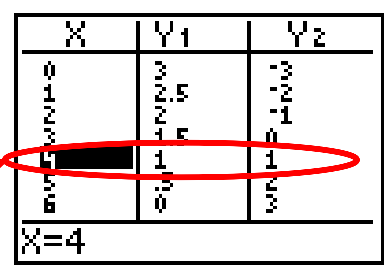
Choose Option 5

ENTER **ENTER** **ENTER**



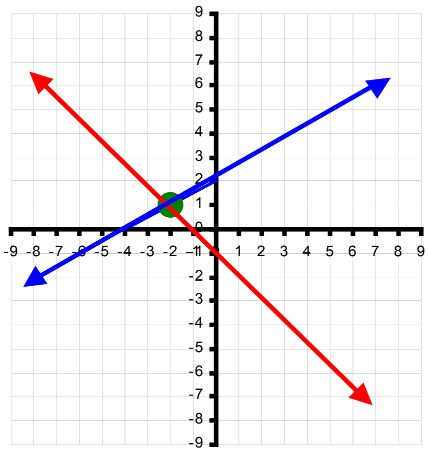
Step 5: Verify using TABLE

2nd **TABLE** **GRAPH**

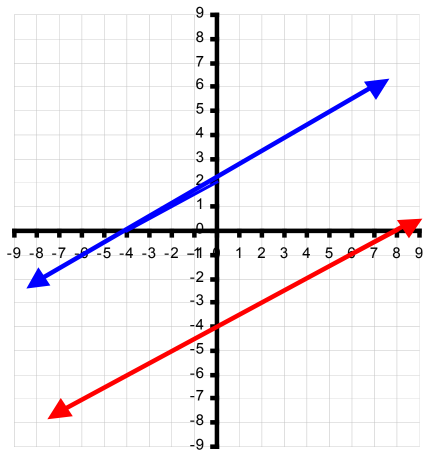


There are three types of solutions to a system of linear equations:

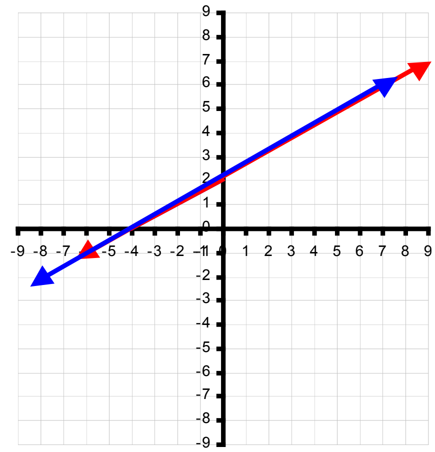
1. Intersecting Lines
One distinct Solution
Different Slopes
Different y-Intercepts



2. Parallel Lines
No Solution
Identical Slopes
Different y-Intercepts



3. Overlapping Lines
Many (Infinite) Solutions
Identical Slopes
Identical y-Intercepts



Example 3: Determine the number of solutions for the systems of equations below.

a. $x + 2y = 10$
$x + 2y = 6$

Line 1:

$$x+2y=10$$
$$2y=-x+10$$
$$y=\frac{-x+10}{2}$$
$$\boldsymbol{y=\frac{-1}{2}x+5}$$
$$\boldsymbol{m=\frac{-1}{2},\ y\text{-int}=5}$$

Line 2:

$$x+2y=6$$
$$2y=-x+6$$
$$y=\frac{-x+6}{2}$$
$$\boldsymbol{y=\frac{-1}{2}x+3}$$
$$\boldsymbol{m=\frac{-1}{2},\ y\text{-int}=5}$$

Identical slopes, but different y-intercepts mean parallel lines. Therefore, this system has NO SOLUTION.

b. $2x + 5y = 15$
$6x + 15y = 45$

Line 1:

$$2x+5y=15$$
$$5y=-2x+15$$
$$y=\frac{-2x+15}{5}$$
$$\boldsymbol{y=\frac{-2}{5}x+3}$$
$$\boldsymbol{m=\frac{-2}{5},\ y\text{-int}=3}$$

Line 2:

$$6x+15y=45$$
$$15y=-6x+45$$
$$y=\frac{-6x+45}{15}$$
$$\boldsymbol{y=\frac{-2}{5}x+3}$$
$$\boldsymbol{m=\frac{-2}{5},\ y\text{-int}=3}$$

Identical slopes and y-intercepts mean overlapping lines. Therefore, this system has MANY SOLUTIONS.

When using the **substitution method** to solve a system of linear equations:

1. **Isolate a variable from one equation. (Always pick the variable with 1 as a coefficient.)**
2. **Substitute the resulting expression into that variable of the other equation.**
3. **Solve for the other variable.**
4. **Substitute the result from the last step into one of the original equation and solve for the remaining variable.**

Example 4: Using the substitution method, algebraically solve the system of equations below.

$$2(x-2y)=5(5-y)$$
$$3(y-x)=-2(y+7)$$

Expand each equation accordingly.

Equation 1

$$2(x-2y)=5(5-y)$$
$$2x-4y=25-5y$$
$$\mathbf{2x+y=25}$$

Equation 2

$$3(y-x)=-2(y+7)$$
$$3y-3x=-2y-14$$
$$\mathbf{5y-3x=-14}$$

Solve for both variables using the substitution method.

Isolate y from the first equation.

$$2x+y=25$$
$$\mathbf{y=25-2x}$$

Substitute expression into y in the second equation.

$$5y-3x=-14$$
$$5(25-2x)-3x=-14$$
$$125-10x-3x=-14$$
$$-13x=-14-125$$
$$-13x=-139$$
$$\mathbf{x=\frac{139}{13}}$$

Solve for the remaining variable. Pick the easier equation of the two.

$$2x+y=25$$
$$2\left(\frac{139}{13}\right)+y=25$$
$$\frac{278}{13}+y=25$$
$$y=25-\frac{278}{13}$$
$$\mathbf{y=\frac{47}{13}}$$

Elimination by Multiplication: - most useful when neither equation has the same like terms.
- by multiplying different numbers (factors of their LCM) on each equation, we can change these equations into their equivalent form with the same like terms.

Example 5: Solve the system of linear equations $\begin{matrix}3x+7y=3\\4y-5x=42\end{matrix}$ by elimination.

First, rearrange the equations so x and y terms will line up.

$$\mathbf{3x+7y=3}$$
$$\mathbf{-5x+4y=42}$$

Eliminating x

LCM of $3x$ and $5x$ = $\mathbf{15x}$

(Multiply Equation 1 by **5** to obtain $15x$)
(Multiply Equation 2 by **3** to obtain $15x$)

$$5\times(3x+7y=3) \rightarrow \mathbf{15x+35y=15}$$
$$3\times(-5x+4y=42) \rightarrow \mathbf{-15x+12y=126}$$

Eliminate x by addition.

$$\cancel{15x}+35y=15$$
$$\boxed{+}\ \underline{(\cancel{-15x}+12y=126)}$$
$$47y=141$$
$$y=\frac{141}{47}$$
$$\mathbf{y=3}$$

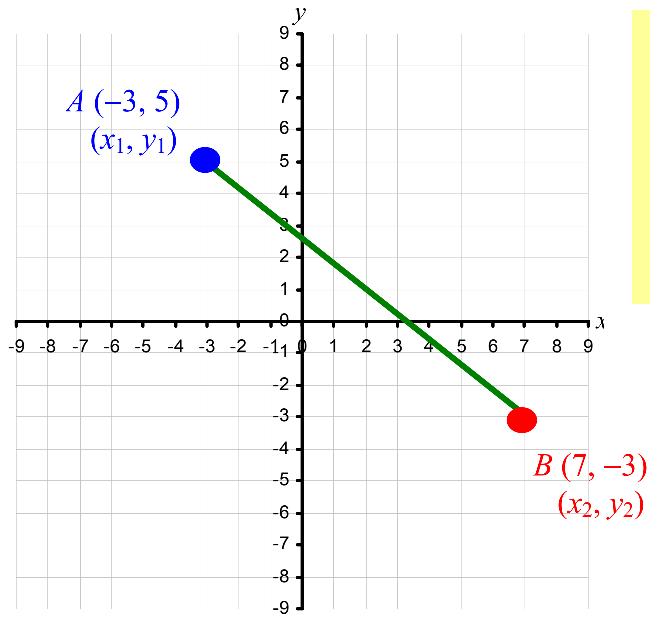
Next, substitute y into one of the equations to solve for x.

$$3x+7y=3$$
$$3x+7(3)=3$$
$$3x=3-21$$
$$3x=-18$$
$$\mathbf{x=-6}$$

Review of Coordinate Geometry

Distance $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **Midpoint** $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Example 6: Find the distance (in exact value) between A (−3, 5) and B (7, −3).



$$d_{\overline{AB}} = \sqrt{(7 - -3)^2 + (-3 - 5)^2}$$
$$= \sqrt{(10)^2 + (-8)^2}$$
$$= \sqrt{100 + 64}$$
$$= \sqrt{164}$$
$$d_{\overline{AB}} = 2\sqrt{41}$$

Example 7: A circle has a diameter with endpoints (−4, −1) and (2, 6). Find the exact length of the radius.

Let A (x_1, y_1) = (−4, −1) and B (x_2, y_2) = (2, 6)

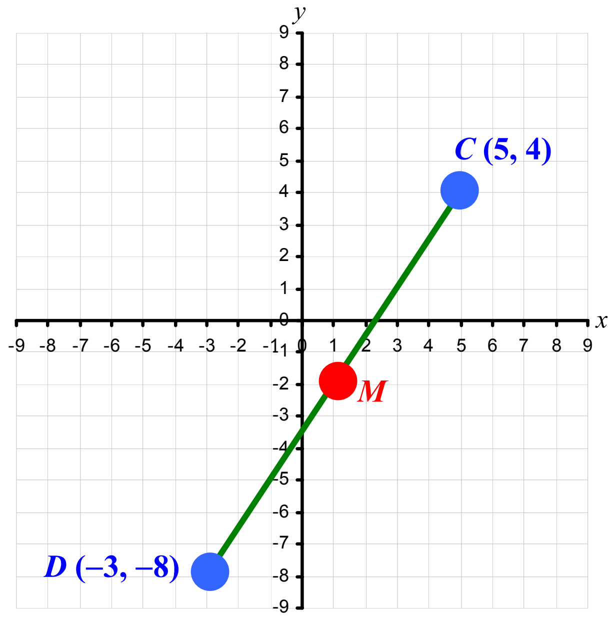
Length of Diameter $\overline{AB}$

$$d_{\overline{AB}} = \sqrt{(2 - -4)^2 + (6 - -1)^2}$$
$$= \sqrt{6^2 + 7^2} = \sqrt{36 + 49}$$
$$d_{\overline{AB}} = \sqrt{85}$$

Length of the Radius $= \dfrac{diameter\,\overline{AB}}{2}$

Length of the Radius $= \dfrac{\sqrt{85}}{2}$

Example 8: Find the midpoints of the following line segments $\overline{CD}$ where C (5, 4) and D (−3, −8).

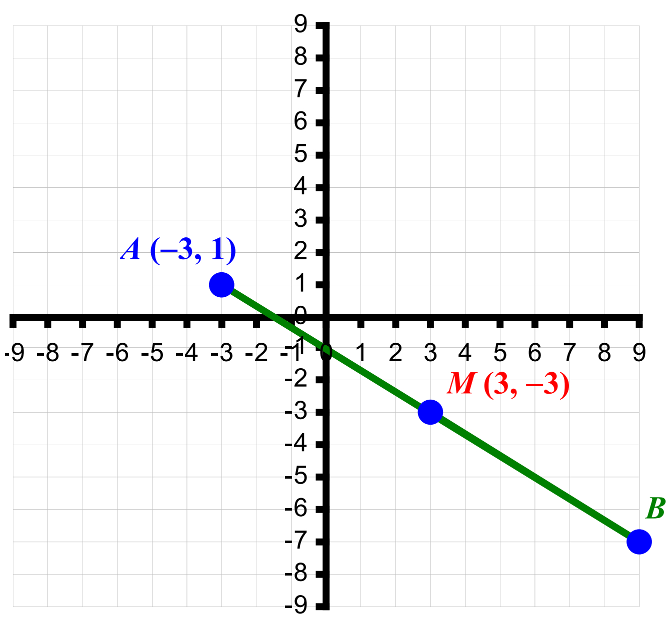


$$M_{\overline{CD}} = \left(\frac{5+-3}{2}, \frac{4+-8}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{-4}{2}\right)$$

$$M_{\overline{CD}} = (1, -2)$$

Example 9: Given that the midpoint of $\overline{AB}$ is M (3, −3). If one of the endpoint of $\overline{AB}$ is A (−3, 1), find the coordinate of endpoint B.



$$M_{\overline{AB}} = \left(\frac{x_2 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$(3, -3) = \left(\frac{-3 + x_2}{2}, \frac{1 + y_2}{2}\right)$$

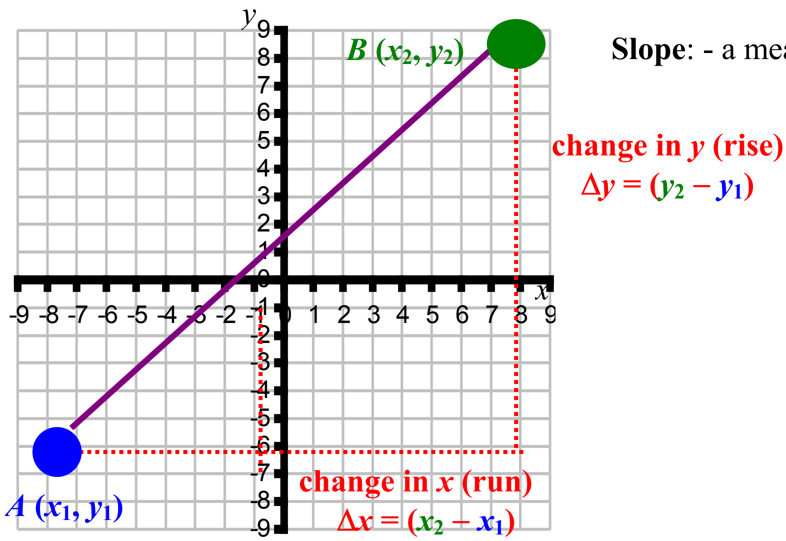
$$3 = \frac{-3 + x_2}{2}$$
$$3 \times 2 = -3 + x_2$$
$$6 = -3 + x_2$$
$$6 + 3 = x_2$$
$$x_2 = 9$$

$$-3 = \frac{1 + y_2}{2}$$
$$-3 \times 2 = 1 + y_2$$
$$-6 = 1 + y_2$$
$$-6 - 1 = y_2$$
$$y_2 = -7$$

***B* = (9, −7)**

1-1 Assignment: pg. 5–7 #5 to 33 (every other odd) (Omit #9c–e, 21)

1-2 and 1-3: Slopes of Lines and Finding Equations of Lines



Slope: - a measure on the steepness of the line segment.

Slope of a Line Segment

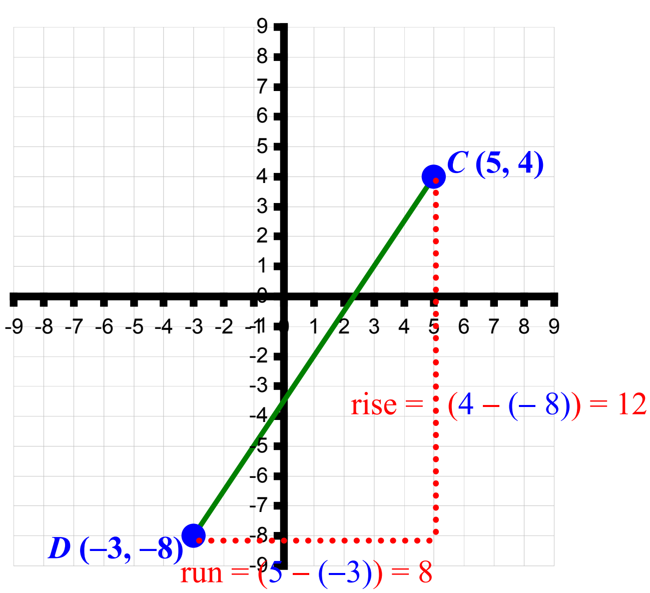
$$m = \frac{rise}{run} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$$

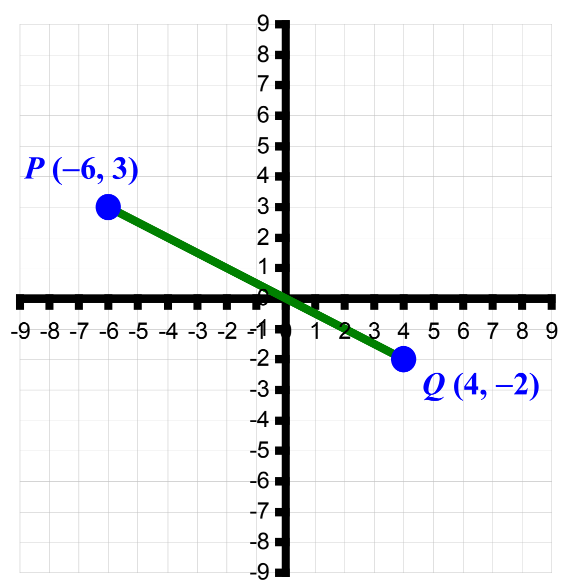
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1: Find the slope of the following line segments.

a. $\overline{CD}$ where C (5, 4) and D (–3, –8)

b. $\overline{PQ}$ where P (–6, 3) and Q (4, –2)





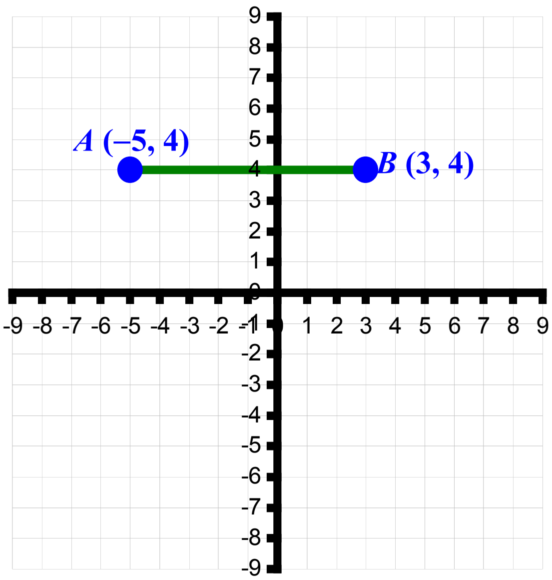
$$m_{\overline{CD}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-8)}{5-(-3)} = \frac{12}{8}$$

$$m_{\overline{CD}} = \frac{3}{2}$$

$$m_{\overline{PQ}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-3}{4-(-6)} = \frac{-5}{10}$$

$$m_{\overline{PQ}} = \frac{-1}{2}$$

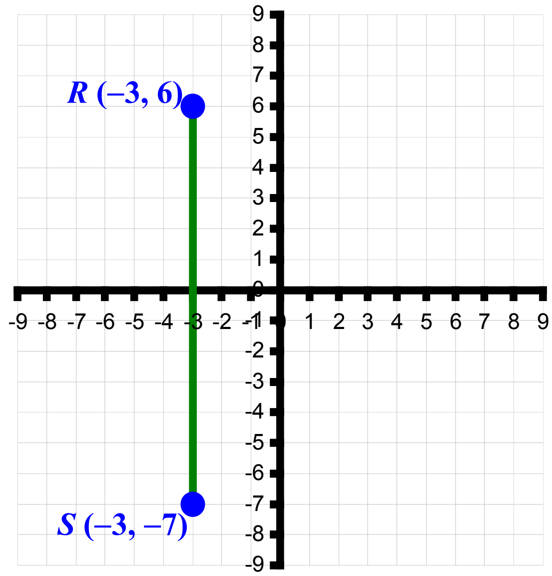
c. $\overline{AB}$ where A (−5, 4) and B (3, 4)



$$m_{\overline{AB}} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{4-4}{3-(-5)}$$
$$= \frac{0}{8}$$

$$m_{\overline{AB}} = \mathbf{0}$$

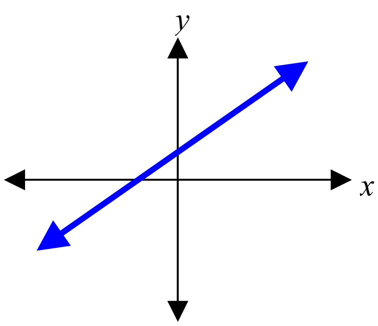
d. $\overline{RS}$ where R (−3, 6) and S (−3, −7)



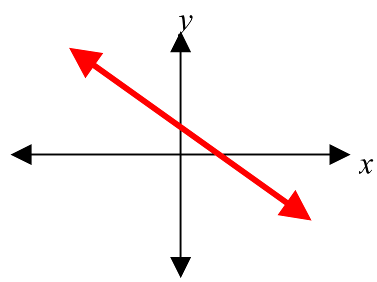
$$m_{\overline{RS}} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-7-6}{-3-(-3)}$$
$$= \frac{-13}{0}$$

$m_{\overline{RS}}$ = **undefined**

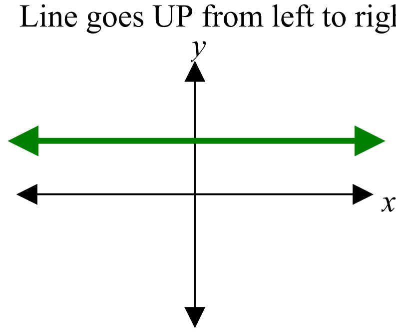
In general, slopes can be classified as follows:



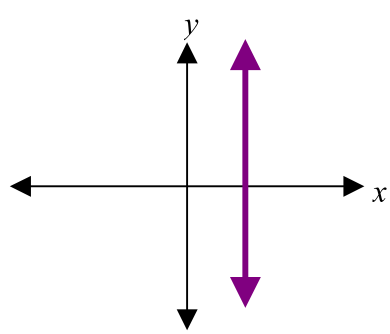
Positive Slope (m > 0)
Line goes UP from left to right.



Negative Slope (m < 0)
Line goes DOWN from left to right.



Zero Slope (m = 0)
Horizontal (Flat) Line [Rise = 0]



Undefined Slope
Vertical Line [Run = 0]

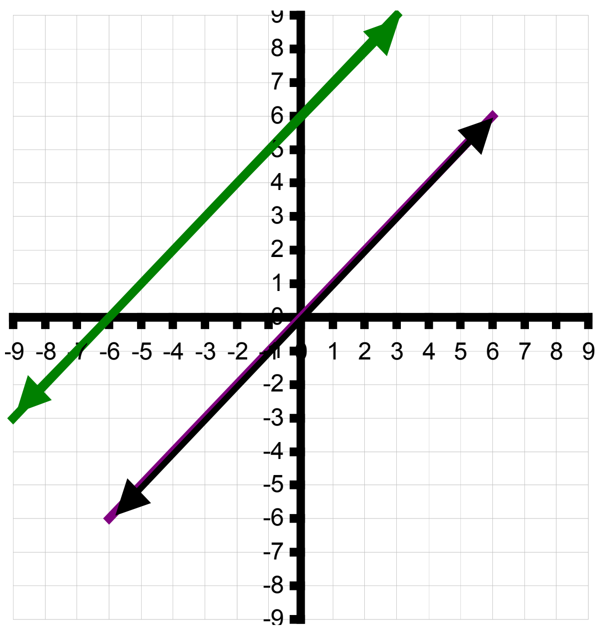
Example 2: If the slope of a line is $\frac{3}{4}$, and it passes through $A\ (2t, t - 2)$ and $B\ (2, -3)$, find the value of t and point A.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$\frac{3}{4} = \frac{-3-(t-2)}{2-2t}$$
$$\frac{3}{4} = \frac{-t-1}{2-2t}$$

$$4(-t-1) = 3(2-2t)$$
$$-4t - 4 = 6 - 6t$$
$$-4t + 6t = 6 + 4$$
$$2t = 10$$
$$t = \frac{10}{2}$$

$t = 5$

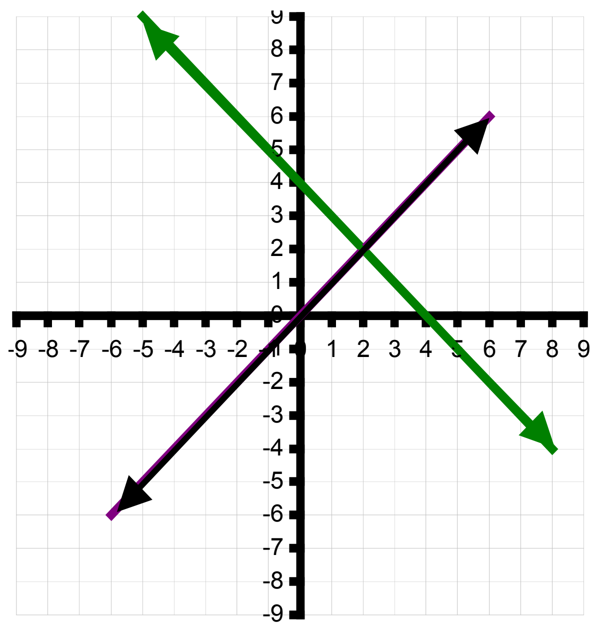
$A\ (2(5), (5) - 2) = A\ (10, 3)$



Parallel Lines

slope of line 1 = slope of line 2

$$m_{l_1} = m_{l_2}$$



Perpendicular Lines

slope of line 1 = negative reciprocal slope of line 2

$$m_{l_1} = \frac{-1}{m_{l_2}}$$

Example 3: Given the slope of two lines below, determine whether the lines are parallel or perpendicular.

a. $m_1 = \frac{-3}{4}$ and $m_2 = \frac{8}{6}$

b. $m_1 = \frac{4}{6}$ and $m_2 = \frac{6}{9}$

c. $m_1 = \frac{1}{2}$ and $m_2 = -\frac{1}{2}$

a. $m_1 = \frac{-3}{4}$ and $m_2 = \frac{4}{3}$

(negative reciprocal slopes)

Perpendicular Lines

b. $m_1 = \frac{2}{3}$ and $m_2 = \frac{2}{3}$

(same slopes)

Parallel Lines

c. $m_1 = \frac{1}{2}$ and $m_2 = \frac{-1}{2}$

(neither the same nor negative reciprocal)

Neither Parallel nor Perpendicular Lines

Example 4: Find the slope of the line given and its perpendicular slope.

a. $y = -3x + 2$

$m = -3$

($\perp$ means perpendicular)

$m_\perp = \frac{1}{3}$ (negative reciprocal)

b. $2x - 5y - 4 = 0$

$-5y = -2x + 4$

$y = \frac{2}{5}x - \frac{4}{5}$ $\quad m = \frac{2}{5}$

$m_\perp = \frac{-5}{2}$ (negative reciprocal)

c. $x - 6 = 0$

$x = 6$ (vertical line)

$m = \text{undefined} = \frac{1}{0}$

$m_\perp = \frac{-0}{1}$ (negative reciprocal)

$m_\perp = 0$

When given a slope (m) and a point (x_1, y_1) on the line, we can find the equation of the line using the **point-slope form**:

$\frac{y - y_1}{x - x_1} = m$ (slope formula) $\qquad$ $y - y_1 = m\,(x - x_1)$ **(Point-Slope form)**

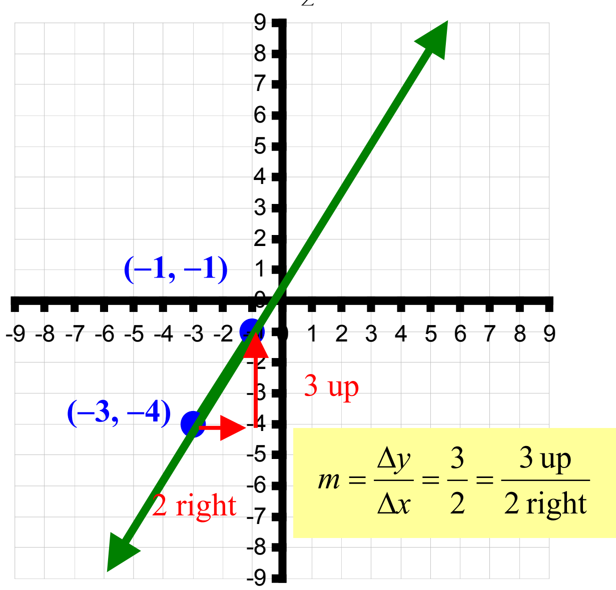
If we rearrange the equations so that all terms are on one side, it will be in **general form**:

$Ax + By + C = 0$ **(General form)**

($A \geq 0$, the leading coefficient for the x term must be positive)

Example 5: Find the equation in point-slope form and general form given the followings.

a. $(-3, -4)$ and $m = \frac{3}{2}$



For Slope-Point form:

$y - y_1 = m(x - x_1)$

$y - (-4) = \frac{3}{2}(x - (-3))$

$y + 4 = \frac{3}{2}(x + 3)$

For General form:

$y + 4 = \frac{3}{2}(x + 3)$

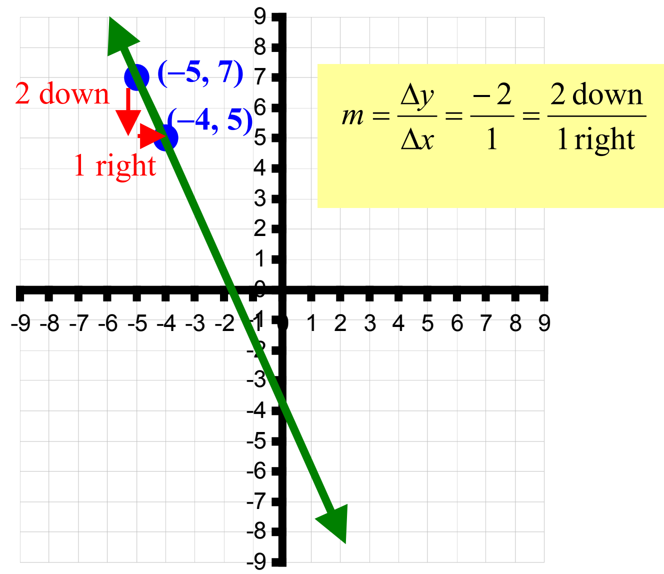
$2(y + 4) = 3(x + 3)$

$2y + 8 = 3x + 9$

$0 = 3x - 2y + 9 - 8$

$\mathbf{0 = 3x - 2y + 1}$

b. $(-5, 7)$ and $m = -2$



For Slope-Point form:

$y - y_1 = m(x - x_1)$

$y - 7 = -2(x - (-5))$

$y - 7 = -2(x + 5)$

For General form:

$y - 7 = -2(x + 5)$

$y - 7 = -2x - 10$

$2x + y - 7 + 10 = 0$

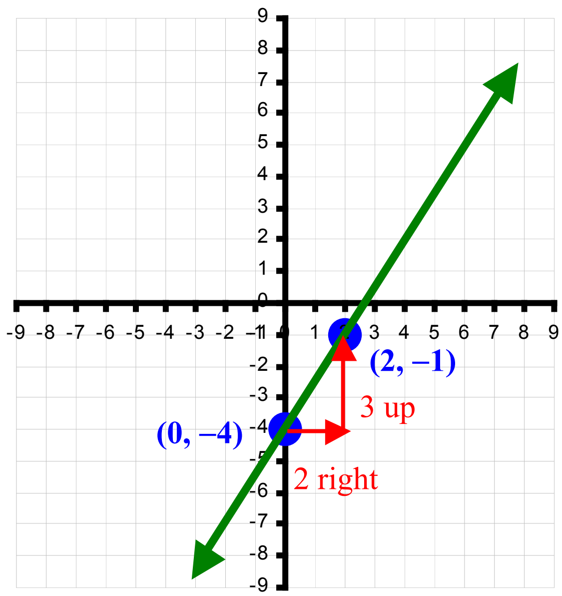
$\mathbf{2x + y + 3 = 0}$

When given a slope (m) and the y-intercept $(0, b)$ of the line, we can find the equation of the line using the **slope and y-intercept form**:

$$y = mx + b \qquad \text{where } m = \text{slope and } b = y\text{-intercept}$$

Example 6: Given the y-intercept and slope, write the equation of the line in slope and y-intercept form, and standard form. Sketch a graph of the resulting equation.

a. $(0, -4)$ and $m = \dfrac{3}{2}$



$$m = \frac{\Delta y}{\Delta x} = \frac{3}{2} = \frac{3\text{ up}}{2\text{ right}}$$

y-intercept $= b = -4$

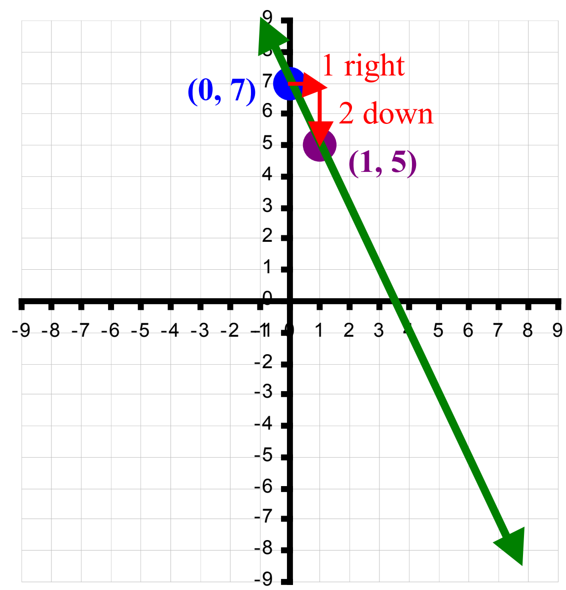
$$y = \frac{3}{2}x - 4$$

Multiply both sides by 2.

$$2y = 3x - 8$$

$$\mathbf{0 = 3x - 2y - 8}$$

b. $b = 7$ and $m = -2$



$$m = \frac{\Delta y}{\Delta x} = \frac{-2}{1} = \frac{2\text{ down}}{1\text{ right}}$$

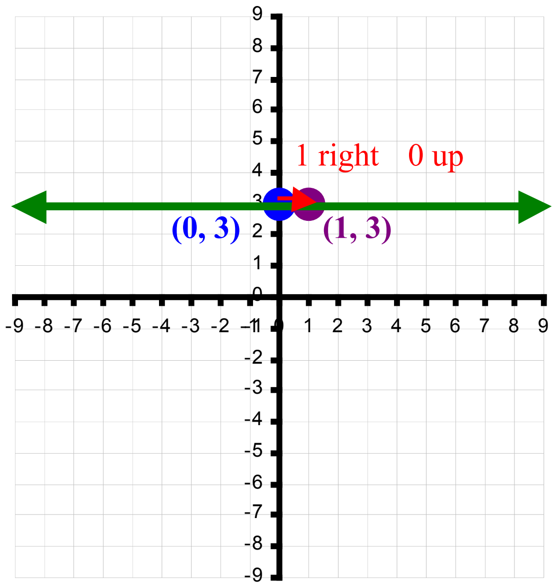
y-intercept $= b = 7$

$$\mathbf{y = -2x + 7}$$

Rearrange to the left side.

$$\mathbf{2x + y - 7 = 0}$$

c. $b = 3$ and $m = 0$



$$m = \frac{\Delta y}{\Delta x} = \frac{0}{1} = \frac{0 \text{ up}}{1 \text{ right}}$$

y-intercept = b = 3

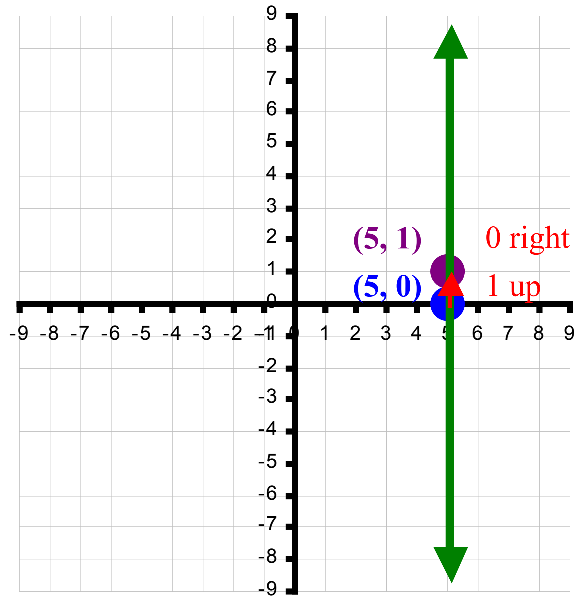
$y = 0x + 3$

$\mathbf{y = 3}$

Rearrange to the left side.

$\mathbf{y - 3 = 0}$

d. x-intercept = 5 and m = undefined



x-intercept = 5 and undefined slope (Vertical Line).

$\mathbf{x = 5}$

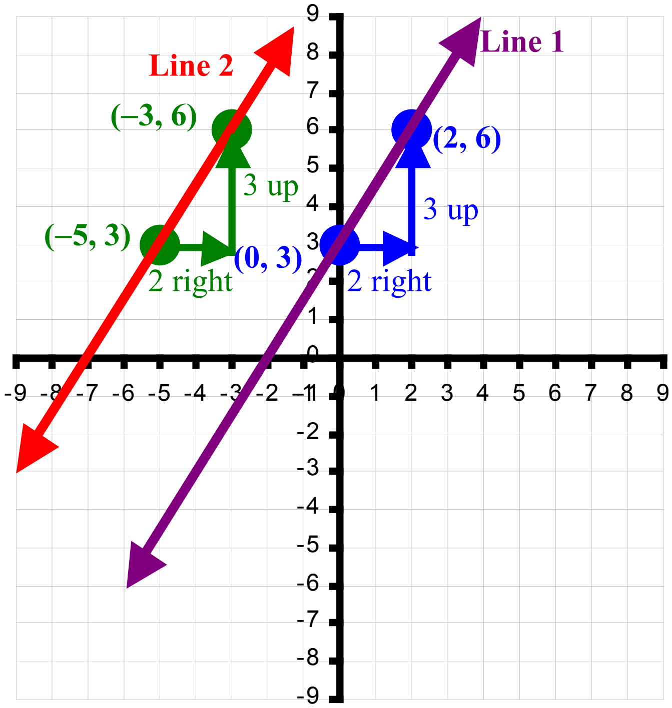
Rearrange to the left side.

$\mathbf{x - 5 = 0}$

Vertical Line – NO x term.

Horizontal Line – NO y term.

Example 7: Find the equation of a line parallel to $3x - 2y + 6 = 0$ and passes through $(-5, 3)$.



Line 1:

$-2y = -3x - 6$

$y = \dfrac{-3x-6}{-2}$

$y = \dfrac{3}{2}x + 3$

$m_1 = \dfrac{3}{2}$

Line 2:

$m_2 = \dfrac{3}{2}$ (parallel lines – same slope as m_1)

Using $(-5, 3)$ as (x, y) and the form $y = mx + b$, we have:

$(3) = \dfrac{3}{2}(-5) + b$

$3 = \dfrac{-15}{2} + b$

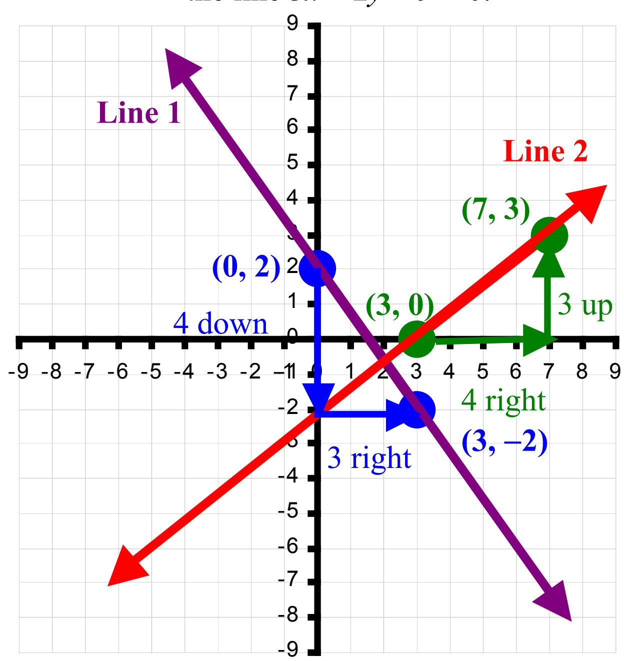
$3 + \dfrac{15}{2} = b$

$\dfrac{6}{2} + \dfrac{15}{2} = b$

$b = \dfrac{21}{2}$

$y = \dfrac{3}{2}x + \dfrac{21}{2}$

Example 8: Find the equation of a line perpendicular to $4x + 3y - 6 = 0$ and having the same x- intercept as the line $3x - 2y - 9 = 0$.



Line 1:

$3y = -4x + 6$ $\quad y = \dfrac{-4x+6}{3}$ $\quad y = \dfrac{-4}{3}x + 2$ $\quad m_1 = \dfrac{-4}{3}$

Line 2:

$m_2 = \dfrac{3}{4}$ (perpendicular lines – negative reciprocal of m_1)

To find x-intercept of $3x - 2y - 9 = 0$, we let $y = 0$.

$3x - 2(0) - 9 = 0$ $\quad 3x = 9$ $\quad x\text{-int} = 3$ means **(3, 0)**

Using $(3, 0)$ as (x, y) and the form $y = mx + b$, we have:

$(0) = \dfrac{3}{4}(3) + b$

$0 = \dfrac{9}{4} + b$

$b = \dfrac{-9}{4}$

$y = \dfrac{3}{4}x - \dfrac{9}{4}$

1-2 Assignment: pg. 11–12 #5 to 25 (every other odd)
1-3 Assignment: pg. 16–18 #1 to 27 (odd) (Omit #5, 13, 17, 21, 23, 25)

1-4: Linear Functions and Models

Linear Function: - a set ordered pairs that exhibit a straight line when plotted on a graph.

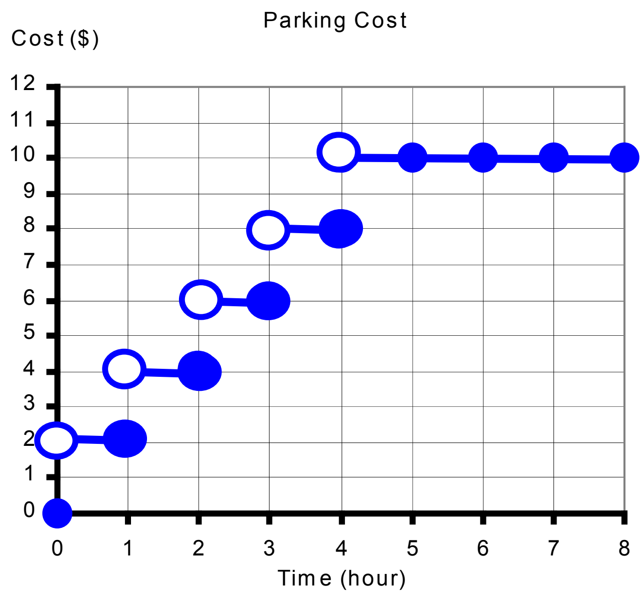
Mathematical Model: - one or more functions, graphs, tables, equations, or inequalities that describe a real-world situation.

There are two types of data

a. **Discrete Data**: - a graph with a series of separated ordered pairs or broken lines.

Example: Cost of Parking is $2.00 every hour with a Daily Maximum of $10.00.

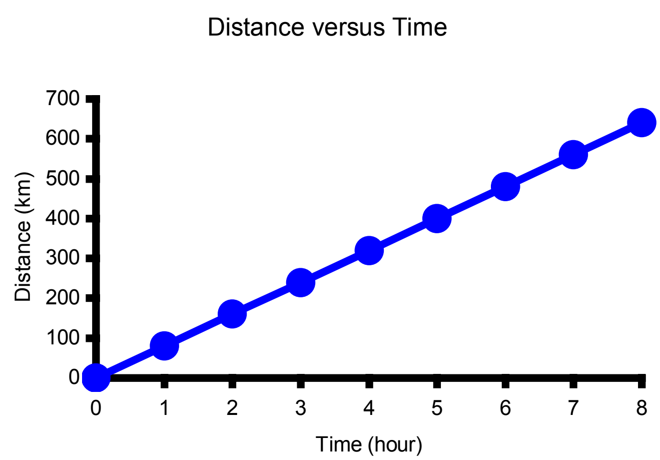
Time (hr)	Cost ($)
0	0
1	2
2	4
3	6
4	8
5	10
6	10
7	10
8	10



b. **Continuous Data**: - a graph with an unbroken line that connects a series of ordered pairs.

Example: Distance versus Time of a car with a constant speed of 80 km/h.

Time (hr)	Distance (km)
0	0
1	80
2	160
3	240
4	320
5	400
6	480
7	560
8	640



Direct Variation: - a variable that ***varies directly*** **(by a rate of change)** with another variable.

$y \propto x$ (**y is directly proportional to x**) or $y = kx$ **where k = rate of change**

Example 1: Gasoline at one time costs \$0.70 per Litre.

a. Write out the cost of gasoline as a function of volume bought.
b. What is the rate of change?
c. Set up a table of values from $V = 20$ L to $V = 50$ L with a scale of 5 L.
d. Graph the function.

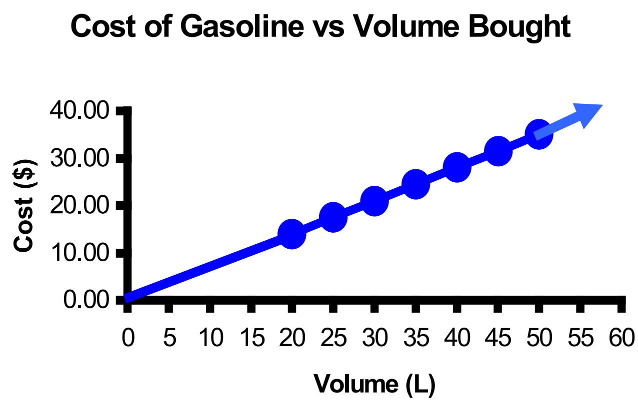
a. $C(V) = 0.7\ V$

b. Rate of Change = \$0.70/L (unit price of gasoline)

c.

V in L	$C(V)$ in \$
20	0.7 (20) = 14.00
25	0.7 (25) = 17.50
30	0.7 (30) = 21.00
35	0.7 (35) = 24.50
40	0.7 (40) = 28.00
45	0.7 (45) = 31.50
50	0.7 (50) = 35.00

d.



Example 2: The amount of fuel used by a vehicle varies directly with the distance travelled. On a particular trip, 42.35 L of gasoline is used for a distance of 516.5 km.

a. Calculate the rate of change.
b. Find the function of volume of gasoline used in terms of distance travelled.
c. What is the distance travelled if 35 L of gasoline is used?
d. How much would it cost to fuel up the car when the price of gasoline was \$65.90 / 100 L if the entire trip was 631 km?

a. $V(d) = kd$

$42.35 = k\,(516.5)$

$\frac{42.35L}{516.5km} = k$

$k = 0.082$ L/km

b. $V(d) = kd$

$V(d) = 0.082d$

c. $V = 35$ L, $d = ?$

$35 = 0.082\ d$

$\frac{35}{0.082} = d$

$d = 426.8$ km

d. $d = 631$ km, $V = ?$

$V = 0.082\ (631)$

$V = 51.742$ L

$\frac{\$65.90}{100L} = \frac{x}{51.742L}$

$x = \$34.10$

Partial Variation: - a variable that ***varies partially*** **(by a constant rate of change and a fixed amount)** with another variable.

$$y = kx + b$$

where k = rate of change and b = fixed amount (initial amount when x = 0)

Example 3: The cost of a school dance organized by the student council \$5.50 per person and \$300 to hire the DJ.

a. What is the rate of change and the fixed amount?
b. Express the cost as a function of the number of people attending.
c. Set up a table of values from n = 0 to n = 100 people with a scale of 20 people.
d. Graph the function.
e. Explain whether the graph should be discrete or continuous.
f. How many people attended if the cost was \$1125?

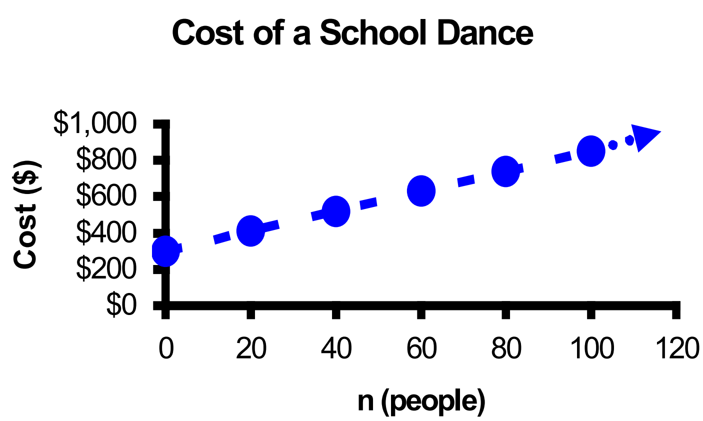
a. Rate of Change = \$5.50 / person (cost per person)
Fixed Amount = \$300 (fixed cost)

b. $C(n) = 5.50n + 300$

c.

n	$C(n)$ in \$
0	5.50 (0) +300 = 300
20	5.50 (20) +300 = 410
40	5.50 (40) +300 = 520
60	5.50 (60) +300 = 630
80	5.50 (80) +300 = 740
100	5.50 (100) +300 = 850

d.



e. The graph should be discrete because you can not have a decimal number to describe the number of people attending.

f. $C(n) = \$1125$, n =?

$$C(n) = 5.50n + 300$$
$$1125 = 5.50n + 300$$
$$1125 - 300 = 5.50n$$
$$825 = 5.50n$$
$$n = \frac{825}{5.50}$$

n = 150 people

Example 4: A taxi ride costs you \$18.00 for 5 km travelled, and \$34.80 if you travelled 12 km.

a. Express the equation as a function of Cost in terms of distance travelled.
b. What is the rate of change and the fixed amount?
c. How much would it cost for a 20 km ride to the airport?
d. If the ride costs \$32.50, what is the distance travelled?

a. There are two order pairs (5 km, \$18.00) and (12 km, \$34.80)

$$m = k = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\$34.80 - \$18.00}{12\text{ km} - 5\text{ km}} \qquad \boldsymbol{k = \$2.40/\text{km}}$$

$\boldsymbol{C = kd + b}$ (Substitute an order pair into d and C)

$\$18.00 = (\$2.40/\text{km})(5\text{ km}) + b$

$\$18.00 = \$12.00 + b$

$\boldsymbol{b = \$12.00}$

Therefore, $\boldsymbol{C(d) = 2.4d + 6}$

b. Rate of Change = \$2.40/km (cost per km travelled)
Fixed Amount = \$6.00 (Flat Rate)

c. d = 20 km, C = ?

$\boldsymbol{C(d) = 2.4d + 6}$

$C(d) = 2.4(20) + 6$

$\boldsymbol{C(d) = \$54.00}$

d. C = \$32.50, d = ?

$\boldsymbol{C(d) = 2.4d + 6}$

$32.50 = 2.4d + 6$

$32.50 - 6 = 2.4d$

$26.5 = 2.4d$

$\dfrac{26.5}{2.4} = d$

$\boldsymbol{d} = 11.04\text{ km}$

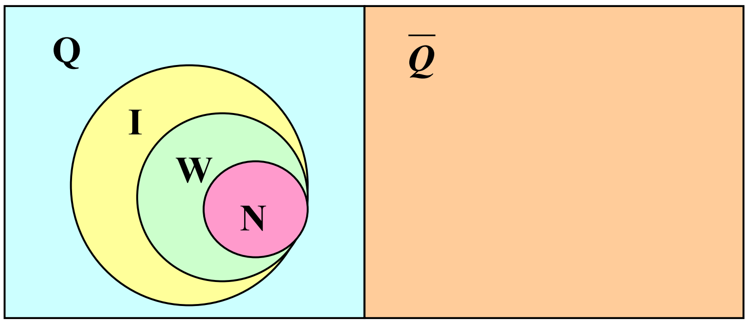
1-4 Assignment: pg. 22–23 #1 to 15 (odd), 20, 23 and 24 (Omit #5)

1-5: The Complex Numbers

Complex Numbers: - ***non-real numbers*** that cannot be represented on a number line or a Cartesian plane.
- also called ***imaginary number***.

$$\boxed{\textbf{If } i^2 = -1\textbf{, then } i = \sqrt{-1} \textbf{ (imaginary unit)}}$$

Real Numbers (R)



Complex Numbers $\left(\overline{R}\right)$

Example:

$\sqrt{-40}$

or

$2\sqrt{-10} = 2\sqrt{-1} \times \sqrt{10}$

or

$2i\sqrt{10}$

Rational Numbers (Q): - numbers that can be turned into a fraction $\frac{a}{b}$, where $a, b \in I$, and $b \neq 0$.
- include all Terminating or Repeating Decimals.
- include all Natural Numbers, Whole Numbers and Integers.
- include any perfect roots (radicals).

Irrational Numbers ($\overline{Q}$): - numbers that **CANNOT** be turned into a fraction $\frac{a}{b}$, where $a, b \in I$, and $b \neq 0$.
- include all non-terminating, non-repeating decimals.
- include any non-perfect roots (radicals).

Real Numbers (R): - any numbers that can be put on a number line.
- include all natural numbers, whole numbers, integers, rational and irrational numbers.

Example 1: Simplify the following.

a. $\sqrt{-25}$

$$\sqrt{-25} = \sqrt{25} \times \sqrt{-1}$$
$$= \mathbf{5i}$$

b. $\sqrt{-441}$

$$\sqrt{-441} = \sqrt{441} \times \sqrt{-1}$$
$$= \mathbf{21i}$$

c. $\sqrt{-50x^7y^4}$

$$\sqrt{-50x^7y^4} = \sqrt{25x^6y^4} \times \sqrt{-1} \times \sqrt{2x}$$
$$= \mathbf{5x^3y^2i\sqrt{2x}}$$

d. $\sqrt{(-18)^2}$

$$\sqrt{(-18)^2} = \sqrt{324}$$
$$= \mathbf{18}$$

e. $\left(\sqrt{-18}\right)^2$

$$\left(\sqrt{-18}\right)^2 = \sqrt{-18} \times \sqrt{-18}$$
$$= \sqrt{18} \times \sqrt{-1} \times \sqrt{18} \times \sqrt{-1}$$
$$= 18i^2$$
$$= \mathbf{-18}$$

Example 2: Simplify the following.

a. $\left(\sqrt{-6}\right)\left(\sqrt{3}\right)$

$= i\sqrt{6}\times\sqrt{3} = i\sqrt{18} = i\ \sqrt{9}\times\sqrt{2}$

$= \mathbf{3i\ \sqrt{2}}$

b. $\left(\sqrt{-8}\right)\left(\sqrt{-2}\right)$

$= i\sqrt{8}\times i\sqrt{2} = i^2\ \sqrt{16} = -1\times 4$

$= \mathbf{-4}$

c. $-(7i)^2$

$= -49i^2 = -49(-1)$

$= \mathbf{49}$

d. $(-7i)^2$

$= 49i^2 = 49(-1)$

$= \mathbf{-49}$

e. $\dfrac{8}{\sqrt{-4}}$

$= \dfrac{8}{2i} = \dfrac{4}{i} = \dfrac{4}{i}\times\dfrac{i}{i}$

$= \dfrac{4i}{i^2} = \dfrac{4i}{-1} = \mathbf{-4i}$

f. i^{-44}

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = i^2\times i = -1i = -i$

$i^4 = i^2\times i^2 = -1\times -1 = 1$

$i^5 = i^4\times i = i$

$i^6 = i^4\times i^2 = -i$

(Pattern continues)

$i^{-44} = \dfrac{1}{i^{44}} = \dfrac{1}{1}$ $\mathbf{1}$

Example 3: Simplify the following.

a. $(4 - 5i) + (-6 - 2i)$

$= 4 - 5i + -6 - 2i$

$= \mathbf{-2 - 7i}$

b. $(7 - 8i) - (9 - 3i)$

$= 7 - 8i - 9 + 3i$

$= \mathbf{-2 - 5i}$

Subtacting a Bracket, change signs in the barcket

Solving Equations with Complex Numbers:

1. **Rewrite the equation into two separate equations**. **One with Real Numbers**, **the other with imaginary numbers**.
2. Solve the two equations separately and combine the results.
3. Verify the answers by substituting each part into the original equation.

Example 4: Solve for x and y for $4x - 2 - (y + 4)i = 8x - 3yi$.

$4x - 2 - (y + 4)i = 8x - 3yi$

(Expand) $4x - 2 - yi - 4i = 8x - 3yi$

(Separate Real from Imaginary)

$4x - 2 = 8x$ $\quad$ $-yi - 4i = -3yi$

$4x - 8x = 2$ $\quad$ $-yi + 3yi = 4i$

$-4x = 2$ $\quad$ $2yi = 4i$

$\mathbf{x = -\dfrac{1}{2}}$ $\quad$ $\mathbf{y = 2}$

Verify: $4\left(-\dfrac{1}{2}\right) - 2 - ((2) + 4)i = 8\left(-\dfrac{1}{2}\right) - 3(2)i$

$-2 - 2 - 6i = -4 - 6i$

$-4 - 6i = -4 - 6i$

Multiplying Complex Numbers: - multiply numerical coefficients and recall that $i^2 = -1$.

Example 5: Simplify the following.

a. $(7i)(4i)$

$= 28i^2 = 28 \times -1$

$= -28$

b. $(6i)^2$

$= 36i^2 = 36 \times -1$

$= -36$

c. $(-5 + 2i)(3 - 8i)$

$= -15 + 40i + 6i - 16i^2$ (Expand)

$= -15 + 46i - 16(-1)$

$= 1 + 46i$

d. $(2 - 9i)^2$

$= (2 - 9i)(2 - 9i)$

$= 4 - 18i - 18i + 81i^2$

$= 4 - 36i + 81(-1)$

$= -77 - 36i$

Conjugate of Complex Number

$(a + bi)$ is a conjugate of $(a - bi)$

Product of Conjugate Complex Numbers

$(a + bi)(a - bi) = a^2 - bi^2 = a^2 - b^2(-1)$

$(a + bi)(a - bi) = a^2 + b^2$

Example 6: Simplify the following.

a. $(4 - 3i)(4 + 3i)$

$= 16 + 12i - 12i - 9i^2$

$= 16 - 9(-1) = 25$

b. $(7 + 6i)(7 - 6i)$

$= 49 + 42i - 42i - 36i^2$

$= 49 - 36(-1) = 85$

c. $(3 - 8i)^2(3 + 8i)^2$

$= [(3 - 8i)(3 + 8i)]^2$

$= [9 + 24i - 24i - 64i^2]^2$

$= [9 - 64(-1)]^2 = [73]^2$

$= 5329$

Rationalizing Complex Numbers: - similar to rationalizing radical binomial expression, we multiply by a fraction consists of the conjugate of the denominator over itself.

Example 7: Simplify the following.

a. $\dfrac{6}{3-5i}$

$$= \frac{6}{(3-5i)} \times \frac{(3+5i)}{(3+5i)}$$

$$= \frac{6(3+5i)}{9+15i-15i-25i^2}$$

$$= \frac{6(3+5i)}{9-25(-1)}$$

$$= \frac{6(3+5i)}{34}$$

$$= \frac{3(3+5i)}{17}$$

$$= \frac{9+15i}{17}$$

b. $\dfrac{5-7i}{4+2i}$

$$= \frac{(5-7i)}{(4+2i)} \times \frac{(4-2i)}{(4-2i)}$$

$$= \frac{20-10i-28i+14i^2}{16-8i+8i-4i^2}$$

$$= \frac{20-38i+14(-1)}{16-4(-1)}$$

$$= \frac{6-38i}{20}$$

$$= \frac{2(3-19i)}{20}$$

$$= \frac{3-19i}{10}$$

c. $\dfrac{-2+3i}{-2-3i}$

$$= \frac{(-2+3i)}{(-2-3i)} \times \frac{(-2+3i)}{(-2+3i)}$$

$$= \frac{4-6i-6i+9i^2}{4-6i+6i-9i^2}$$

$$= \frac{4-12i+9(-1)}{4-9(-1)}$$

$$= \frac{-5-12i}{13}$$

1-5 Assignment: pg. 28–29 #1 to 39 (Omit #7, 13, 27)

1-6: Solving Quadratic Equations

Roots: - solution to an algebraic equation; commonly called **zeros** or ***x*-intercepts**.

There are several ways of solving quadratic equations

Solve Quadratic Equations by Factoring:

1. **Expand, Simplify, Cross-Multiply, and /or Rearrange the quadratic equation into the general form $ax^2 + bx + c = 0$ (with the preference for $a > 0$).**
2. **FACTOR the resulting general quadratic equation.**
3. **EQUATE each factor to 0 and solve for x.**

$$A \times B = 0$$
means
$$A = 0 \quad \text{and / or} \quad B = 0$$

Example 1: Solve the following quadratic equations.

a. $y - \dfrac{1}{y+4} = -4$

$$y + 4 = \frac{1}{y+4} \qquad \text{(Rearrange Terms)}$$
$$(y + 4)(y + 4) = 1$$
$$y^2 + 8y + 16 = 1$$
$$y^2 + 8y + 15 = 0$$
$$(y + 3)(y + 5) = 0$$
$$y = -3 \qquad y = -5$$

b. $\dfrac{2m}{3m+4} = \dfrac{5}{m+6}$

$$2m(m + 6) = 5(3m + 4)$$
$$2m^2 + 12m = 15m + 20$$
$$2m^2 - 3m - 20 = 0$$
$$(2m + 5)(m - 4) = 0$$
$$m = \frac{5}{2} \qquad m = 4$$

Sometimes, when the quadratic equation is already in standard form, you may find that it cannot be factored by conventional approach. In those cases, we might need to solve by completing the square.

Expand and Simplify

$0 = a(x - h)^2 + k$ **Standard Form** $\qquad$ $0 = ax^2 + bx + c$ **General Form**

Complete the Square

To Complete the Squares from General Form to Standard Form

1. **Factor out the leading coefficient if *a* is NOT 1 out of the ax^2 and bx terms.**
2. **Group the first two terms in a bracket.**
3. **Complete the Square in the bracket by adding another constant. Be sure to subtract this constant at the end of the equation. Also be aware that sometimes we have to subtract the PRODUCT of this constant and a (when a is NOT 1).**
4. **Factor the perfect trinomial and combine the like terms at the end of the equation.**
5. **Solve for x.**

Example 2: Solve by completing the square. All solutions must be in the simplest radical form.

a. $5x^2 - 1 = 6x$

$$5x^2 - 6x - 1 = 0$$
$$5(x^2 - \frac{6}{5}x \quad) - 1 = 0$$
$$5(x^2 - \frac{6}{5}x + \frac{9}{25}) - 1 - \frac{9}{5} = 0$$
$$5(x - \frac{3}{5})^2 - \frac{14}{5} = 0$$
$$5(x - \frac{3}{5})^2 = \frac{14}{5}$$
$$(x - \frac{3}{5})^2 = \frac{14}{25}$$
$$(x - \frac{3}{5}) = \pm\sqrt{\frac{14}{25}}$$
$$x = \frac{3 \pm \sqrt{14}}{5}$$

b. $\frac{4}{2x-1} - \frac{3}{x+2} = 3$

$$\frac{4(x+2) - 3(2x-1)}{(2x-1)(x+2)} = 3$$
$$4x + 8 - 6x + 3 = 3(2x - 1)(x + 2)$$
$$-2x + 11 = 6x^2 + 9x - 6$$
$$0 = 6x^2 + 11x - 17$$
$$0 = 6(x^2 + \frac{11}{6}x \quad) - 17$$
$$0 = 6(x^2 + \frac{11}{6}x + \frac{121}{144}) - 17 - \frac{121}{24}$$
$$0 = 6(x + \frac{11}{12})^2 - \frac{529}{24}$$
$$\frac{529}{24} = 6(x + \frac{11}{12})^2$$
$$\frac{529}{144} = (x + \frac{11}{12})^2$$
$$\pm\sqrt{\frac{529}{144}} = x + \frac{11}{12}$$
$$-\frac{11}{12} \pm \frac{23}{12} = x$$
$$x = -1 \qquad x = -\frac{17}{6}$$

When we need to find the solution to a non-factorable quadratic equation algebraically, we need to complete the square. As we have seen before, the algebra can get quite long and tedious. In order to speed up this process, we need to find a better way to solve for non-factorable quadratic equations.

To determine EXACT SOLUTIONS of a NON-Factorable Quadratic Equation, use the

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **(MEMORIZE!!)**

Example 3: Use the quadratic formula to find the solutions of the following quadratic equations.

a. $x^2 + 6x = -2$

$$x^2 + 6x + 2 = 0 \qquad a = 1 \quad b = 6 \quad c = 2$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(2)}}{2(1)}$$
$$x = \frac{-6 \pm \sqrt{28}}{2} = \frac{-6 \pm 2\sqrt{7}}{2}$$
$$x = -3 \pm \sqrt{7}$$

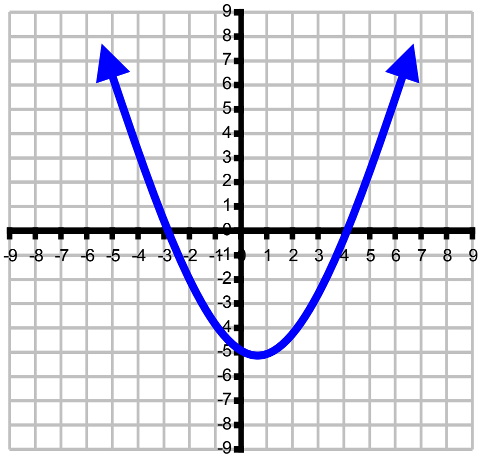
b. $-\frac{1}{3}x^2 = 3 - \frac{3}{2}x$

$$6\left(-\frac{1}{3}x^2\right) = 6(3) - 6\left(\frac{3}{2}x\right)$$
$$-2x^2 = 18 - 9x$$
$$0 = 2x^2 - 9x + 18 \qquad a = 2 \quad b = -9 \quad c = 18$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(18)}}{2(2)}$$
$$x = \frac{9 \pm \sqrt{-63}}{4}$$

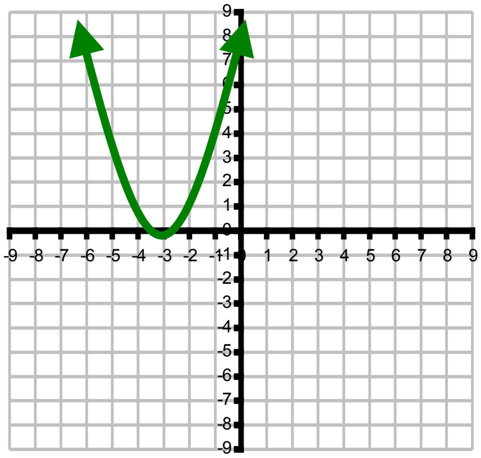
No Real Solutions

Discriminant: - the part of the quadratic formula that determine the type of solution(s) of the equation.

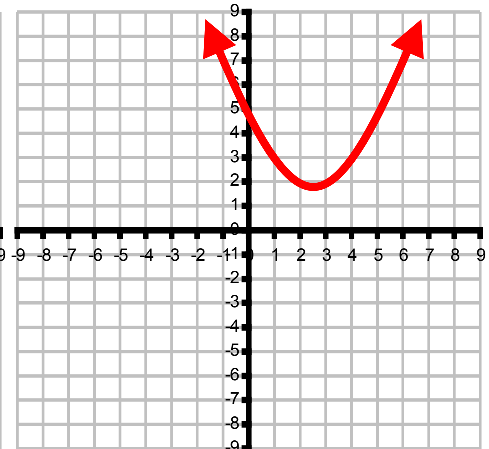
Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **Determinant** $= b^2 - 4ac$



$b^2 - 4ac > 0$
(Discriminant is Positive)
Two Distinct Real Solutions



$b^2 - 4ac = 0$
(Discriminant = 0)
Equal Real Solutions



$b^2 - 4ac < 0$
(Discriminant is Negative)
Imaginary Solutions
(No Real Solutions)

Sometimes, due to various algebraic steps involved in solving an equation, a root can be omitted or an extra root may appeared.

Gaining a Root: - when both sides of the equations are squared.
- **all roots must be verified against the** *original equation* **and all extraneous roots must be rejected.**

Example 4: Solve $\dfrac{3-2x}{4x^2-1} = \dfrac{2x-1}{2x+1} + \dfrac{2x+1}{2x-1}$

$$\frac{3-2x}{4x^2-1} = \frac{2x-1}{2x+1} + \frac{2x+1}{2x-1}$$
$$\frac{3-2x}{4x^2-1} = \frac{(2x-1)(2x-1)+(2x+1)(2x+1)}{(2x+1)(2x-1)}$$
$$\frac{3-2x}{(4x^2-1)} = \frac{(4x^2-4x+1)+(4x^2+4x+1)}{(4x^2-1)}$$
$$3 - 2x = 8x^2 + 2$$
$$0 = 8x^2 + 2x - 1$$
$$0 = (4x - 1)(2x + 1)$$

$x = \dfrac{1}{4}$ $\cancel{x = -\dfrac{1}{2}}$

Verify by substitution into the *original* equation.

For $x = \dfrac{1}{4}$:

$$\frac{3-2\left(\frac{1}{4}\right)}{4\left(\frac{1}{4}\right)^2-1} = \frac{2\left(\frac{1}{4}\right)-1}{2\left(\frac{1}{4}\right)+1} + \frac{2\left(\frac{1}{4}\right)+1}{2\left(\frac{1}{4}\right)-1}$$
$$-\frac{10}{3} = -\frac{10}{3} \checkmark$$

For $x = -\dfrac{1}{2}$:

$$\frac{3-2\left(\frac{-1}{2}\right)}{4\left(\frac{-1}{2}\right)^2-1} = \frac{2\left(\frac{-1}{2}\right)-1}{2\left(\frac{-1}{2}\right)+1} + \frac{2\left(\frac{-1}{2}\right)+1}{2\left(\frac{-1}{2}\right)-1}$$

undefined = undefined ✖

$-\dfrac{1}{2}$ **is an Extraneous Root**

Losing a Root: - commonly occur when dividing both sides of the equation by a common factor
- when the **same variable common factor appears on both sides** of the equation, one **must expand each sides before solving**.
- the only exception to the above rule is if the common factor was numerical only (without variable).

Example 5: Solve $(x + 4)(3x - 1) = 2x(3x - 1)$

The Correct Method:

$(x + 4)(3x - 1) = 2x(3x - 1)$	(Expand both sides)
$3x^2 - x + 12x - 4 = 6x^2 - 2x$	(Quadratic Equation)
$0 = 3x^2 - 13x + 4$	(Collect all terms on one side)
$0 = (3x - 1)(x - 4)$	(Factor resulting equation)

$$x = 4 \qquad x = \frac{1}{3}$$

Verify Both Roots:

For $x = 4$,

$$((4) + 4)(3(4) - 1) = 2(4)(3(4) - 1)$$
$$(8)(11) = 2(4)(11)$$
$$88 = 88 \checkmark$$

For $x = \frac{1}{3}$,

$$\left(\left(\frac{1}{3}\right) + 4\right)\left(3\left(\frac{1}{3}\right) - 1\right) = 2\left(\frac{1}{3}\right)\left(3\left(\frac{1}{3}\right) - 1\right)$$
$$\left(\frac{13}{3}\right)(0) = \left(\frac{2}{3}\right)(0)$$
$$0 = 0 \checkmark$$

The Incorrect Method:

$(x + 4)\cancel{(3x - 1)} = 2x\cancel{(3x - 1)}$	(Cancel Variable Common Factor on both sides – **WRONG!**)
$x + 4 = 2x$	
$4 = x$	
$x = 4$	**(Missing a Root!)**

1-6 Assignment: pg. 35–36 #1 to 35 (odd) and 41 (Omit #25)

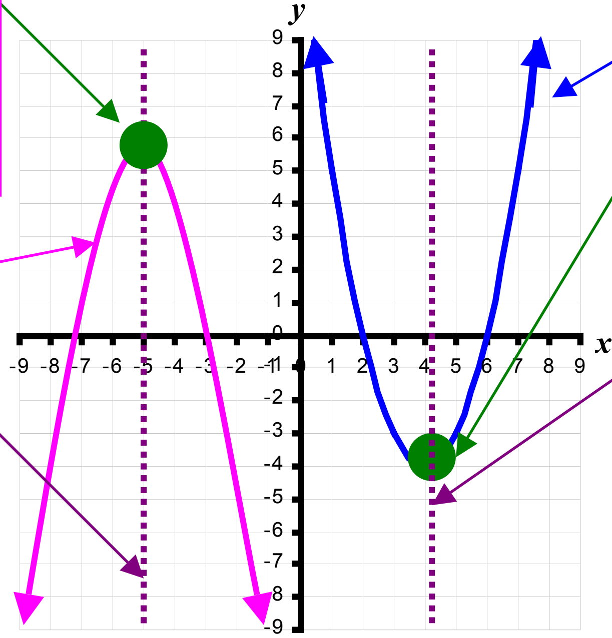
1-7: Quadratic Functions and their Graphs

Vertex at (p, q)
$g(x)$ is opening DOWN
$(a < 0)$
(Vertex is a Maximum)
Maximum at q

$g(x) = a(x - p)^2 + q$

Equation for Axis of Symmetry
$x = p$

Domain: $x \in R$
Range: $g(x) \leq q$



$f(x) = a(x - p)^2 + q$

Vertex at (p, q)
$f(x)$ is opening UP
$(a > 0)$
(Vertex is a Minimum)
Minimum at q

Equation for Axis of Symmetry
$x = p$

Domain: $x \in R$
Range: $g(x) \geq q$

The process of completing the square from the general form, $y = ax^2 + bx + c$, to the standard form, $y = a(x - p)^2 + q$, has the following generalization.

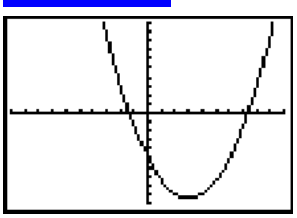
$$y = a\left(x - \frac{-b}{2a}\right)^2 + \frac{4ac - b^2}{4a} \qquad p = \frac{-b}{2a} \text{ and } q = \frac{4ac - b^2}{4a}$$

Example 1: Using a graphing calculator, find the coordinate of the vertex of $y = \frac{1}{2}x^2 - 3x - 5$. Determine the direction of the opening. State the maximum or minimum value of the parabola and the range. Indicate the equation of the axis of symmetry. Find the x- and y-intercepts.

Using the graphing calculator to find the vertex.

1. Graph the Equation

GRAPH



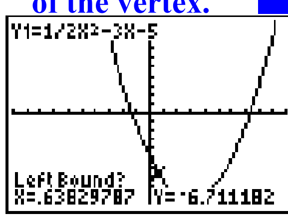
Be sure the vertex is on the screen.

2. Run Minimum function (if $a < 0$, then run the Maximum function).

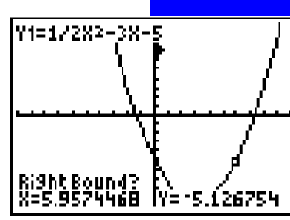
2nd CALC TRACE

ENTER

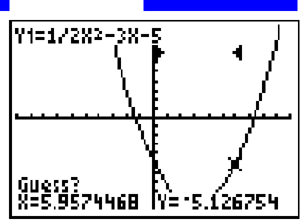
a. Take cursor left of the vertex. ENTER



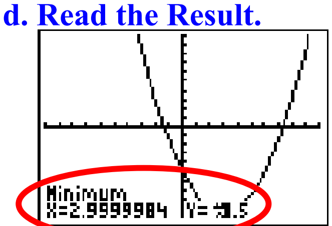
b. Take cursor right of the vertex. ENTER



c. Press ENTER again



d. Read the Result.

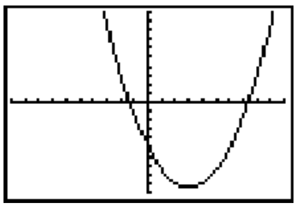


$V(3, -9.5)$ Range: $y \geq -9.5$
Minimum at -9.5 opens up $(a > 0)$
Axis of Symmetry: $x = 3$

Using the graphing calculator to find x-intercepts.

1. Graph the Equation

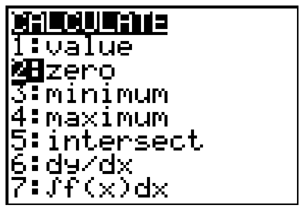
GRAPH



Be sure all x-intercepts are on the screen.

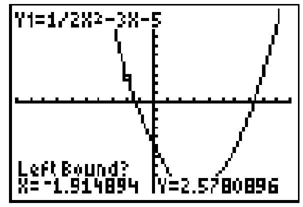
2. Run Zero (x-intercept) function

2nd **CALC** **TRACE**



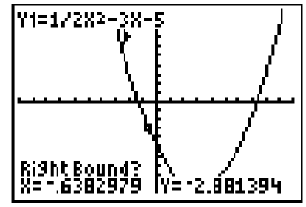
ENTER

a. Take cursor left of the desired x-intercept.



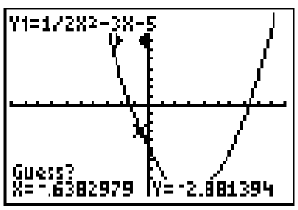
ENTER

b. Take cursor right of the desired x-intercept.

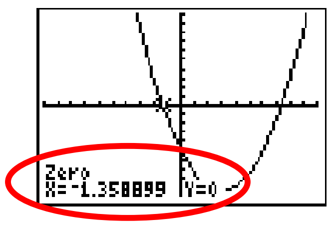


ENTER

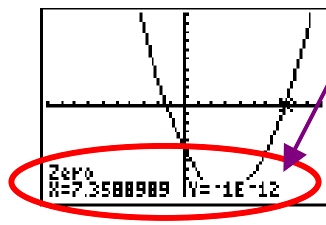
c. Press ENTER again



d. Read the Result.



3. Redo the same steps for the other x-intercept.



very near zero

y-intercept (let $x = 0$)

$y = \frac{1}{2}(0)^2 - 3(0) - 5$

y-int = –5

Example 2: Turn the general equation, $y = -2x^2 - 14x - 17$, into the standard form, $y = a(x - p)^2 + q$. Find the coordinate of the vertex. Determine the direction of the opening. State the maximum or minimum value of the parabola and the range. Indicate the equation of the axis of symmetry. Find the x- and y-intercepts.

$y = -2x^2 - 14x - 17$

$y = -2(x^2 \boxed{+} 7x \quad) - 17$

$y = -2(x^2 + 7x + \frac{49}{4}) - 17 + \frac{49}{2}$

$y = -2(x + \frac{7}{2})^2 + \frac{15}{2}$

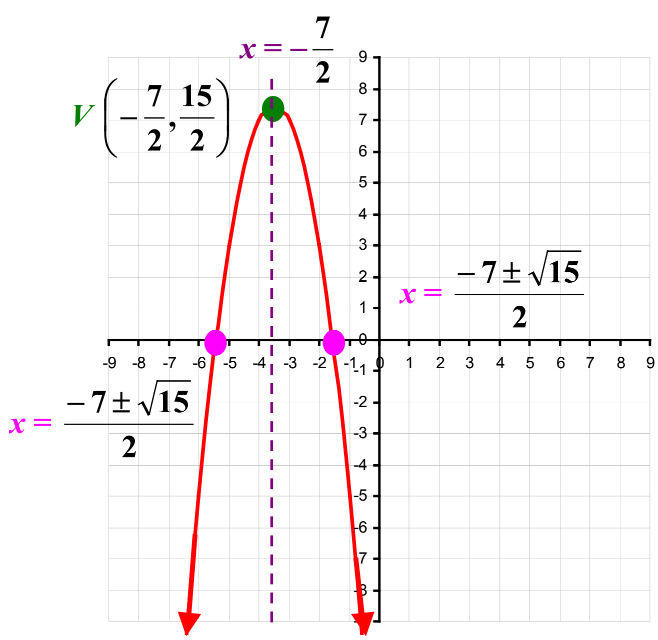
$V\left(-\frac{7}{2}, \frac{15}{2}\right)$ **opens down ($a < 0$)**

Maximum at $\frac{15}{2}$ **Range: $y \le \frac{15}{2}$**

Axis of Symmetry: $x = -\frac{7}{2}$

y-intercept (let $x = 0$)

$y = -2(0)^2 - 14(0) - 17$ **y-int = –17**



x-intercepts (let $y = 0$) $0 = -2x^2 - 14x - 17$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(-2)(-17)}}{2(-2)} = \frac{14 \pm \sqrt{60}}{-4}$ **x-ints = $\frac{-7 \pm \sqrt{15}}{2}$**

Example 3: Find the intersecting points between $y = 2x^2 + 3x - 5$ and $y = x + 7$ using
a. an algebraic method.
b. a graphing calculator.

a. Using the substituting method by equating y (since both equations have already isolated y).

$y = 2x^2 + 3x - 5$ and $y = x + 7$

$$2x^2 + 3x - 5 = x + 7$$
$$2x^2 + 2x - 12 = 0$$
$$2(x^2 + x - 6) = 0$$
$$2(x + 3)(x - 2) = 0$$

$x = -3$ and $x = 2$

For $x = -3$:
$y = (-3) + 7$
$y = 4$

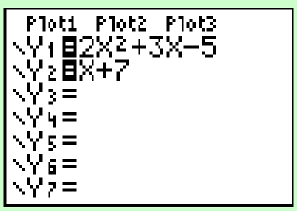
For $x = 2$:
$y = (2) + 7$
$y = 9$

Intersecting Points: $(-3, 4)$ and $(2, 9)$

b. Using a graphing calculator,

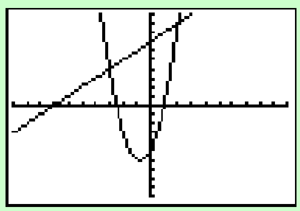
1. Enter the equations

Y=



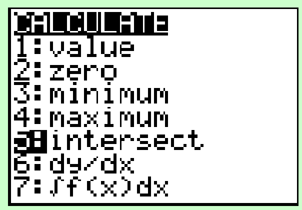
2. Graph

GRAPH

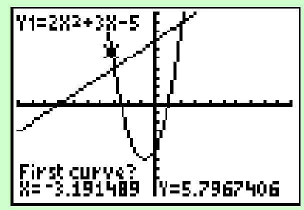


3. Run INTERSECT

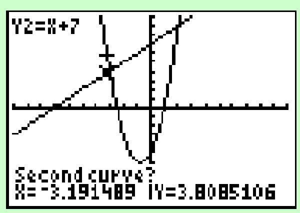
2nd **CALC** **TRACE**

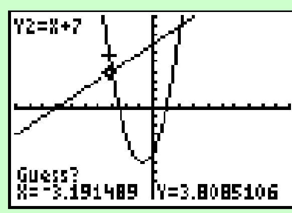


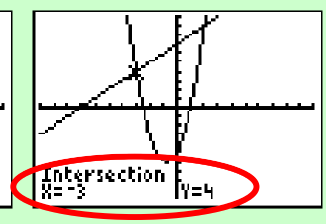
a. Bring Cursor near the left intersecting point



b. Press **ENTER** **three times**

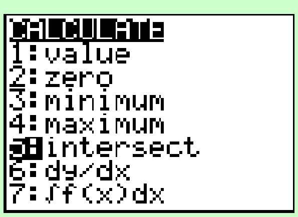




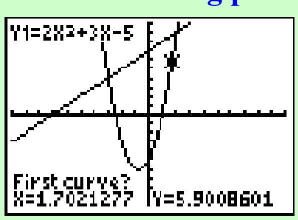


4. Run INTERSECT

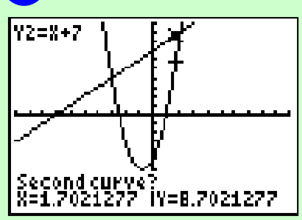
2nd **CALC** **TRACE**

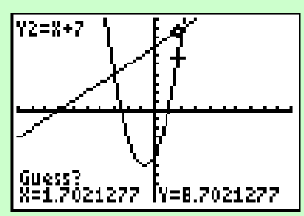


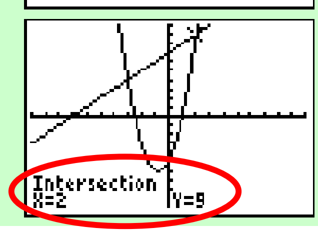
a. Bring Cursor near the right of intersecting point



b. Press **ENTER** **three times**







1-7 Assignment: pg. 41–42 #1a & b, 7 to 39 (odd) (Omit #33)

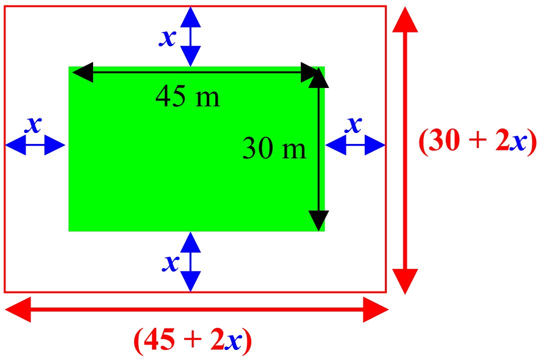
1-8: Quadratic Models

Example 1: The dimensions of a garden are 45 m by 30 m. It is to be surrounded by a brick path that has the same width on all sides of the garden. The area of the brick path itself has to be the same as the area of the garden. Determine the width of the garden.

Let x = the width of the brick path

Garden Area = 45 m × 30 m = 1350 m^2

Brick Path Area = $(30 + 2x)(45 + 2x) - 1350$



Garden Area = Brick Path Area

$$1350 = (30 + 2x)(45 + 2x) - 1350$$
$$1350 = (1350 + 60x + 90x + 4x^2) - 1350$$
$$0 = 4x^2 + 150x - 1350 \quad \textbf{(Solve for } x \rightarrow \textbf{factor)}$$
$$0 = 2(2x^2 + 75x - 675)$$
$$0 = 2(2x - 15)(x + 45)$$

$x = -45$ (not possible – negative width) $\qquad x = \dfrac{15}{2} = 7.5$ **m**

The width of the brick path is 7.5 m

Example 2: A marketing firm for the Santa Clara Transit has determined that there will be 7,500 less people riding the light rail for every five cents increased on the fare. There are currently 300,000 people ride the light rail at \$1.50 on a daily basis. At what price should the Santa Clara Transit charge per fare to yield the maximum profit?

Let x the number of 5 cents increase on the fare.
(If $x = 1$, the new fare is \$1.55; if $x = 2$, the new fare is \$1.60, and so on.)

Revenue = Fare × Number of Riders

Max Revenue = $(1.50 + 0.05x)(300{,}000 - 7{,}500x)$

Max Revenue = $450{,}000 - 11{,}250x + 15{,}000x - 375x^2$

Max Revenue = $-375x^2 + 3750x + 450{,}000$ **(Max/Min Problem – Complete the Square)**

Max Revenue = $-375(x^2$ [redacted] $10x + \quad) + 450{,}000$

Max Revenue = $-375(x^2 - 10x + 25) + 450{,}000 + 9375$

Max Revenue = $-375(x - 5)^2 + 459{,}375$

V(5, \$459,375) **Since $x = 5$, the amount of fare increase is 5(\$0.05) = \$0.25**

Hence, the new fare is \$1.50 + \$0.25 = \$1.75

Example 3: The height of an object thrown upward in metres can be expressed as a function of time in seconds by $h(t) = -5t^2 + v_it + h_i$. The initial velocity is represented by v_i in m/s and h_i is the initial height in metres. A cannon ball was fired with a height of 10 m and a initial velocity of 60 m/s.

a. State the function of height in terms of time and sketch the graph representing the function.
b. Find the maximum height of the cannon ball and the time it happened.
c. Determine the time at which the object landed on the ground.

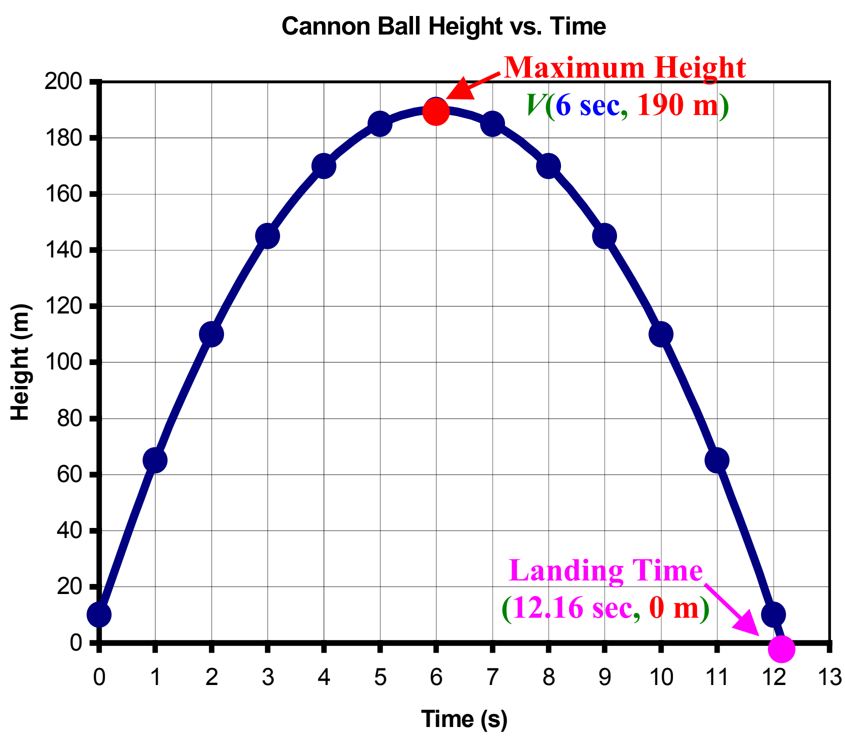
a. $h(t) = -5t^2 + v_it + h_i$ $\qquad v_i = 60$ m/s
$h_i = 10$ m

$h(t) = -5t^2 + 60t + 10$

b. Maximum Height – need to Complete the Square.
$h(t) = -5t^2 + 60t + 10$
$h(t) = -5(t^2$ [redacted] $12t \qquad) + 10$
$h(t) = -5(t^2 - 12t + 36) + 10 + 180$
$h(t) = -5(t^2 - 6)^2 + 190$ $\qquad V(6$ sec, 190 m$)$

The maximum height is at 190 m in 6 seconds.

Time (s)	Height (m)
0	10
1	65
2	110
3	145
4	170
5	185
6	190
7	185
8	170
9	145
10	110
11	65
12	10
13	-55



c. Time takes to land – need to find t-intercept (at 0 m, t = ?)
$h(t) = -5t^2 + 60t + 10$
$0 = -5(t^2$ [redacted] $12t$ [redacted] $2)$
$0 = t^2 - 12t - 2$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(-2)}}{2(1)} = \frac{12 \pm \sqrt{152}}{2} = \frac{12 \pm 2\sqrt{38}}{2} = 6 \pm \sqrt{38}$$

$t \approx 12.16$ seconds and -0.16 seconds (time cannot be negative)

The time to land is 12.16 seconds.

Example 4: Three-tiers wedding cakes are sold in sizes of 10 inches, 12 inches and 14 inches in diameter. Their prices are \$626, \$870 and \$1154 respectively. Determine the quadratic function that models the prices of the wedding cakes in terms of their diameters by

a. an algebraic approach.
b. using the graphing calculator.

a. using an algebraic approach.

$P(d) = ad^2 + bd + c$ $\quad\quad$ (10, 626), (12, 870), (14, 1154)

$$626 = a(10)^2 + b(10) + c \qquad 626 = 100a + 10b + c$$
$$870 = a(12)^2 + b(12) + c \qquad 870 = 144a + 12b + c$$
$$1154 = a(14)^2 + b(14) + c \qquad 1154 = 196a + 14b + c$$

Combining the first and second equations:

$$\begin{aligned} 870 &= 144a + 12b + c \\ -\ (626 &= 100a + 10b + c) \\ \hline 244 &= 44a + 2b \end{aligned}$$

Combining the second and third equations:

$$\begin{aligned} 1154 &= 196a + 14b + c \\ -\ (870 &= 144a + 12b + c) \\ \hline 284 &= 52a + 2b \end{aligned}$$

$$\begin{aligned} 284 &= 52a + 2b \\ -\ (244 &= 44a + 2b) \\ \hline 40 &= 8a \end{aligned}$$

$a = 5$

Substitute $a = 5$:
$$244 = 44a + 2b$$
$$244 = 44(5) + 2b$$
$$24 = 2b$$

$b = 12$

Substitute $a = 5$ and $b = 12$:
$$626 = 100a + 10b + c$$
$$626 = 100(5) + 10(12) + c$$
$$626 = 500 + 120 + c$$

$c = 6$

$$P(d) = 5d^2 + 12d + 6$$

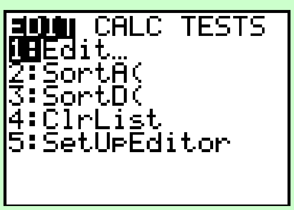
b. Using a Graphing Calculator,

1. To Enter a Table of values, use the STAT menu.

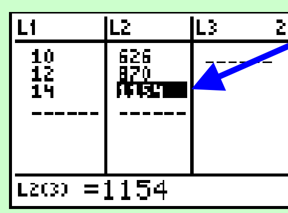
STAT

Choose Option 1

ENTER



Enter Coordinates



When finish entering,

2nd

QUIT
MODE

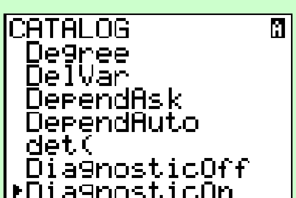
2. Obtaining Equation with Correlation of Determination:

a. Setting DiagnosticOn for Correlation of Determination

2nd

CATALOG
0

and Move Down using



ENTER

Select DiagnosticOn

ENTER

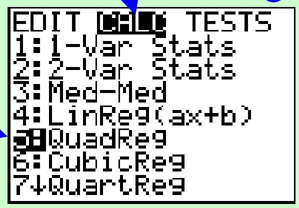
Again

Note: After DiagnosticOn is selected; it will remain ON even when the calculator is turned Off. However, resetting the calculator will turn the Diagnostic Off (factory setting).

b. Use Quadratic Regression to obtain equation

STAT **Select CALC, use ▶**

Choose Option 5 QuadReg



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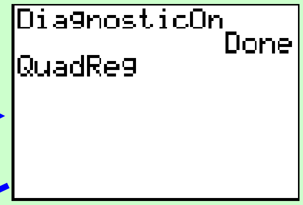
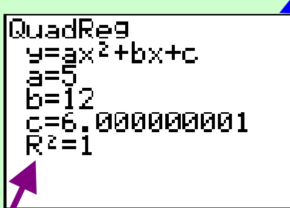


ENTER **Again**



The quadratic equation is $y = 5x^2 + 12x + 6$.

The function that models the price of a wedding cake in terms of its diameter is $P(d) = 5d^2 + 12d + 6$

Correlation of Determination (r^2) is 1, which means the equation is an excellent fit for the coordinates. (The closer it is to 1, the better the fit.)

1-8 Assignment: pg. 45–47 #1 to 15 (odd) (Omit #3)