Unit 2: Rational Expressions

4-2: Simplifying Rational Expressions

Rational Expressions: - fractions with polynomials as numerator and / or denominator.

To Simplify (Reduce) Rational Expressions: (Factor all polynomials and reduce)

Example 1: Simplify the followings.

a.
$$\frac{9x^{2}-2x}{9x-2}$$

b.
$$\frac{3x}{x^{2}+6x}$$

$$=\frac{x(9x-2)}{(9x-2)}$$

$$=\frac{x(9x-2)}{(9x-2)}$$

$$=\frac{x(9x-2)}{(9x-2)}$$

$$=\frac{x}{(9x-2)}$$

$$=\frac{3x}{x(x+6)}$$

$$=\frac{$$

c.
$$\frac{6x^{2}y + 12xy}{8x^{2} - 8x}$$
d.
$$\frac{x^{2} - 9}{x^{2} - 6x + 9}$$

$$= \frac{6xy(x+2)}{8x(x-1)}$$

$$= \frac{6xy(x+2)}{8y(x-1)}$$

$$= \frac{3y(x+2)}{4x(x-1)}$$
d.
$$\frac{x^{2} - 9}{x^{2} - 6x + 9}$$
NPV:

$$8x(x-1) = 0$$

$$8x = 0 \quad x - 1 = 0$$

$$x = 0 \quad x = 1$$

$$x = 0 \text{ and } 1$$

$$= \frac{(x-3)(x+3)}{(x-3)(x-3)}$$

$$= \frac{(x-3)(x+3)}{(x-3)(x-3)}$$

$$x = 3$$

e.
$$\frac{2y^2 - 7y + 3}{y^2 - 5y + 6}$$
 f. $\frac{x - 2}{x + 2}$

$$= \frac{(y - 3)(2y - 1)}{(y - 2)(y - 3)}$$

$$= \frac{(x - 3)(2y - 1)}{(y - 2)(y - 3)}$$

$$= \frac{(2y - 1)}{(y - 2)(y - 3)}$$

$$= \frac{(2y - 1)}{(y - 2)}$$
Provide the set of the set

<u>4-2 Homework Assignments</u>

Regular: pg. 158-159 #1 to 49 (odd), 50, 51 (a to e), 53

AP: pg. 158-159 #2 to 48 (even), 50 to 55

4-3: Multiplying and Dividing Rational Expressions





AP: pg. 163-164 #2 to 48 (even), 49 to 54, 55 (a, b, c), 56 to 59

<u>To /</u>	Add and Subtract Rational Expressions	3 5	3(3)+2(5)	LCM and
1.	Find Common Denominator.	$\frac{-+-}{4}$	$=$ $\frac{12}{12}$	Equivalent Fractions
2.	Obtain Equivalent Fractions.		9 + 10	
3.	Add or Subtract Numerators.		$=\frac{3+10}{10}$	
4.	Reduce if Possible.		12	
			19	
			$= \frac{1}{12}$	

Example 1: Simplify the followings.

LCM = x

x = 0

NPV:

 $\frac{6}{x} + \frac{2}{x} - \frac{9}{x}$

 $=\frac{6+2-9}{2}$

a.

c.



$$\frac{5x}{3} + \frac{9x}{2} - \frac{7x}{4} + 1$$
d.
$$\frac{3x-2}{6} + \frac{x+5}{4} - \frac{2x-3}{3}$$

$$= \frac{4(5x) + 6(9x) - 3(7x) + 12}{12}$$
LCM = 12
$$= \frac{20x + 54x - 21x + 12}{12}$$

$$= \underbrace{\frac{53x + 12}{12}}_{12}$$

$$= \underbrace{\frac{53x + 12}{12}}_{12}$$

$$= \underbrace{\frac{53x + 12}{12}}_{12}$$

Example 2: A rectangle has a dimensions $\frac{3x+1}{2}$ and $\frac{2x-3}{5}$. Find its perimeter.

Width =
$$\frac{2x-3}{5}$$

 $P = 2\left(\frac{3x+1}{2}\right) + 2\left(\frac{2x-3}{5}\right)$
 $P = (3x+1) + \frac{4x-6}{5}$
 $P = \frac{5(3x+1) + (4x-6)}{5}$
 $P = \frac{15x+5+4x-6}{5}$
 $P = \frac{19x-1}{5}$

4-4 Homework Assignments

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Regular: pg. 167 #1 to 25 (odd), 26, 27, 29a

AP: pg. 167 #2 to 24 (even), 26 to 28, 29a, 30

4-5: Adding and Subtracting Rational Expressions (Part 2)

LCM of Monomial: - LCM of monomial coefficient, and the variable(s) with its / their highest exponent(s).

Example 1: LCM of
$$3a^2$$
, $5a$, $6a^3$
LCM of 3, 5, $6 = 30$ Variable with Highest Exponent $= a^3$
LCM $= 30a^3$
Example 2: LCM of $3x^2y$, $21x^3y^2$, $7xy^3$
LCM of 3, 21 , $7 = 21$ Variables with Highest Exponents $= x^3y^3$
LCM $= 21x^3y^3$

LCM of Polynomial: - common factor(s) (written once) along with any uncommon (leftover) factor(s).

Example 3: LCM of
$$x^2 - 2x - 3$$
 and $x^2 - x - 2$
Factors of $x^2 - 2x - 3 = (x - 3)(x + 1)$ and Factors of $x^2 - x - 2 = (x + 1)(x - 2)$
Common Factor Leftover Factors
LCM = $(x + 1)(x - 3)(x - 2)$

Example 4: Simplify the followings.

a.
$$\frac{4}{3x^2} - \frac{5}{2x} + 3$$

$$= \frac{2(4) - 3x(5) + 6x^2(3)}{6x^2}$$

$$= \frac{8 - 15x + 18x^2}{6x^2}$$

$$= \underbrace{\frac{18x^2 - 15x + 8}{6x^2}}_{6x^2}$$

$$x = 0$$
b.
$$\frac{4}{a^2b} + \frac{3}{ab^2} - \frac{2}{a^2b^3}$$

$$= \underbrace{\frac{b^2(4) + ab(3) - (2)}{a^2b^3}}_{2b^3}$$

$$= \underbrace{\frac{4b^2 + 3ab - 2}{a^2b^3}}_{2b^3}$$

c.
$$\frac{5}{x+2} + \frac{3x+1}{3x+6}$$
d.
$$\frac{2x}{9x^2-4} - \frac{3x}{9x^2-12x+4}$$
LCM = $(3x-2)(3x-2)(3x-2)(3x+2)$

$$= \frac{5}{x+2} + \frac{3x+1}{3(x+2)}$$
LCM = $3(x+2)$

$$= \frac{3(5)+(3x+1)}{3(x+2)}$$

$$= \frac{15+3x+1}{3(x+2)}$$

$$= \frac{(3x+16)}{(3x+2)}$$
NPV:

$$\frac{3(x+2)=0}{x+2=0}$$

$$x=-2$$

$$= \frac{-3x^2-10x}{(3x-2)^2(3x+2)}$$

$$= \frac{-3x^2-10x}{(3x-2)^2(3x+2)}$$

$$= \frac{-3x^2-10x}{(3x-2)^2(3x+2)}$$
NPV:

$$(3x-2)=0$$

$$(3x+2)=0$$

$$3x=2$$

$$x=\frac{2}{3}$$

$$x=-\frac{2}{3}$$

$$x=\pm\frac{2}{3}$$
c.
$$\frac{y^2+\frac{y-3}{2}}{3}$$
LCM = 2

$$= \frac{\frac{2(y^2+y-3}{3}}{3y^2-5y+2}$$
Switch Signs!

$$= \frac{(2y+3)(y-1)}{2}$$

$$= \frac{(2y+3)(y-1)}{2}$$

$$= \frac{(2y+3)(y-1)}{2(3y-2)(y-1)}$$

$$= \frac{(2y+3)(y-1)}{2(3y-2)(y-1)}$$
NPV:

$$(3y-2)(y-1)=0$$

$$(3y-2)(y$$

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a.

Example 5: Car A takes *x* hours to travel 100 km while car B takes 1 hour less than car A to travel 100 km.

- a. Write an algebraic expression for each car A and car B that represent the speed at which they travel in terms of x.
- b. Write an algebraic expression for their difference in speed.

For questions involving Distance, Speed and Time, make a table to organize any data & expressions.

		Recall that: spa	$eed = \frac{distance}{time}$	
		Distance (km)	Speed (km/hr)	Time (hr)
	Car A	100	$\frac{100}{x}$	x
	Car B	100	$\frac{100}{x-1}$	<i>x</i> – 1
'				

b. Difference in Speed (Car B is faster) = Speed of Car B – Speed of Car A

$$= \frac{100}{x-1} - \frac{100}{x}$$

$$= \frac{100x - 100(x-1)}{x(x-1)} \quad LCM = x (x-1)$$
NPV:

$$x(x-1) = 0$$

$$x = 0 \quad (x-1) = 0$$

$$x = 1$$
In reality, x, the time it takes for
Car A has to be greater than 1. = \frac{100}{x(x-1)} \text{ km/hr}
$$= \frac{100}{x(x-1)} \text{ km/hr}$$

<u>4-6: Review of Solving Linear Equations</u>

When Solving Linear Equations:

- 1. Expand and Simplify each side.
- 2. Make Sure you LINE UP the Equal signs as you work downward.
- 3. Collect Like-Terms with the Unknown terms on the Left Hand Side.
- 4. When Moving Terms to the other side of the Equal sign, do Reverse Order of Operation (SAMDEB) and Reverse Operations.

b.

5. Solve for the solution by isolating the variable.

Example 1: Solve the following equations.

a.
$$3x + 2 = x - 8$$

$$3x - x = -8 - 2$$
$$2x = -10$$
$$x = \frac{-10}{2}$$
$$x = -5$$

Collect Like Terms 9x and Reverse Operations 9x

$$9(x-4) = 2(x+17)$$

$$-36 = 2x + 34$$

$$-2x = 34 + 36$$

$$7x = 70$$

$$x = \frac{70}{7}$$

$$x = 10$$

Collect Like Terms and Reverse Operations

c.
$$3(2x+1) - (2-3x) + 7 = 5x - 6$$

$$6x + 3 - 2 + 3x + 7 = 5x - 6$$
$$9x + 8 = 5x - 6$$
$$9x - 5x = -6 - 8$$
$$4x = -14$$
$$x = \frac{-14}{4}$$
$$x = \frac{-7}{2}$$

Expand and Simplify each side

Collect Like Terms and Reverse Operations

<u>4-6 Homework Assignments</u>

Regular: pg. 179 - 180 #1 to 37 (odd), 39 to 47

4-7: Solving Rational Equations

When solving rational equations, there are two methods you can use.

Method 1: Multiply Both Sides with LCM

- doing so will eliminate all denominators
- solve the remaining equation

Method 2: Cross-Multiplication

- only do so when there is a single fraction equals to another single fraction.
- solve the remaining equation.

Example 1: Solve the following equations.

a.
$$\frac{3x}{2} - 5 = 4$$

Method 1: Multiply Both Sides with LCMMethod 2: Cross-Multiplication
$$\frac{3x}{2} - 5 = 4$$
 $\frac{3x}{2} - 5 = 4$ $2(\frac{3x}{2} - 5) = 2(4)$ Multiply
Both Sides
by the LCM $3x - 10 = 8$ $\frac{3x}{2} \times \frac{9}{11}$ Cross Multiply
after
Rearrangement $3x = 8 + 10$ $3x = 18$ $x = 6$ $3x = 18$

b. $\frac{3}{x} + 4 = \frac{3}{2x}$	
<u>Method 1: Multiply Both Sides with LCM</u> $\frac{3}{x} + 4 = \frac{3}{2x}$	$\frac{\text{Method 2: Cross-Multiplication}}{\frac{3}{x} + 4 = \frac{3}{2x}}$
$2x\left(\frac{3}{x}+4\right) = 2x\left(\frac{9}{2x}\right)$ Multiply Both Sides by the LCM	Obtain LCM and Equivalent Fraction $\frac{3+4x}{x} = \frac{9}{2x}$ x in the denominator can cancel on both sides
6 + 8x = 9	$\frac{3+4x}{1}$ $\frac{9}{2}$ Cross Multiply
8x = 9 - 6	2(3+4x)=9
8x = 3	6 + 8x = 9
$x = \frac{3}{8}$	8x = 9 - 6
NPV = 0	$2x = 0$ $x = 0$ $x = \frac{3}{8}$

c. $\frac{5}{x+2} = \frac{2}{x-3}$

Method 1: Multiply Both Sides with LCM	Method 2: Cross-Multiplication		
Multiply Both Sides by the LCM $\frac{5}{x+2} = \frac{2}{x-3}$	Cross Multiply $\frac{5}{x+2}$ $\xrightarrow{2}{x-3}$		
$(x+2)(x-3)\left(\frac{5}{x+2}\right) = (x+2)(x-3)\left(\frac{2}{x-3}\right)$ Reduce	5(x-3) = 2(x+2)		
when possible. $5(x-3) = 2(x+2)$ 5x-15 = 2x+4 5x-2x = 4+15	5x - 15 = 2x + 4 5x - 2x = 4 + 15 3x - 19		
3x = 19	$x = \frac{19}{3}$		
$x = \frac{19}{3}$ NPV x +2 =	= 0 x - 3 = 0		
$\chi = -$	-2 x = 3		

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d.
$$\frac{6}{3x-2} - 3 = \frac{4}{3x-2}$$

Method 1: Multiply Both Sides with LCM			Method 2: Cross-Multiplication		
$\frac{6}{3x-2} - 3 = \frac{4}{3x-2}$ $\frac{6}{3x-2} - 3 = \frac{4}{3x-2}$ $\frac{6}{3x-2} - 3 = \frac{4}{3x-2} + \frac{4}{3x-2}$ $\frac{4}{3x-2} + \frac{4}{3x-2} + \frac{4}$	$\frac{\frac{6}{3x-2} - 3 = \frac{4}{3x-2}$ Obtain LCM and Equivalent $\frac{6-3(3x-2)}{(3x-2)} = \frac{4}{(3x-2)}$ Fraction				
6-3(3x-2)=4		6-9x+6=4 $(3x-2)$ in the denominator			
6 - 9x + 6 = 4	-9x+12 = 4 can cancel on both sides				
-9x + 12 = 4	-9x = 4 - 12				
-9x = 4 - 12	-9x = -8				
-9x = -8			$x = \frac{-8}{-9}$		
$x = \frac{-8}{-9}$	NPV 3x - 2 3x	2 = 0 $c = 2$	$x = \frac{8}{9}$		
$x=\frac{8}{9}$	ς	$x = \frac{2}{3}$			

<u>4-7 Homework Assignments</u>

Regular: pg. 185 - 186 #1 to 69 (odd), 70 to 78

AP: pg. 185 - 186 #2 to 68 (even), 70 to 78

4-8: Problem Solving with Equations

When Solving Word Problems:

- 1. Decide what the variable represents (usually the unknown or the smaller item).
- 2. Set up an equation by reading the questions bit by bit, or organize the information on a table.
- 3. Solve and Verify.
- 4. Write out a final statement indicating the solution(s).

Example 1: Find three consecutive odd integers that have a sum of -51.



Example 2: Mary has \$7.85 in quarters and dimes. If she has 50 coins, how many coins of each type does she have?

Let	x = number of quarters (50 - x) = number of dimes	25x + 10(50 - x) = 785
	Each Quarter is worth 25 cents	25x + 500 - 25x = 785
	Each Dime is worth 10 cents	15x + 500 = 785
		15x = 785 - 500
Ν	fary has 19 quarters and (50 – 19) dimes.	15x = 285
N	Ary has 19 quarters and 31 dimes. $(19 \times \$0.25 + 31 \times \$0.10 = \$7.85)$	$x = \frac{285}{15}$
		x = 19

Example 3: John went for a 199 km mountain biking trip for two days. On day 1, he biked 3 km/h faster than day 2. If he biked for 9 hours on day 1 while on day 2 he biked for 11 hours, how fast was he travelling on each day?

R	ecall that:	spe	$eed = \frac{distance}{time}$	or	speed	l = distance × time
			Distance (km)	Speed (ki	n/hr)	Time (hr)
	Day 1		9(x+3)	<i>x</i> + 3		9
	Day 2		11 <i>x</i>	x		11
	TOTAL		199			
9x + 27 + 11x = 199 $20x + 27 = 199$ $20x = 199 - 27$ $20x = 172$						
x = 8.6			John 1, and	would h l 8.6 km	nave biked 11.6 km/h on n/h on day 2.	

4-8 Homework Assignments

Regular: pg. 190 - 191 #1 to 11 (odd), 13 to 18, 20 to 30

(AP) Example 4: Mary and Jane each left Calgary and Edmonton respectively at the same time, and drove towards Red Deer 150 km away. If Mary drove 10 km/h faster than Jane and she had to wait 10 minutes before Jane arrived at Red Deer, how fast were both of them driving?

	Distance (km)	Speed (km/hr)	,	Time (hr)			
Mary	150	<i>x</i> + 10	$\frac{150}{x+10}$	(Faster – Less Time)			
Jane	150	x	$\frac{150}{x}$ (8)	Slower – More Time)			
DIFFERENCE	,		10 m	inutes = $\frac{1}{6}hr$			
$\frac{150(x-x)}{x(x-x)}$ $\frac{150x+x}{x(x-x)}$ Speed can be Negative	$\frac{50}{x} - \frac{150}{x+10} = \frac{1}{6}$ $\frac{+10) - 150x}{(x+10)} = \frac{1}{6}$ $\frac{1500 - 150x}{(x+10)} = \frac{1}{6}$ $\frac{1500}{x(x+10)} = \frac{1}{6}$ $(6)(1500) = x(x+10)$ $9000 = x^{2} + 10$ $0 = x^{2} + 10$ $0 = (x - 90)$ $x + 100 = 0$ not $x = -1$	we Quadratic ons, bring ing to one d Factor! -90 = 100 x = 90	Jane drove at 90 km/h, while Jary drove at 100 km/h				
Г							
	(even), 13 to 31						

4-9: Equations with Literal Coefficients

Literal Coefficient: - the variable part of a monomial.

Formula: - an equation where the variables represent certain measurements or constant.

Example 1: For d = st, solve for s and t.



When manipulating (rearranging) formulas, follow the rule of Reverse Order of Operation (SAMDEB) and Reverse Operations.

Example 2: Given the formulas below, solve for the variables indicated.



Example 3: Use the formula, $C = \frac{5}{9}(F - 32)$, where *C* is temperature in degree Celsius and *F* is temperature in Fahrenheit, to find the equivalent of 25°C in Fahrenheit.

First, we have to manipulate (rearrange) the formula to solve for *F*.

$$C = \frac{5}{9} \left(F - 32 \right)$$

 $\frac{9C}{5} = F - 32$ $\frac{9C}{5} + 32 = F$

For 25° C, *C* = 25 and *F* = ?

$$F = \frac{9(25)}{5} + 32$$
$$F = 77$$

4-9 Homework Assignments

Regular: pg. 193 #1 to 16