Unit 3: Relations and Functions

5-1: Binary Relations

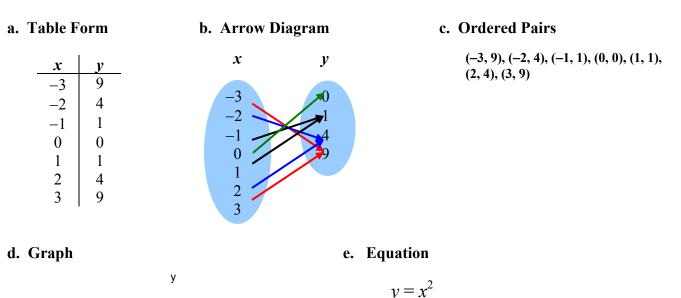
Binary Relation: - a set ordered pairs (coordinates) that include two variables (elements).

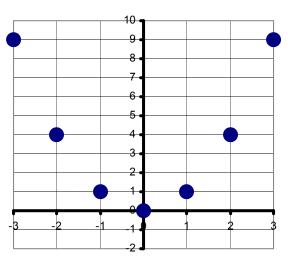
(x, y) x = horizontal y = vertical

Domain: - all the *x*-values (first elements) of a relation.

Range: - all the *y*-values (second elements) of a relation.

Different Ways to Describe a Relation.





f. Words

The second number is equal to the square of the first number.

Domain {-3, -2, -1, 0, 1, 2, 3}

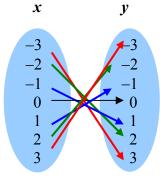
Range {0, 1, 4, 9}

Pure Math 10 Notes

Example 1: Use the graph on the right to express the relations as

- a. a set of ordered pairs.
- b. an arrow diagram.
- c. in words.
- d. an equation.
- e. Find the Domain and Range.



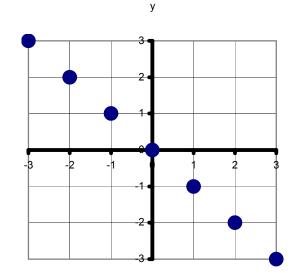


- c. The second number is the negative of the first number.
- d. y = -x
- e. Domain $\{-3, -2, -1, 0, 1, 2, 3\}$ Range $\{-3, -2, -1, 0, 1, 2, 3\}$

5-1 Homework Assignments

Regular: pg. 214 to 215 #1 to 20 (except 19d, 20d, and 20e)

AP: pg. 214 to 215 # 1 to 21



5-2: Linear Relation and Line of Best Fit

Linear Relation: - a set ordered pairs that exhibit a straight line when plotted on a graph.

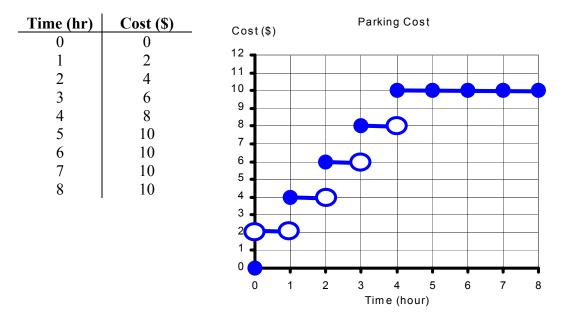
Scatter Plot: - a graph that only has ordered pairs showed.

Line of Best Fit: - a line that will best describe the general relation of the ordered pairs on the graph.

There are two types of data

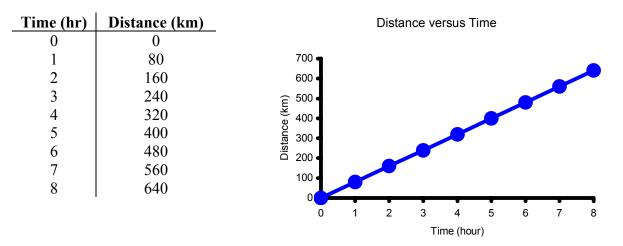
a. Discrete Data: - a graph with a series of separated ordered pairs or broken lines.

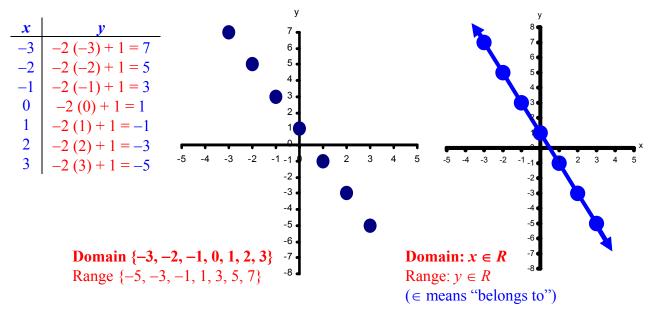
Example: Cost of Parking is \$2.00 every hour with a Daily Maximum of \$10.00.



b. Continuous Data: - a graph with an unbroken line that connects a series of ordered pairs.

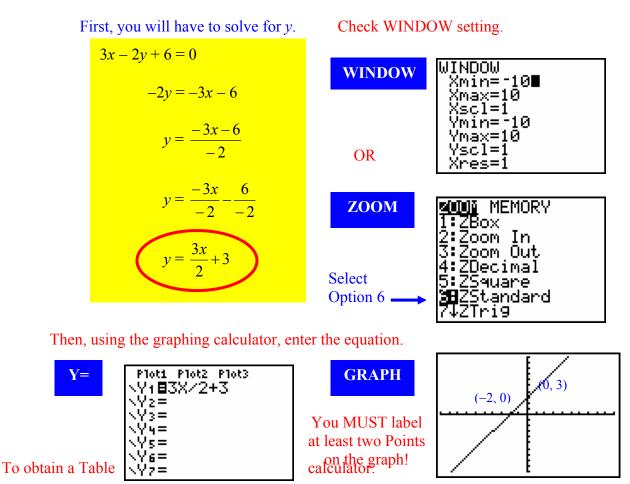
Example: Distance versus Time of a car with a constant speed of 80 km/h.

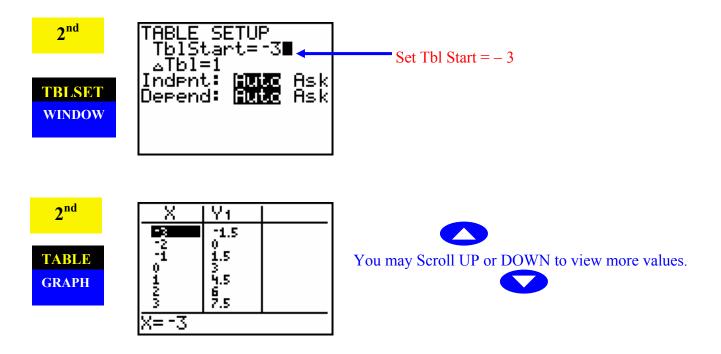




Example 1: Graph y = -2x + 1 for the domains $\{-3, -2, -1, 0, 1, 2, 3\}$ and the real number set, *R*.

Example 2: Use a graphing calculator to graph the equation 3x - 2y + 6 = 0 (copy it with axes properly labelled) and obtain a table of values from x = -3 to x = 2.





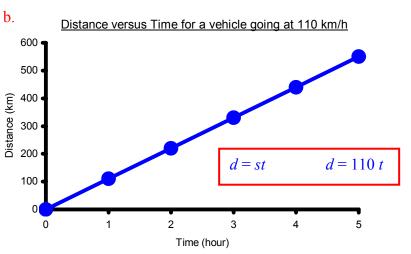
Example 3: A vehicle is going at 110 km/h.

- a. Set up a table of values from t = 0 hours to t = 5 hours for every hour and the distances travelled.
- b. Plot the distance versus time graph from the table of values above and obtain an equation relating distance and time.
- c. Explain whether the graph should be continuous or discrete.

a.

Time (hr)	Distance (km)
0	0
1	110
2	220
3	330
4	440
5	550

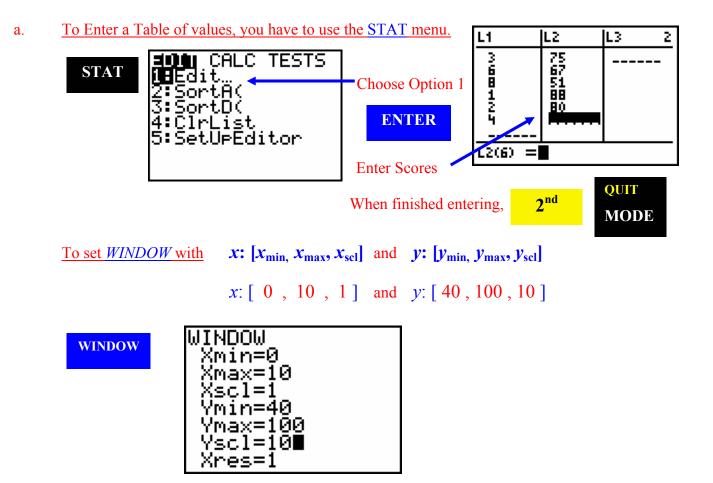
c. Data should be continuous because you may have *t*, *time*, between the whole number intervals (example 2.5 hours is allowed).



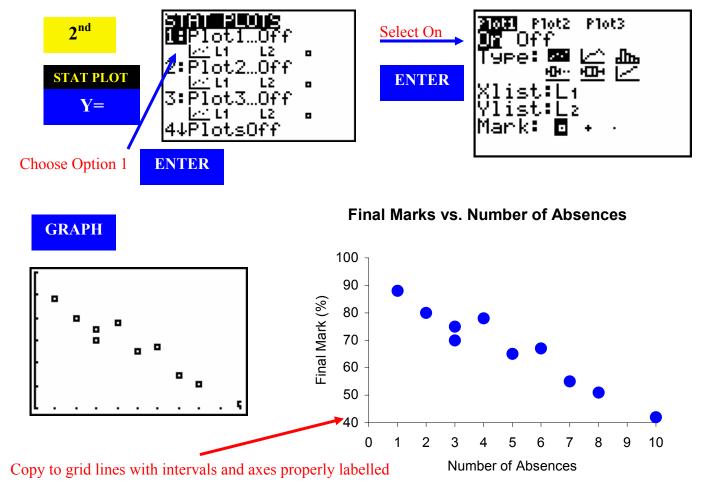
Student Number	Number of Absences	Final Marks (%)
1	3	75
2	6	67
3	8	51
4	1	88
5	2	80
6	4	78
7	10	42
8	7	55
9	3	70
10	5	65

Example 4: A school tracked 10 students' attendance records and their final marks in the class of Applied Math 10.

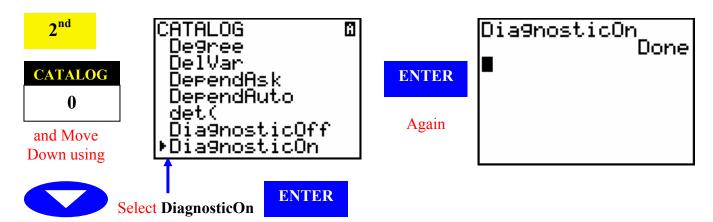
- a. Using the graphing calculator, decide on an appropriate *WINDOW* setting. Plot the final marks versus number of absences. Copy this graph with axes properly labelled.
- b. Using the graphing calculator, obtain the line of best fit and its correlation coefficient.
- c. Using the equation of the line of best fit; predict what final mark a student will likely get with 9 absences.
- d. Explain whether the final graph should be discrete or continuous.



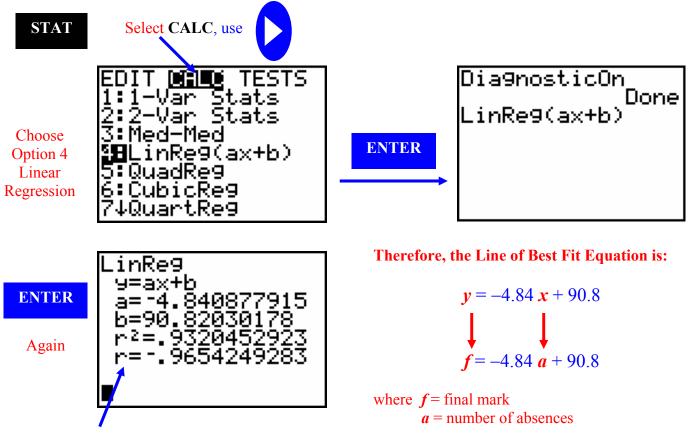
To Graph from Data in STAT Lists, you must turn the STAT PLOT On and be sure there is <u>NO</u> equation in the <u>Y</u> = Screen.



b. <u>Obtaining Equations with Correlation Coefficient:</u>

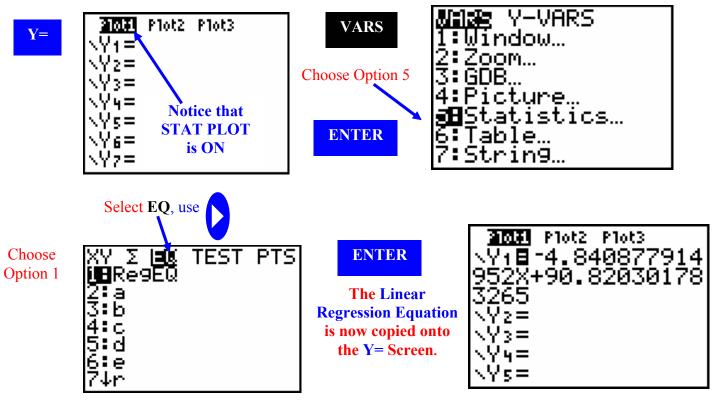


Note: After DiagnosticOn is selected; it will remain ON even when the calculator is turned Off. However, resetting the calculator will turn the Diagnostic Off (factory setting).

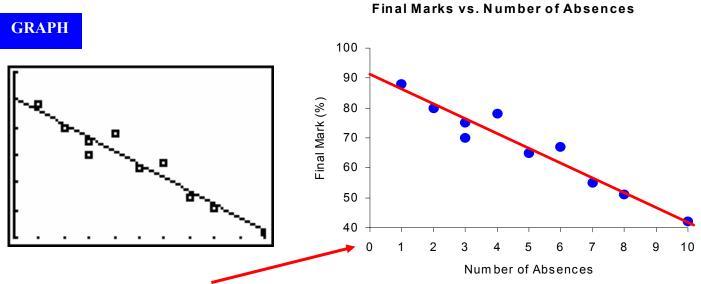


Correlation Coefficient is -0.965, which means a good fit, close to 1 or -1, and as x increases, y decreases.

To Draw the Line of best Fit on the Graph: (Need to Copy the Equation to the Y= Screen)



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Copy graph to actual grid with best fit line, intervals and axes properly labelled

- At 9 Absences, a = 9, f = ?or Using Graphing Calculator c. TRACE Y1=-4.840877914952X+90.8_ f = -4.84 a + 90.8Select Equation of f = -4.84(9) + 90.8the Best Fit Line to Trace by = 47% Press 9 for x = 9X=9 Y=47.252401
- **d.** The data is discrete because you cannot have a decimal as the number of absences.

5-2 Homework Assignments

Regular: pg. 219 to 220 #1 to 26d, 27, 28a to 28d, 29 to 31

AP: pg. 219 to 220 # 1 to 26d, 27 to 31

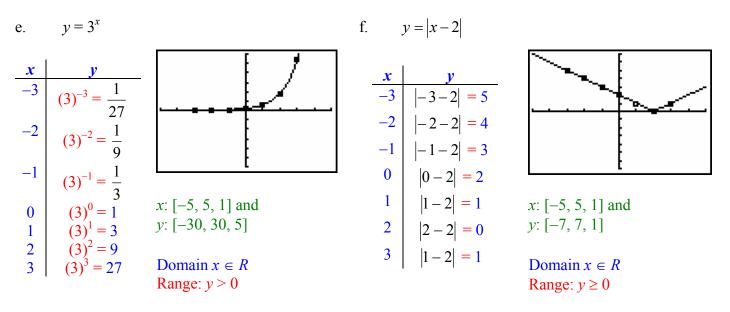
5-3: Non-Linear Relations

Non-Linear Relation: - a set ordered pairs when graphs, exhibit a curve as the line of best fit instead of a straight line.

- usually when the *x* and/or *y* variable(s) have an exponent other than 0 or 1.

Example 1: Provide the table of values and a graph for the following equations. For each graph, state the domain and range.

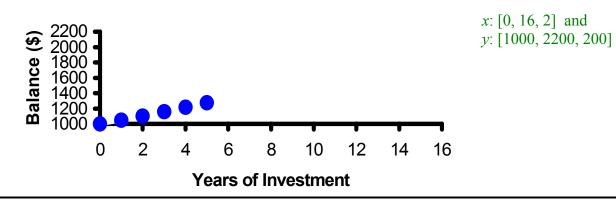
 $v = x^2 + 3$ $v = 2x^3$ b. a. $(-3)^2 + 3 = 12$ $(-2)^2 + 3 = 7$ -2 $(-2)^{+} + 5 = 7$ $(-1)^{2} + 3 = 4$ $(0)^{2} + 3 = 3$ $(1)^{2} + 3 = 4$ $(2)^{2} + 3 = 7$ $(3)^{2} + 3 = 12$ -1 0 1 2 *x*: [-5, 5, 1] and *y*: [-20, 20, 2] x: [-5, 5, 1] and y: [-60, 60, 10] 3 Domain $x \in R$ Domain $x \in R$ Range: $y \ge 3$ Range: $y \in R$ d. $y = \frac{6}{r}$ $v = 3\sqrt{x}$ c. $\frac{y}{3\sqrt{(-3)}} = \text{no solution}$ $\frac{6}{(-3)} = -2$ -3 $3\sqrt{(-2)} = \text{no solution}$ -2 $3\sqrt{(-1)} =$ no solution -1 $\frac{6}{(-2)} = -3$ -2 $3\sqrt{(0)} = 0$ 0 $\frac{6}{(-1)} = -6$ $\frac{6}{(0)} =$ undefined $3\sqrt{(1)} = 3$ 1 -1 $3\sqrt{(2)} = 4.2426$ 2 *x*: [-5, 5, 1] and 3 $3\sqrt{(3)} = 5.1962$ 0 *y*: [-10, 10, 1] $\frac{6}{(1)} = 6$ *x*: [-5, 5, 1] and Domain $x \neq 0$ 1 *y*: [-10, 10, 1] Range: $v \neq 0$ $\frac{6}{(2)} = 3$ $\frac{6}{(3)} = 2$ 2 Domain $x \ge 0$ Range: $y \ge 0$ 3



Example 2: A \$1000 investment was left in an account that pay 5%/a (a = annum = year). The table below shows the balance at the end of each year, assuming no withdrawal is made in anytime.

Year	Balance	a.	Graph the balance versus the number of years on a graphing calculator.
0	\$1000.00		Show the <i>WINDOW</i> settings and label the axes properly.
1	\$1050.00		
2	\$1102.50	b.	Explain the pattern in words.
3	\$1157.63		
4	\$1215.51	C.	Write out a possible equation. Verify this equation with your graphing
5	\$1276.28		calculator. Did the graph of your equation give a curve that goes through
			most of the points?

- d. Explain whether the data is discrete or continuous.
- e. Use the equation to predict the balance at the end of the 10 years.
- f. Using the graphing calculator, predict the minimum number of years for the initial investment of \$1000 to double.

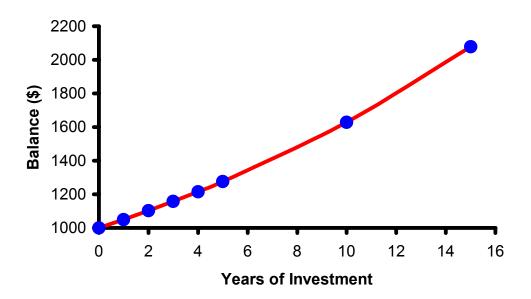


Balance vs Years of Investment

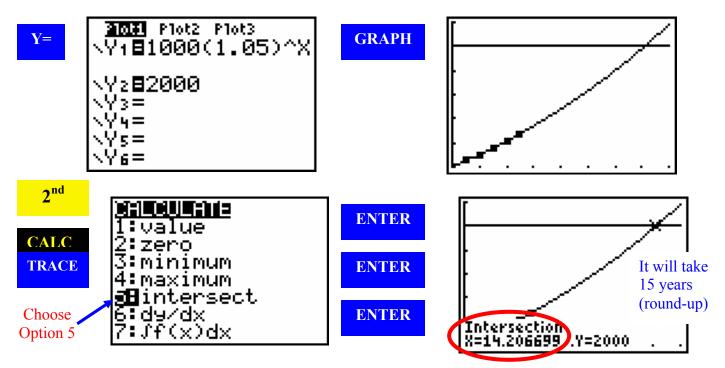
a.

- b. For every year it is invested, the balance increases by 5% of the previous year's balance.
- c. $A = \$1000 (1.05)^n$ where A = Balance and n = number of years invested.

Balance vs Years of Investment



- d. The data is discrete because interest is paid out at the end of the year.
- e. Using the equation, $A = \$1000 (1.05)^n A = \$1000 (1.05)^{10} = \$1628.89$
- f. By drawing another line, $Y_1 = 2000$ and Run the Intersect Function,



a.

(AP) Example 3: For the relation $(x - 3)^2 + (y + 5)^2 = 16$

- a. Solve for *y*
- b. Graph the resulting equations.
- c. Describe the shape, domain, and range of the graph.

b.

- d. If x = 2, what could be the value(s) for y?
- e. If y = -4, what could be the value(s) for x?

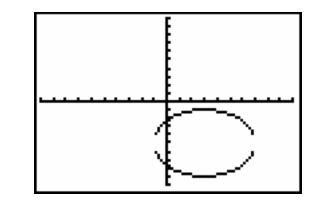
$$(x-3)^{2} + (y+5)^{2} = 16$$
$$(y+5)^{2} = 16 - (x-3)^{2}$$
$$y+5 = \pm \sqrt{(16 - (x-3)^{2})}$$
$$y = \pm \sqrt{(16 - (x-3)^{2})} - 5$$
$$\mathbf{V}_{1} = \sqrt{(16 - (x-3)^{2})} - 5$$

$$Y_1 = \sqrt{\left(16 - (x - 3)^2\right)} - 5$$

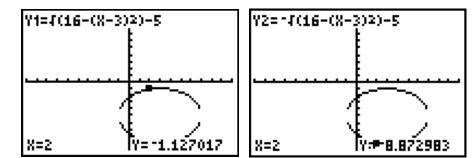
$$Y_2 = -\sqrt{\left(16 - (x - 3)^2\right)} - 5$$

c. CIRCLE (the graph appears to be an ellipse because the calculator screen is a rectangle; each x interval is longer than the equivalent y interval).

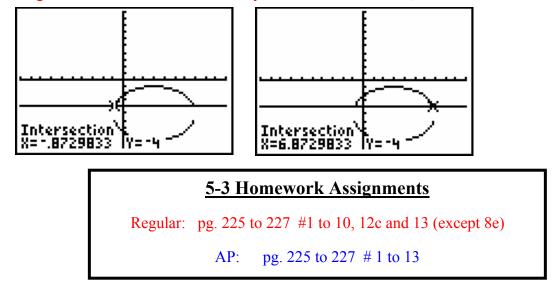
> Domain: $-1 \le x \le 7$ Range: $-9 \le y \le -1$



d. Using the TRACE function for both equations, y = -1.13 and -8.87



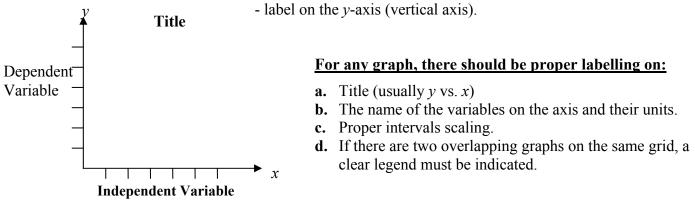
e. Using the INTERSECT function for equations Y_1 and Y_3 twice, x = -0.87 and 6.87



5-4: General Relations

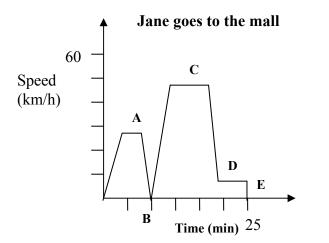
Independent (Manipulated) Variable: - a variable that you change in a situation to cause an effect. - label on the *x*-axis (horizontal axis).

Dependent (Responding) Variable: - a variable that you measure because of the changes you caused with the manipulated variable.

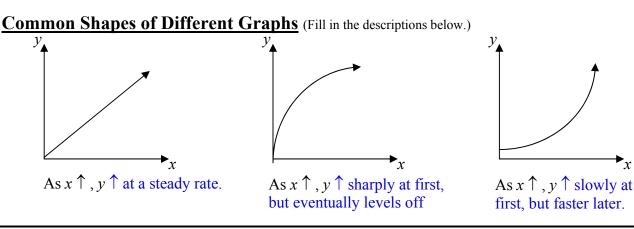


Creating a Scenario to Match a Graph

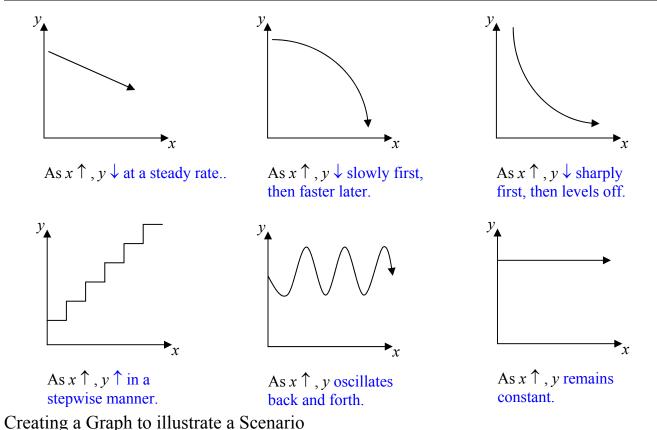
Example 1: Jane is driving to the shopping mall from her house. Using the graph, write a scenario that would describe her travel.



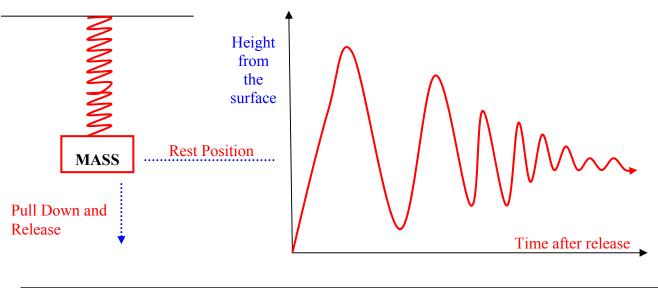
- A Jane Left her house and drove at 27 km/h (playground zone).
- **B** She stopped at the stop sign.
- **C** Jane drove at 50 km/h (residential zone).
- **D** She entered the parking lot of the mall and was looking for a parking spot (10 km/h).
- **E** Jane parked and stopped.



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Example 2: Imagine a mass attached to a spring. It occupies a position at rest above a level surface. If the mass is pulled down and then released, it will move up and down. Sketch the graph to represent the relationship between the height of the mass above the surface and the time after its release.



<u>5-4 Homework</u>	Assignments
Regular: pg. 229 to 231 #1 to 11a, 11c, 12	AP: pg. 229 to 231 # 1 to 11a, 11c, 12

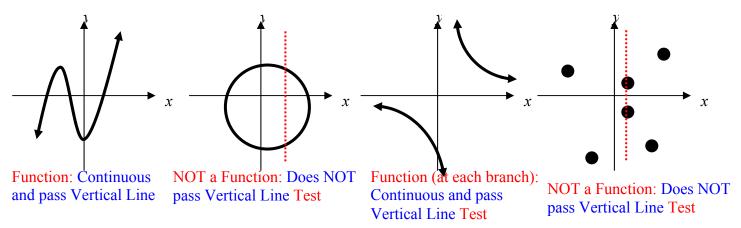
5-5: Functions

Relation: - an equation that explains how one variable (input *x*) can turn into another variable (output *y*).

Function: - a special relation that must satisfy the following two conditions:

- **a.** The graph is **continuous** (no break unless stated in the domain and range).
- b. For each input, there is only one unique output.
 (Vertical Line Test If a vertical line moves from left to right of the graph and it did not cross the graph at two different points, then we can say the graph passed the vertical line test).

Example 1: State whether each of the graphs below is a function. Provide reasons.



Function Notation: - a way to express an equation to denote that it is a function (satisfies requirements of continuity and vertical line test).

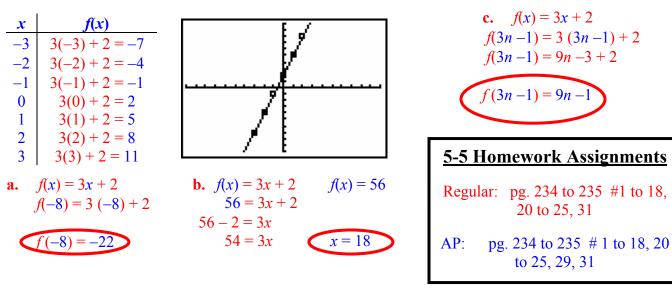
- instead of writing y, we can write f(x).
- we can say "Function f of x" or "f as a function of x".
- a number to be put in to replace $x \inf f(x)$ for the purpose of substitution.

c. Find f(3n-1).

Example 2: Given f(x) = 3x + 2, set up a Table of Values for x = -3 to x = 3. Graph f(x)

a. Find f(-8).

b. Find x when f(x) = 56



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<u>5-6: Applications of Linear Functions</u>

Direct Variation: - a variable that varies directly (by a constant rate of change) with another variable.

$$y \propto x$$
 (y is directly proportional to x) or $y = kx$

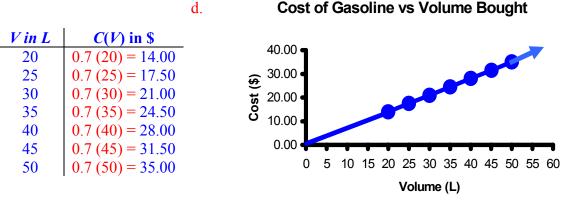
Example 1: Gasoline at one time costs \$0.70 per Litre.

- a. Write out the cost of gasoline as a function of volume bought.
- b. What is the constant of variation?
- c. Set up a table of values from V = 20 L to V = 50 L with a scale of 5 L.
- d. Graph the function.
- e. Find the cost of 63 L of gasoline.
- f. How much gasoline can you buy with \$26.43?

a. C(V) = 0.7 V

b. Constant of Variation = 0.70/L (unit price of gasoline)





Example 2: The amount of fuel used by a vehicle varies directly with the distance travelled. On a particular trip, 42.35 L of gasoline is used for a distance of 516.5 km.

- a. Calculate the constant of variation.
- b. Find the function of volume of gasoline used in terms of distance travelled.
- c. What is the distance travelled if 35 L of gasoline is used?
- d. How much would it cost to fuel up the car when the price of gasoline was \$65.90 / 100 L if the entire trip was 631 km?

a.
$$V(d) = kd$$

 $42.35 = k (516.5)$
 $\frac{42.35L}{516.5km} = k$
 $k = 0.082 L/km$
b. $V(d) = kd$
 $V(d) = 0.082d$
c. $V = 35 L, d = ?$
 $35 = 0.082 d$
 $\frac{35}{0.082} = d$
 $d. d = 631 km, V = ?$
 $V = 0.082 (631)$
 $V = 51.742 L$
 $\frac{$65.90}{100L} = \frac{x}{51.742L}$
 $x = 34.10

Partial Variation: - a variable that *varies partially* (by a constant rate of change and a fixed amount) with another variable.

$$y = kx + b$$

where k = constant of variation (constant of proportionality – rate of change) b = fixed amount (initial amount when x = 0)

- Example 3: The cost of a school dance organized by the student council \$5.50 per person and \$300 to hire the DJ.
 - a. What is the constant of variation and the fixed amount?
 - b. Express the cost as a function of the number of people attending.
 - c. Set up a table of values from n = 0 to n = 100 people with a scale of 20 people.
 - d. Graph the function.
 - e. Explain whether the graph should be discrete or continuous.
 - f. State the Domain and Range.
 - g. How many people attended if the cost was \$1125?
- a. Constant of Variation = 5.50 / person (cost per person) Fixed Amount = 300 (fixed cost) b. C(n) = 5.50n + 300

c.			d.			Cost of	a Sch	ool D	ance		
-	<i>n</i> 0 20 40 60 80 100	C(n) in \$ 5.50 (0) +300 = 300 5.50 (20) +300 = 410 5.50 (40) +300 = 520 5.50 (60) +300 = 630 5.50 (80) +300 = 740 5.50 (100) +300 = 850		Cost (\$)	\$1,000 \$800 \$600 \$400 \$200 \$0	, -•	•-	•	•	-•••	
					0	20	40	60	80	100	120
							n	(peopl	e)		

- e. The graph should be discrete because you can not have a decimal number to describe the number of people attending.
- f. Domain: $n \in W$ Range: C = 5.50n + 300 where $n \in W$ or $\{300, 305.5, 311, 316.5, 322 \dots\}$

g.
$$C(n) = $1125, n =?$$

$$C(n) = 5.50n + 300$$

$$1125 = 5.50n + 300$$

$$1125 - 300 = 5.50n$$

$$825 = 5.50n$$

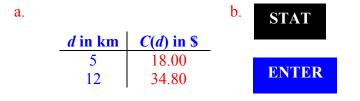
$$n = \frac{825}{5.50}$$

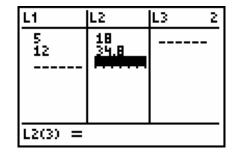
$$n = 150 \text{ people}$$

Relations and Functions

Example 4: A taxi ride costs you \$18.00 for 5 km travelled, and \$34.80 if you travelled 12 km.

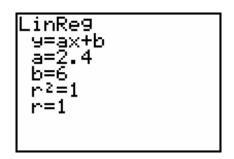
- a. Express the above information in a table.
- b. Enter the table into the STAT menu of your graphing calculator.
- c. Using the Appropriate *WINDOW* settings, graph the data.
- d. Run LINEAR REGRESSION to obtain an equation.
- e. Express the equation as a function of Cost in terms of distance travelled.
- f. What is the constant of variation and the fixed amount?
- g. How much would it cost for a 20 km ride to the airport?
- h. If the ride costs \$32.50, what is the distance travelled?





c. x: [0, 15, 1] and y: [0, 40, 5] d.





- C(d) = 2.4d + 6 f. Constant of Variation = \$2.40/km (cost per km travelled) Fixed Amount = \$6.00 (Flat Rate)
- g. d = 20 km, C = ? C(d) = 2.4d + 6 C(d) = 2.4(20) + 6 C(d) = \$54.00 C(d) = \$54.00 C(d) = \$54.00 C(d) = \$2.4d + 6 32.50 = 2.4d + 6 32.50 - 6 = 2.4d 26.5 = 2.4d $\frac{26.5}{2.4} = d$ d = 11.04 km

<u>5-6 Ho</u>	omework Assignments
Regular:	pg. 238 to 240 #1 to 21, 24
AP:	pg. 238 to 240 # 1 to 26

e.