Unit 5: Sequences

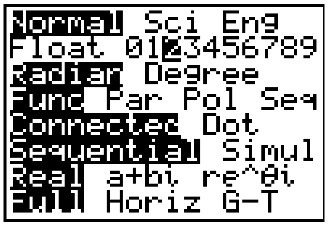
2-1A: Patterns in Table

Non-Recursive Tables: - tables where the result at the end of one row does **NOT** affect the beginning of the next row.

Example 1: John bought the following items in Vancouver. In the province of British Columbia, the 8% PST (Provincial Sales Tax) and the 7% GST (Goods and Service Tax –Federal Sales Tax) are calculated on the sales price separately. Complete the following table below.

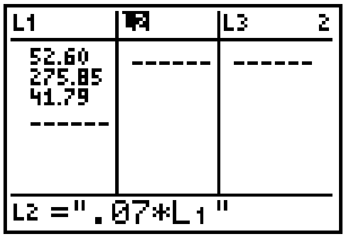
Item	**Sales Price**	**GST**	**PST**	**Total**
4 CDs	$52.60	**$3.68**	**$4.20**	**$60.49**
Boom Box	$275.85	**$19.31**	**$22.07**	**$317.23**
2 DVDs	$41.79	**$2.93**	**$3.34**	**$48.06**
			Total	**$425.78**

We can use the calculator to compute values in a non-recursive table. Since we are dealing with money, we can set our decimal places to 2.



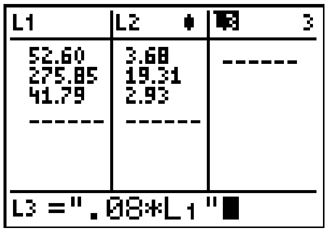
```
Normal Sci Eng
Float 0123456789
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi re^θi
Full Horiz G-T
```

Next, go to the **STAT** menu, and select **Edit...** Enter the prices in **L**$_1$. Take the cursor to the top of L$_2$, and enter the formula to find the GST. Formulas have to be entered with quotation marks.



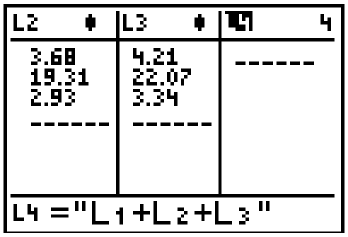
```
L1      L2      L3      2
52.60   ------  ------
275.85
41.79
------
L2 =".07*L1"
```

Repeat with the heading of **L**$_3$. Enter the formula to find the PST.



```
L1      L2      L3      3
52.60   3.68    ------
275.85  19.31
41.79   2.93
------  ------
L3 =".08*L1"
```

Finally, enter the formula to find the total of each row under the heading of **L**$_4$.

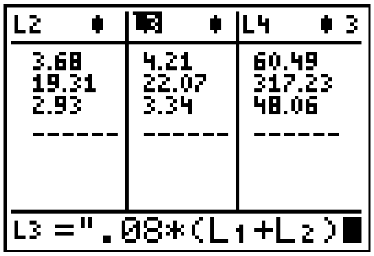


```
L2      L3      L4      4
3.68    4.21    ------
19.31   22.07
2.93    3.34
------  ------
L4 ="L1+L2+L3"
```

Example 2: Jacque bought the following items in Montreal. In the province of Quebec, the 8% PST is calculated on the price with the 7% GST. Complete the following table below.

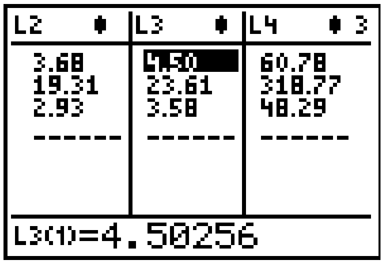
Item	**Sales Price**	**GST**	**PST**	**Total**
4 CDs	$52.60	**$3.68**	**$4.50**	**$60.78**
Boom Box	$275.85	**$19.31**	**$23.61**	**$318.77**
2 DVDs	$41.79	**$2.93**	**$3.58**	**$48.29**
			Total	**$427.84**

Since the PST is calculated on the price with the GST, we need to modify the formula in **L**$_3$.



```
L2      L3      L4      3
3.68    4.21    60.49
19.31   22.07   317.23
2.93    3.34    48.06
------  ------  ------
L3 =".08*(L1+L2)
```

Press **ENTER** will modify the table accordingly.



```
L2      L3      L4      3
3.68    4.50    60.78
19.31   23.61   318.77
2.93    3.58    48.29
------  ------  ------
L3(1)=4.50256
```

Example 3: John invested $2000 in a simple interest saving bond. How much money will he have in 3 years if the interest rate is 4%/a?

Year	Opening Balance	Interest	Amount Withdrawal	Closing Balance
1	$2000	**$80**	**$80**	**$2000**
2	**$2000**	**$80**	**$80**	**$2000**
3	**$2000**	**$80**	**$80**	**$2000**

Total after 3 years = Total Withdrawals + Final Closing Balance
= $240 + $2000

$$= \boxed{\$2240}$$

Recursive Tables: - tables where the result at the end of one row **DOES** affect the beginning of the next row.

Compound Interest: - interests earned in every term are not withdrawn, but accumulated.
- the closing balance of each term is the opening balance of the next term.

$$A = P\,(1 + r)^{n}$$

A = Final Amount **P = Principal**
r = Rate Per Term **n = Total Number of Terms**

Term: - the period of time spent before interest is calculated.

Compound Term	Number of times interest is calculated in a year	Interest Rate per term (r = interest rate quoted per annum)
Annually	1	r
Semi-annually	2	$\frac{r}{2}$
Quarterly	4	$\frac{r}{4}$
Monthly	12	$\frac{r}{12}$
Daily	365	$\frac{r}{365}$

Example 4: Mary invested \$2000 compounded semi-annually for 3 years at 4%/a. Using the compound interest formula and the table below, calculate the value of her investment and the total interest earned at the end of the three years.

$A = P(1 + r)^n$ $P = \$2000$ $r = \frac{4\%}{2} = \frac{0.04}{2} = 0.02$ $n = 3$ years × 2 terms/year = 6 terms

$A = \$2000(1 + 0.02)^6$ $A = \$2252.32$

Total Interest Earned = \$2252.32 – \$2000.00 **Net Gain = \$252.32**

Year	Opening Balance	Interest	Additional Investment	Closing Balance
0.5	\$2000.00	**\$40.00**	–	**\$2040.00**
1.0	**\$2040.00**	**\$40.80**	–	**\$2080.80**
1.5	**\$2080.80**	**\$41.62**	–	**\$2122.42**
2.0	**\$2122.42**	**\$42.45**	–	**\$2164.87**
2.5	**\$2164.87**	**\$43.30**	–	**\$2208.17**
3.0	**\$2208.17**	**\$44.16**	–	**\$2252.33**
	TOTAL	**\$252.33**		

The difference between the table value and the value calculated by the formula is due to the successive rounding off in the table.

Example 5: Using the compound interest formula, calculate the value of her investment and the total interest earned at the end of the three years, if Mary was to invest \$2000 for 3 years at 4%/a

a. compounded quarterly.

$P = \$2000$ $r = \frac{4\%}{4} = \frac{0.04}{4} = 0.01$

$n = 3$ years × 4 terms/year = 12 terms

$A = \$2000(1 + 0.01)^{12}$ $A = \$2253.65$

Total Interest = \$2253.65 – \$2000.00

Net Gain = \$253.65

b. compounded monthly.

$P = \$2000$ $r = \frac{4\%}{12} = \frac{0.04}{12}$

$n = 3$ years × 12 terms/year = 36 terms

$A = \$2000\left(1 + \frac{0.04}{12}\right)^{36}$ $A = \$2254.54$

Total Interest = \$2254.54 – \$2000.00

Net Gain = \$254.54

2-1A Homework Assignments

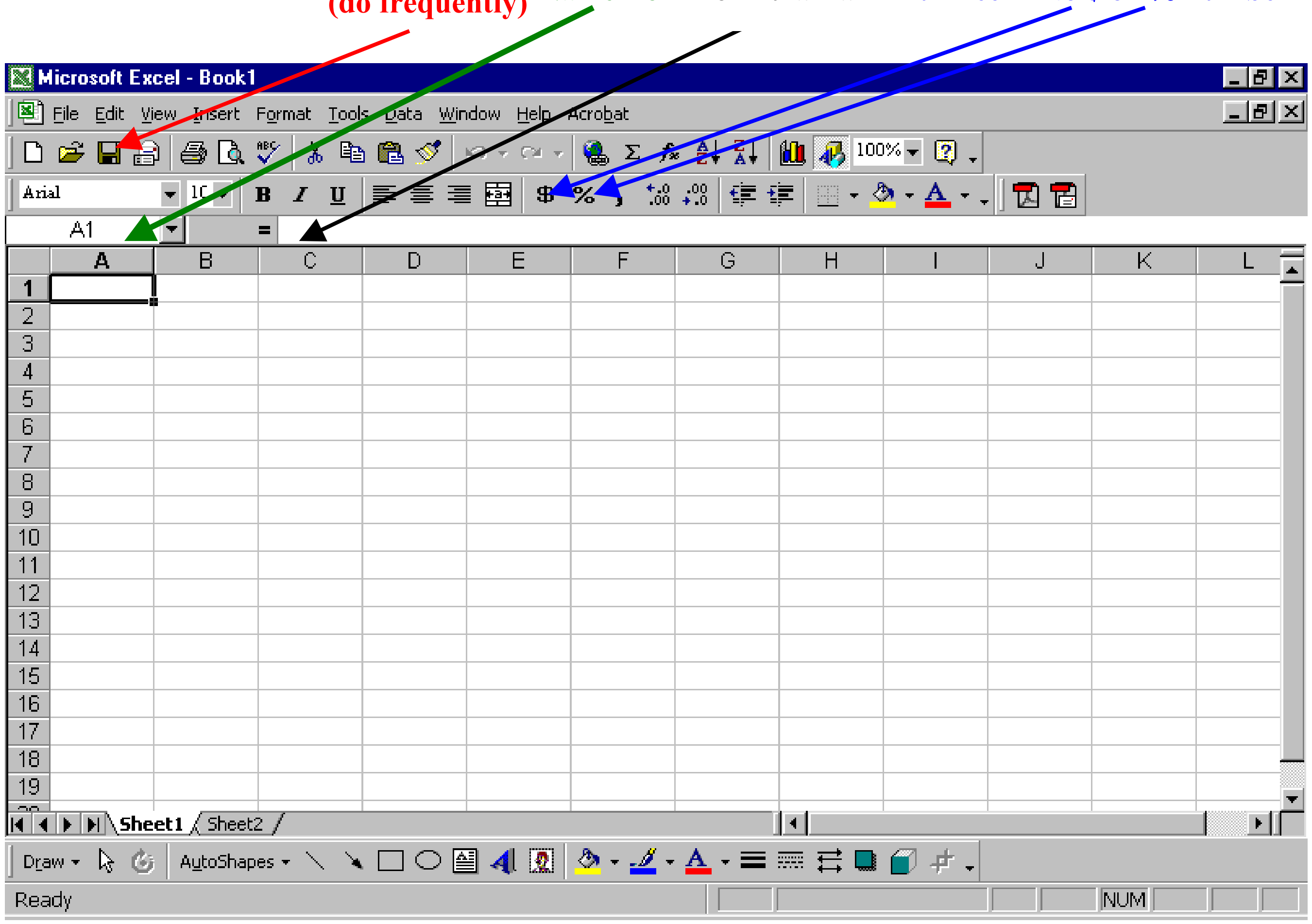
Regular: pg. 57 to 58 #1 to 9, 11 to 14, 15, 16a to 16c.

AP: pg. 57 to 58 #1 to 9, 11 to 14, 15, 16a to 16c.

2-1B: Patterns in Spreadsheets

Spreadsheet: - a computer table software that allows the user to manage and calculate on columns and rows of data.

Using Microsoft EXCEL



SAVE (do frequently)

Name Box

Formula Bar

Turn cell into $ or % number

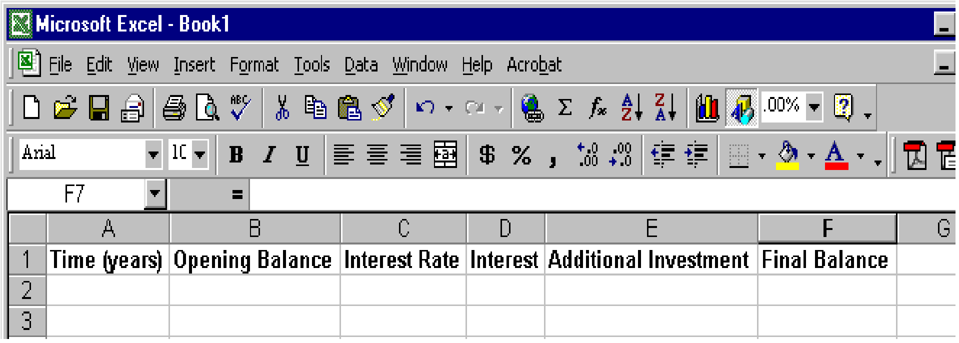
Different Forms of Investment and Loan Payments

Lump Sum: - a form of investment where the investor put a sum of money at the beginning of the term but makes no additional contribution.
- a form of loan payment where the entire loan and interest are paid at the end of the loan period.
- you can use the Compound Interest Formula $A = (1 + r)^n$.

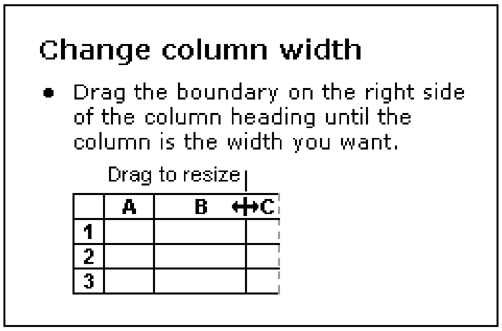
Annuity: - a form of investment where the investor puts in money at a regular interval.
- a form of loan repayment where the lender makes payments at a regular interval until the loan is paid off.
- you **CANNOT** use the Compound Interest Formula. You must **USE SPREADSHEET** or work it out on a table by hand.

Example 1: John has $5000 to save initially and he would like to make an additional investment of $1200 to his investment account every three months for 3 years. If his investment earns him 8%/a compounded quarterly, create a spreadsheet to find his final balance at the end of the 3 years.

1. Type in the column headings. You can bold the fonts (highlight all the cells and click the Bold icon). Adjust you column width (see below).



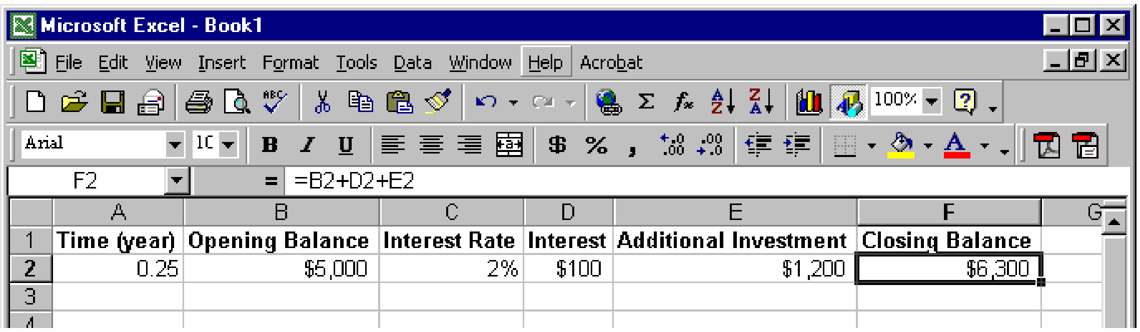
	A	B	C	D	E	F	G
1	Time (years)	Opening Balance	Interest Rate	Interest	Additional Investment	Final Balance	
2							
3							



Change column width

- Drag the boundary on the right side of the column heading until the column is the width you want.

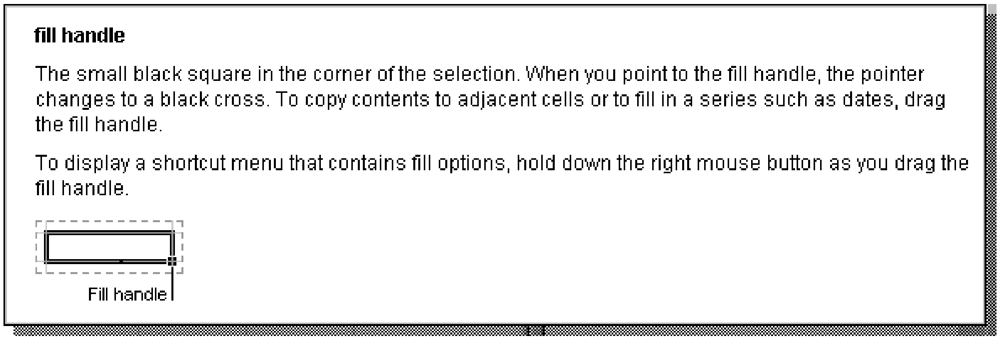
2. In A2, type **0.25**. In B2, type **$5000**. In C2, type **2%**. In D2, a formula will be entered into the cell. Since *I* = *Prt*, the calculation is the **Interest = Opening Balance * Interest Rate**. In D2, type **=B2*C2** (all formulas in EXCEL require an equal sign at the beginning) Since C2 is already in percentage, we do not need to divide by 100 in the formula for D2. Press Enter. In E2, type **$1200**. In F2, type **=B2+D2+E2**. Adjust column width if needed (if a series of #### appears or the number has been rounded, this is because the column is not wide enough).



= =B2+D2+E2

	A	B	C	D	E	F	G
1	Time (year)	Opening Balance	Interest Rate	Interest	Additional Investment	Closing Balance	
2	0.25	$5,000	2%	$100	$1,200	$6,300	
3							
4							

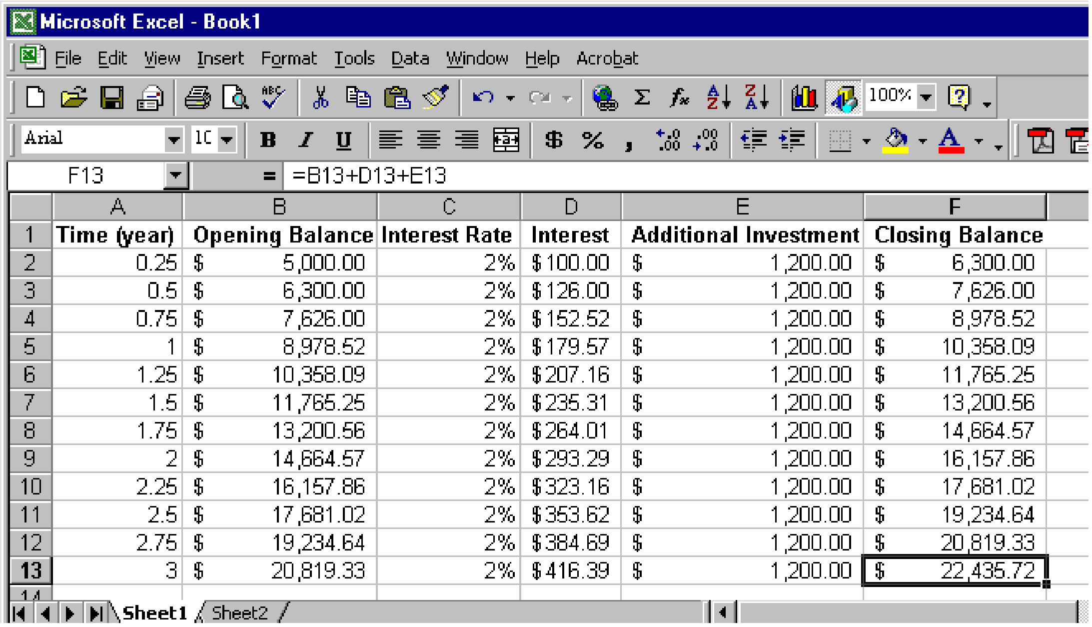
3. Columns B, D, E and F deal with money. To change the appearance to two decimal places, click on the dollar icon **($)** when each of the columns is highlighted. In A3, **Type = A2+0.25**. Go to the right bottom corner of the cell A3. Drag down the fill handle to A13 (see below). In B3, type **=F2**. Do the fill handle again in column B until you reach B13. Drag C2 using the fill handle to C13. Repeat with D2 and drag the fill handle to D13. Drag E2 using the fill handle to E13. Repeat with F2 and drag the fill handle to F13. The spreadsheet should look as follows on the next page.



fill handle

The small black square in the corner of the selection. When you point to the fill handle, the pointer changes to a black cross. To copy contents to adjacent cells or to fill in a series such as dates, drag the fill handle.

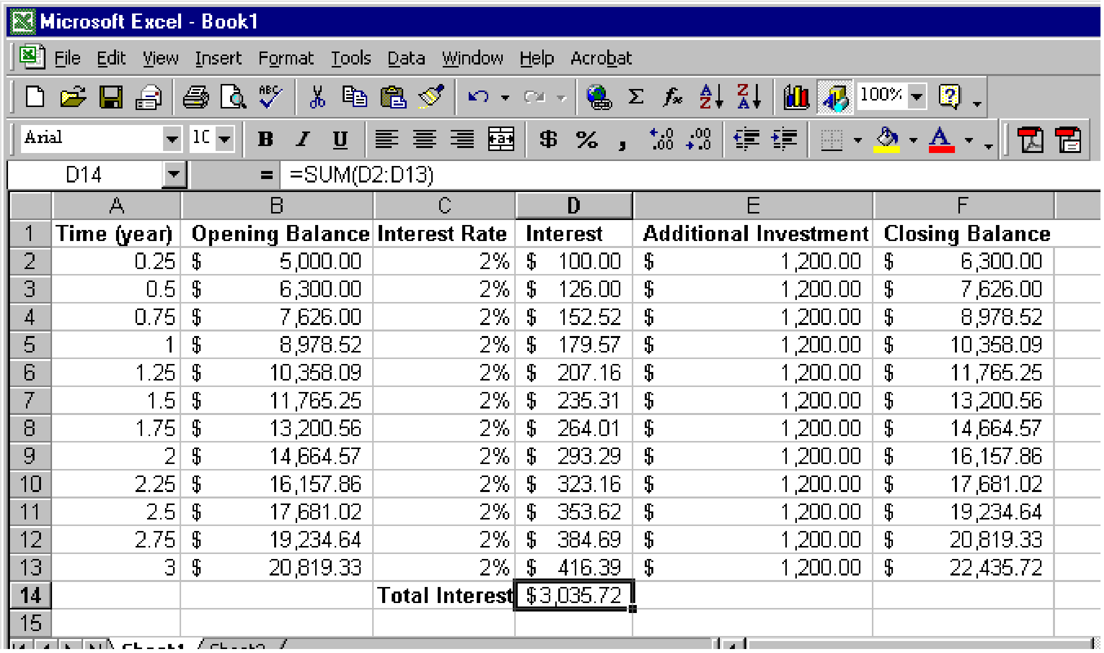
To display a shortcut menu that contains fill options, hold down the right mouse button as you drag the fill handle.



F13 = =B13+D13+E13

	A	B	C	D	E	F
1	Time (year)	Opening Balance	Interest Rate	Interest	Additional Investment	Closing Balance
2	0.25	$ 5,000.00	2%	$100.00	$ 1,200.00	$ 6,300.00
3	0.5	$ 6,300.00	2%	$126.00	$ 1,200.00	$ 7,626.00
4	0.75	$ 7,626.00	2%	$152.52	$ 1,200.00	$ 8,978.52
5	1	$ 8,978.52	2%	$179.57	$ 1,200.00	$ 10,358.09
6	1.25	$ 10,358.09	2%	$207.16	$ 1,200.00	$ 11,765.25
7	1.5	$ 11,765.25	2%	$235.31	$ 1,200.00	$ 13,200.56
8	1.75	$ 13,200.56	2%	$264.01	$ 1,200.00	$ 14,664.57
9	2	$ 14,664.57	2%	$293.29	$ 1,200.00	$ 16,157.86
10	2.25	$ 16,157.86	2%	$323.16	$ 1,200.00	$ 17,681.02
11	2.5	$ 17,681.02	2%	$353.62	$ 1,200.00	$ 19,234.64
12	2.75	$ 19,234.64	2%	$384.69	$ 1,200.00	$ 20,819.33
13	3	$ 20,819.33	2%	$416.39	$ 1,200.00	$ 22,435.72

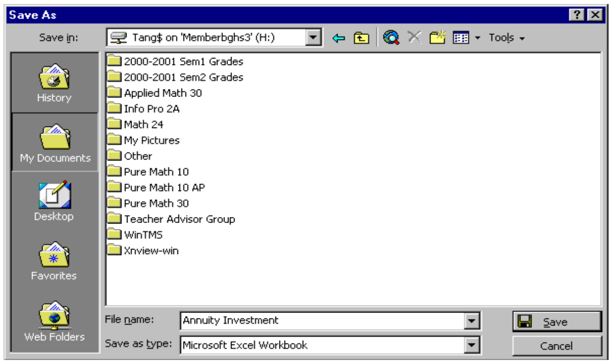
4. To calculate the Total Interest Earned, in C14, type Total Interest. In D14, type = SUM(D2:D13). This will sum cell D2 to D13. Your final spreadsheet should look as follows.



D14 = =SUM(D2:D13)

	A	B	C	D	E	F
1	Time (year)	Opening Balance	Interest Rate	Interest	Additional Investment	Closing Balance
2	0.25	$ 5,000.00	2%	$ 100.00	$ 1,200.00	$ 6,300.00
3	0.5	$ 6,300.00	2%	$ 126.00	$ 1,200.00	$ 7,626.00
4	0.75	$ 7,626.00	2%	$ 152.52	$ 1,200.00	$ 8,978.52
5	1	$ 8,978.52	2%	$ 179.57	$ 1,200.00	$ 10,358.09
6	1.25	$ 10,358.09	2%	$ 207.16	$ 1,200.00	$ 11,765.25
7	1.5	$ 11,765.25	2%	$ 235.31	$ 1,200.00	$ 13,200.56
8	1.75	$ 13,200.56	2%	$ 264.01	$ 1,200.00	$ 14,664.57
9	2	$ 14,664.57	2%	$ 293.29	$ 1,200.00	$ 16,157.86
10	2.25	$ 16,157.86	2%	$ 323.16	$ 1,200.00	$ 17,681.02
11	2.5	$ 17,681.02	2%	$ 353.62	$ 1,200.00	$ 19,234.64
12	2.75	$ 19,234.64	2%	$ 384.69	$ 1,200.00	$ 20,819.33
13	3	$ 20,819.33	2%	$ 416.39	$ 1,200.00	$ 22,435.72
14			Total Interest	$3,035.72		
15						

5. **Save** the spreadsheet in your **H:** Drive and name it **Annuity Investment**.

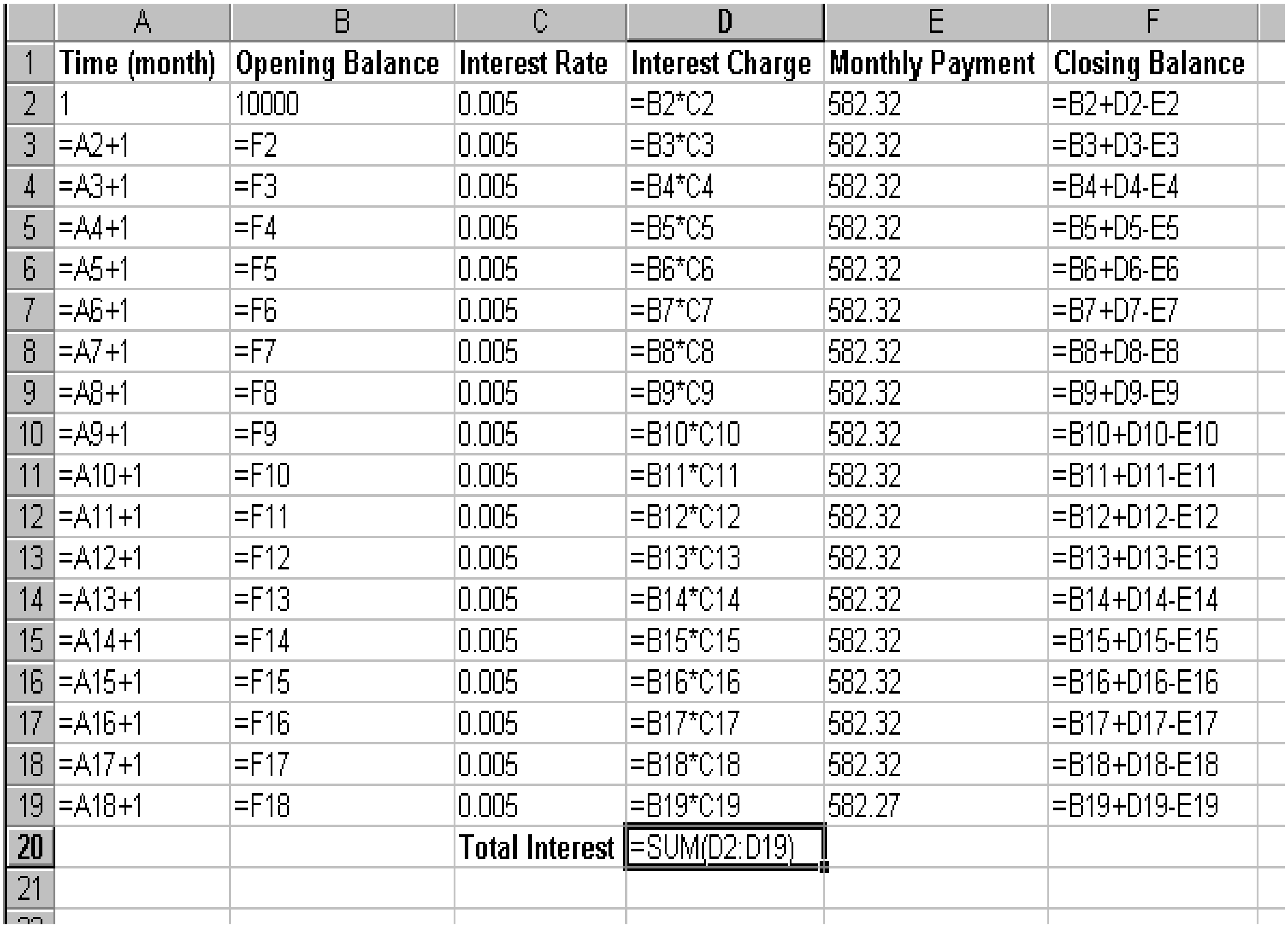


Save in: Tang$ on 'Memberbghs3' (H:)

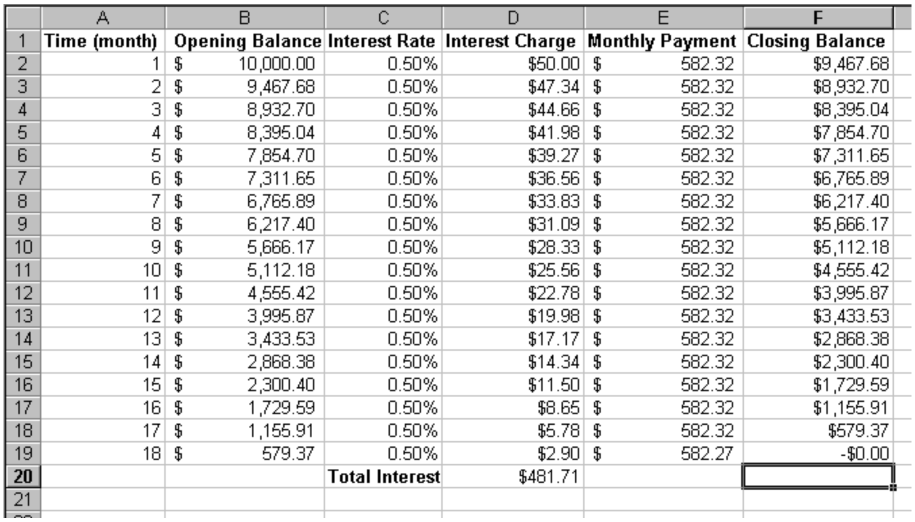
2000-2001 Sem1 Grades
2000-2001 Sem2 Grades
Applied Math 30
Info Pro 2A
Math 24
My Pictures
Other
Pure Math 10
Pure Math 10 AP
Pure Math 30
Teacher Advisor Group
WinTMS
Xnview-win

File name: Annuity Investment
Save as type: Microsoft Excel Workbook

Example 2: Mary borrowed $10000 for her last year in college at 6%/a compounded monthly. She would like to repay it in 18 months by making monthly payment. Using a spreadsheet, find out the amount of her monthly payment and the total interest she paid over the 18 months. Write down any numbers and formulas you have input onto the table below. Save the resulting file in your H: drive as **Loan Repayment Schedule**.



	A	B	C	D	E	F
1	Time (month)	Opening Balance	Interest Rate	Interest Charge	Monthly Payment	Closing Balance
2	1	10000	0.005	=B2*C2	582.32	=B2+D2-E2
3	=A2+1	=F2	0.005	=B3*C3	582.32	=B3+D3-E3
4	=A3+1	=F3	0.005	=B4*C4	582.32	=B4+D4-E4
5	=A4+1	=F4	0.005	=B5*C5	582.32	=B5+D5-E5
6	=A5+1	=F5	0.005	=B6*C6	582.32	=B6+D6-E6
7	=A6+1	=F6	0.005	=B7*C7	582.32	=B7+D7-E7
8	=A7+1	=F7	0.005	=B8*C8	582.32	=B8+D8-E8
9	=A8+1	=F8	0.005	=B9*C9	582.32	=B9+D9-E9
10	=A9+1	=F9	0.005	=B10*C10	582.32	=B10+D10-E10
11	=A10+1	=F10	0.005	=B11*C11	582.32	=B11+D11-E11
12	=A11+1	=F11	0.005	=B12*C12	582.32	=B12+D12-E12
13	=A12+1	=F12	0.005	=B13*C13	582.32	=B13+D13-E13
14	=A13+1	=F13	0.005	=B14*C14	582.32	=B14+D14-E14
15	=A14+1	=F14	0.005	=B15*C15	582.32	=B15+D15-E15
16	=A15+1	=F15	0.005	=B16*C16	582.32	=B16+D16-E16
17	=A16+1	=F16	0.005	=B17*C17	582.32	=B17+D17-E17
18	=A17+1	=F17	0.005	=B18*C18	582.32	=B18+D18-E18
19	=A18+1	=F18	0.005	=B19*C19	582.27	=B19+D19-E19
20			**Total Interest**	=SUM(D2:D19)		
21						



	A	B	C	D	E	F
1	**Time (month)**	**Opening Balance**	**Interest Rate**	**Interest Charge**	**Monthly Payment**	**Closing Balance**
2	1	$ 10,000.00	0.50%	$50.00	$ 582.32	$9,467.68
3	2	$ 9,467.68	0.50%	$47.34	$ 582.32	$8,932.70
4	3	$ 8,932.70	0.50%	$44.66	$ 582.32	$8,395.04
5	4	$ 8,395.04	0.50%	$41.98	$ 582.32	$7,854.70
6	5	$ 7,854.70	0.50%	$39.27	$ 582.32	$7,311.65
7	6	$ 7,311.65	0.50%	$36.56	$ 582.32	$6,765.89
8	7	$ 6,765.89	0.50%	$33.83	$ 582.32	$6,217.40
9	8	$ 6,217.40	0.50%	$31.09	$ 582.32	$5,666.17
10	9	$ 5,666.17	0.50%	$28.33	$ 582.32	$5,112.18
11	10	$ 5,112.18	0.50%	$25.56	$ 582.32	$4,555.42
12	11	$ 4,555.42	0.50%	$22.78	$ 582.32	$3,995.87
13	12	$ 3,995.87	0.50%	$19.98	$ 582.32	$3,433.53
14	13	$ 3,433.53	0.50%	$17.17	$ 582.32	$2,868.38
15	14	$ 2,868.38	0.50%	$14.34	$ 582.32	$2,300.40
16	15	$ 2,300.40	0.50%	$11.50	$ 582.32	$1,729.59
17	16	$ 1,729.59	0.50%	$8.65	$ 582.32	$1,155.91
18	17	$ 1,155.91	0.50%	$5.78	$ 582.32	$579.37
19	18	$ 579.37	0.50%	$2.90	$ 582.27	-$0.00
20			**Total Interest**	$481.71		
21						

2-1B Homework Assignments

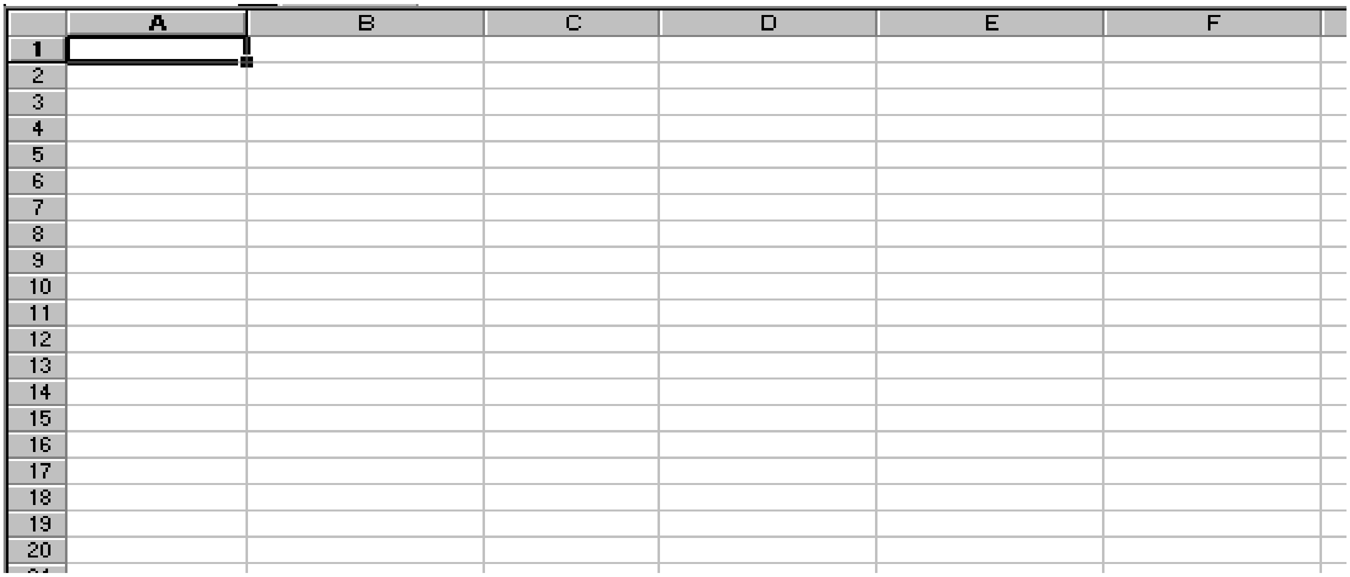
Regular: pg. 59 #17 and 2-1B Worksheet: Patterns in Spreadsheet

AP: pg. 59 #17 and 2-1B Worksheet: Patterns in Spreadsheet

2-1 B Worksheet: Patterns in Spreadsheet

1. Find the closing balance and the total interest earned at the end of 2 years, if the opening balance is \$6500 and the monthly investment is \$500 with the investment compounded monthly at 8%/a.

2. Bob got a loan of \$5000. He is charged 9%/a compounded monthly on his loan. Find the total interest charge and the monthly payment he has to make in order to pay is loan off in 1.5 years.



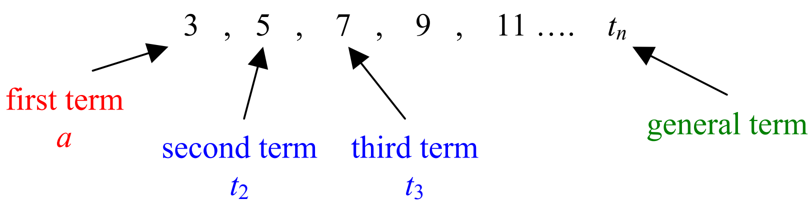
Answers: **1)** Final Balance = \$20590.37 Total Interest Earned = \$2090.37
2) Monthly Payment = \$297.99 Total Interest Charged = \$363.82

2-3: Sequences

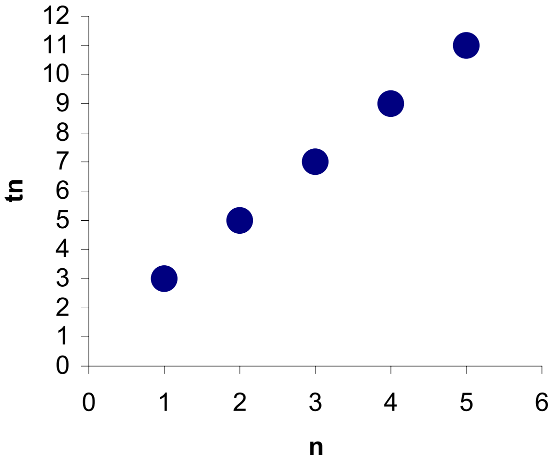
Sequence: - a list of numbers, terms, where it follows a certain pattern.

General Term (t_n): - the general rule that allows us to find the value of any particular term in the sequence.

Example 1: Given 3, 5, 7, 9, 11 …. t_n, find the equation for t_n.



We can express the above sequence as a graph.



n	t_n
0	1
1	3
2	5
3	7
4	9
5	11

−2

+2

+2

+2

t_n and n are replacing y and x

***y*-intercept**
***b* = 1**

Rate of Change
***k* = 2**

Discrete Data; Domain $n \in N$ (Natural Numbers)

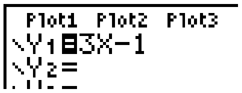
Using the format of Partial Variation, $y = kx + b$, we have

$$y = 2x + 1 \longrightarrow \boxed{t_n = 2n + 1}$$

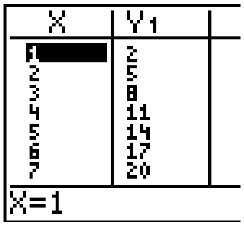
Example 2: Given the following general terms, find the first 5 terms of the sequence.

a. $t_n = 3n - 1$

$t_1 = 3\,(1) - 1$ $a = 2$
$t_2 = 3\,(2) - 1$ $t_2 = 5$
$t_3 = 3\,(3) - 1$ $t_3 = 8$
$t_4 = 3\,(4) - 1$ $t_4 = 11$
$t_5 = 3\,(5) - 1$ $t_5 = 14$



```
Plot1 Plot2 Plot3
\Y1■3X-1
\Y2=
```



```
  X   | Y1
  1   | 2
  2   | 5
  3   | 8
  4   | 11
  5   | 14
  6   | 17
  7   | 20
X=1
```

b. $t_n = -2n + 5$

$t_1 = -2\,(1) + 5$ $a = 3$
$t_2 = -2\,(2) + 5$ $t_2 = 1$
$t_3 = -2\,(3) + 5$ $t_3 = -1$
$t_4 = -2\,(4) + 5$ $t_4 = -3$
$t_5 = -2\,(5) + 5$ $t_5 = -5$

c. $t_n = n^2 - 4$

$t_1 = (1)^2 - 4$
$t_2 = (2)^2 - 4$
$t_3 = (3)^2 - 4$
$t_4 = (4)^2 - 4$
$t_5 = (5)^2 - 4$

$a = -3$
$t_2 = 0$
$t_3 = 5$
$t_4 = 12$
$t_5 = 21$

d. $t_n = 3^n$

$t_1 = 3^{(1)}$
$t_2 = 3^{(2)}$
$t_3 = 3^{(3)}$
$t_4 = 3^{(4)}$
$t_5 = 3^{(5)}$

$a = 3$
$t_2 = 9$
$t_3 = 27$
$t_4 = 81$
$t_5 = 243$

Example 3: Find the general term of the following sequences

a. 7, 4, 1, −2 …

n	**0**	1	2	3	4
t_n	**10**	7	4	1	−2

+3 −3 −3

Linear Relation

$y = -3x + 10$

$\boxed{t_n = -3n + 10}$

b. 1, 8, 27, 64 …

Non-Linear

Guess and Check

Square Numbers – No!
Cube Numbers – Yes!

$\boxed{t_n = n^3}$

c. $x - 2,\ 2x - 2,\ 3x - 2,\ 4x - 2$

(x) n	**0**	1	2	3	4
(y) t_n	**−2**	$x - 2$	$2x - 2$	$3x - 2$	$4x - 2$

$-x$ $+x$ $+x$

Linear Relation

$y = kx + b$

$\boxed{t_n = xn - 2}$

Recursive Sequence: - a sequence where the next term depends on the value of the previous term, t_{n-1}.

Example 4: Write the first five terms of the following recursive sequences.

a. $t_1 = 4$
$t_n = -2t_{n-1} + 5$

$t_1 = 4$
$t_2 = -2\,(4) + 5$
$t_3 = -2\,(-3) + 5$
$t_4 = -2\,(11) + 5$
$t_5 = -2\,(-17) + 5$

$a = 4$
$t_2 = -3$
$t_3 = 11$
$t_4 = -17$
$t_5 = 39$

b. $t_1 = -2$
$t_n = 3t_{n-1} - n$

$t_1 = -2$
$t_2 = 3\,(-2) - (2)$
$t_3 = 3\,(-8) - (3)$
$t_4 = 3\,(-27) - (4)$
$t_5 = 3\,(-85) - (5)$

$a = -2$
$t_2 = -8$
$t_3 = -27$
$t_4 = -85$
$t_5 = -260$

2-3 Homework Assignments

Regular: pg. 66 to 68 #1 to 13 (odd), 14, 15, 16 to 26 (even), 29 to 33 (odd) 35 to 49

AP: pg. 66 to 68 #2 to 12 (even), 14, 15, 17 to 27 (odd), 28 to 34 (even) 35 to 51, 53

2-5: Arithmetic Sequences

Arithmetic Sequence: - a sequence where the pattern is adding a fixed number (common difference).

$$t_n = a + (n - 1)d$$

t_n = value at the n^{th} term
n = number of terms
a = first term
d = common difference

Example 1: For the following sequences, find the next two terms, t_{25}, and, the general term.

a. 9, 13, 17, …

$a = 9$

$d = 4$ (add 4 to get the next term)

$\boxed{t_4 = 21 \text{ and } t_5 = 25}$

$$t_n = a + (n - 1)d$$

For $n = 25$:

$t_{25} = 9 + (25 - 1)(4)$

$= 9 + (24)(4)$

$= 9 + 96$

$\boxed{t_{25} = 105}$

For the general term t_n:

$t_n = 9 + (n - 1)(4)$

$= 9 + 4n - 4$

$\boxed{t_n = 4n + 5}$

b. 6, 1, −4, …

$a = 6$

$d = -5$ (subtract 5 or add −5 to get the next term)

$\boxed{t_4 = -9 \text{ and } t_5 = -14}$

$$t_n = a + (n - 1)d$$

For $n = 25$:

$t_{25} = 6 + (25 - 1)(-5)$

$= 6 + (24)(-5)$

$= 6 - 120$

$\boxed{t_{25} = -114}$

For the general term t_n:

$t_n = 6 + (n - 1)(-5)$

$= 6 - 5n + 5$

$\boxed{t_n = -5n + 11}$

c. $x - 1, x - 3, x - 5, \ldots$

$a = (x - 1)$

$d = -2$ (subtract 2 or add −2 to get the next term)

$\boxed{t_4 = (x - 7) \text{ and } t_5 = (x - 9)}$

$$t_n = a + (n - 1)d$$

For $n = 25$:

$t_{25} = (x - 1) + (25 - 1)(-2)$

$= (x - 1) + (24)(-2)$

$= x - 1 - 48$

$\boxed{t_{25} = (x - 49)}$

For the general term t_n:

$t_n = (x - 1) + (n - 1)(-2)$

$= (x - 1) - 2n + 2$

$\boxed{t_n = x - 2n + 1}$

d. $2y + 3, 3y - 1, 4y - 5, \ldots$

$a = (2y + 3)$

$d = (y - 4)$ (add $y - 4$ to get the next term)

$\boxed{t_4 = (5y - 9) \text{ and } t_5 = (6y - 13)}$

$$t_n = a + (n - 1)d$$

For $n = 25$:

$t_{25} = (2y + 3) + (25 - 1)(y - 4)$

$= (2y + 3) + (24)(y - 4)$

$= 2y + 3 + 24y - 96$

$\boxed{t_{25} = (26y - 93)}$

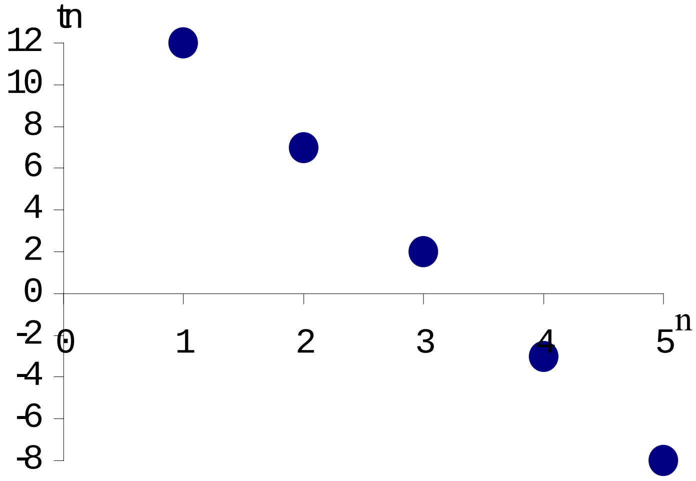
For the general term t_n:

$t_n = (2y + 3) + (n - 1)(y - 4)$

$= (2y + 3) + (ny - 4n - y + 4)$

$\boxed{t_n = ny + y - 4n + 7}$

Example 2: Using the graph below, find t_{38} and the general term.



$a = 12$

$d = -5$ (subtract 5 or add -5 to get the next term)

$$\boldsymbol{t_n = a + (n - 1)d}$$

For $n = 38$:

$t_{38} = 12 + (38 - 1)(-5)$

$= 12 + (37)(-5)$

$= 12 - 185$

$\boxed{\boldsymbol{t_{38} = -173}}$

For the general term t_n:

$t_n = 12 + (n - 1)(-5)$

$= 12 - 5n + 5$

$\boxed{\boldsymbol{t_n = -5n + 17}}$

Example 3: Find the first term of the sequence if $t_{10} = 15$ and $d = -3$.

$t_{10} = -15$ $n = 10$ $d = -3$ $a = ?$

$$\boldsymbol{t_n = a + (n - 1)d}$$

$t_{10} = \;\; -15 = a + (10 - 1)(-3)$

$-15 = a + (9)(-3)$

$-15 = a - 27$

$-15 + 27 = a$

$\boxed{\boldsymbol{a = 12}}$

Example 4: Every year, Mary gets a \$3000 raise in her annual salary (definitely not a teacher). If her starting salary was \$35000 and she is now making \$74000 as a senior manager, how long has she worked at this company?

$t_n = 74000$ $n = ?$ $d = 3000$ $a = 35000$

$$\boldsymbol{t_n = a + (n - 1)d}$$

$74000 = 35000 + (n - 1)(3000)$

$74000 = 35000 + 3000n - 3000$

$74000 = 3000n + 32000$

$74000 - 32000 = 3000n$

$42000 = 3000n$

$\dfrac{42000}{3000} = n$

$\boxed{\boldsymbol{n = 14 \text{ years}}}$

Arithmetic Means: - the terms between a pair of non-consecutive terms in an arithmetic sequence.

Example 5: Find the three arithmetic means between −12 and 10.

$$-12, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, 10$$

$a = -12$ $t_5 = 10$ $n = 5$ $d = ?$

$$\mathbf{t_n = a + (n-1)d}$$
$$t_5 = 10 = -12 + (5-1)\,d$$
$$10 = -12 + 4d$$
$$10 + 12 = 4d$$
$$22 = 4d$$
$$\frac{22}{4} = d$$

d = 5.5 (add 5.5 to get to the next term)

$$\boxed{\mathbf{-12, \underline{-6.5}, \underline{-1}, \underline{4.5}, 10}}$$

Example 6: In an arithmetic sequence, the third term is 12 and the eighth term is 47. Find the first term and the common difference.

$$\_\_\_\_\_, \_\_\_\_\_, 12, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, 47$$

We will pretend that the first term is 12, and 47 is the sixth term.

$a = 12$ $t_6 = 47$ $n = 6$ $d = ?$

$$\mathbf{t_n = a + (n-1)d}$$
$$t_6 = 47 = 12 + (6-1)\,d$$
$$47 = 12 + 5d$$
$$47 - 12 = 5d$$
$$35 = 5d$$
$$\frac{35}{5} = d$$

d = 7 (add 7 to get to the next term)
(subtract 7 to get to the previous term)

$$\mathbf{\underline{-2}, \underline{5}, 12, \underline{19}, \underline{26}, \underline{33}, \underline{40}, 47}$$

the real a

(AP) Example 7: In an arithmetic sequence, $t_{11} = 53$ and the sum of the 5th and the 7th terms is 56, Find the first term, common difference.

$a = ?$ $t_{11} = 53$ $t_5 + t_7 = 56$
$d = ?$

$$\mathbf{t_n = a + (n-1)d}$$
$$t_{11} = 53 = a + (11-1)\,d$$
$$53 = a + 10d$$

$$t_5 + t_7 = 56$$
$$[a + (5-1)d] + [a + (7-1)d] = 56$$
$$[a + 4d] + [a + 6d] = 56$$
$$2a + 10d = 56$$

$$\mathbf{53 - a = 10d} \xleftrightarrow{\textbf{EQUATE}} \mathbf{10d = 56 - 2a}$$

$$\mathbf{53 - a = 56 - 2a}$$
$$-a + 2a = 56 - 53$$
$$\boxed{\mathbf{a = 3}}$$

$$53 = a + 10d$$
$$53 = \mathbf{(3)} + 10d$$
$$50 = 10d$$
$$\boxed{\mathbf{d = 5}}$$

2-5 Homework Assignments

Regular: pg. 74 to 76 #1 to 45 (odd), 46 to 60

AP: pg. 74 to 76 #2 to 30 (even), 31 to 60 (except 33, 35, 39 and 41)

2-7: Arithmetic Series

Series: - the sum of the terms in a sequence.

Example 1: Find the sum of the sequence 6, 10, 14, 18, … , t_n up to the fifth term.

Sum of the first one term	$S_1 = 6$	$S_1 = 6$
Sum of the first two terms	$S_2 = 6 + 10$	$S_2 = 16$
Sum of the first three terms	$S_3 = 6 + 10 + 14$	$S_3 = 30$
Sum of the first four terms	$S_4 = 6 + 10 + 14 + 18$	$S_4 = 48$
Sum of the first five terms	$S_5 = 6 + 10 + 14 + 18 + 22$	$S_5 = 70$

Arithmetic Series: - a sum of the terms in an arithmetic sequence.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] \quad \text{or} \quad S_n = \frac{n}{2}\left[a + t_n\right]$$

S_n = value of the series to the n^{th} term

a = first term

n = number of terms

d = common difference

t_n = value of the sequence at the n^{th} term

Example 2: For the following series, find the value of S_{15} and the general series, S_n.

a. $9 + 13 + 17 + \ldots$

$a = 9$

$d = 4$ (add 4 to get the next term)

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

For $n = 15$:

$\boxed{S_{15} = 555}$

For the general series S_n:

$\boxed{S_n = 2n^2 + 7n}$

b. $6 + 1 - 4 + \ldots$

$a = 6$

$d = -5$ (subtract 5 or add -5 to get the next term)

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

For $n = 15$:

$\boxed{S_{15} = -435}$

For the general series S_n:

$$S_n = \frac{n}{2}\left[2(6) + (n-1)(-5)\right]$$

$$S_n = \frac{n}{2}\left[12 - 5n + 5\right]$$

$$S_n = \frac{n(-5n + 17)}{2}$$

$$\boxed{S_n = \frac{-5n^2 + 17n}{2}}$$

c. $(2y + 3) + (3y - 1) + (4y - 5) + \ldots$

$a = (2y + 3)$ $d = (y - 4)$ (add $y - 4$ to get the next term)

$$\boldsymbol{S_n = \frac{n}{2}\left[2a + (n-1)d\right]}$$

For $n = 15$:

$$S_{15} = \frac{15}{2}\left[2(2y+3) + (15-1)(y-4)\right]$$

$$S_{15} = \frac{15}{2}\left[4y + 6 + (14)(y-4)\right]$$

$$S_{15} = \frac{15}{2}\left[4y + 6 + 14y - 56\right]$$

$$S_{15} = \frac{15(18y - 50)}{2}$$

$$S_{15} = \frac{270y - 750}{2}$$

$$\boldsymbol{S_{15} = (135y - 375)}$$

For the general series S_n:

Example 3: Find the sum of the arithmetic series for 8 + 11 + 14 + …. + 245

$a = 8$ $d = 3$ $t_n = 245$ $n = ?$ $S_n = ?$

We will have to use the arithmetic sequence formula to find n first.

$$t_n = a + (n - 1)d$$
$$245 = 8 + (n - 1)(3)$$
$$245 = 8 + 3n - 3$$
$$245 = 3n + 5$$
$$245 - 5 = 3n$$
$$240 = 3n$$
$$\frac{240}{3} = n$$

$$\boldsymbol{n = 80}$$

Since we have t_n, we can use the second formula of the arithmetic series.

$$S_n = \frac{n}{2}(a + t_n)$$

$$S_{80} = \frac{80}{2}(8 + 245)$$

$$S_{80} = 40(253)$$

$$\boldsymbol{S_{80} = 10120}$$

Example 4: The front row of an auditorium has 20 seats, and each successive row has 5 more seats. Find the number of seats in the last row and the maximum seating capacity of this auditorium if it has a total of 40 rows.

$a = 20$ $d = 5$ $t_n = ?$ $n = 40$ $S_{40} = ?$

$$t_n = a + (n - 1)d$$
$$t_{40} = 20 + (40 - 1)(5)$$
$$t_{40} = 20 + (39)(5)$$
$$t_{40} = 20 + 195$$

$$\boldsymbol{t_{40} = 215}$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{40} = \frac{40}{2}\left[2(20) + (40-1)(5)\right]$$

$$S_{40} = 20\left[40 + (39)(5)\right]$$

$$S_{40} = 20\left[40 + 195\right]$$

$$S_{40} = 20\left[235\right]$$

OR

$$S_n = \frac{n}{2}(a + t_n)$$

$$S_{40} = \frac{40}{2}(20 + 215)$$

$$S_{40} = 20(235)$$

$$\boldsymbol{S_{40} = 4700}$$

Example 5: A store decided to display the latest flavour of can soup by stacking the items in a pyramid. If the top layer has one can of soup, and the bottom most layer has 75 cans of soup with each successive layer having two more cans than the layer above, find the total number of soups needed to complete the display.

$a = 1$ $d = 2$ $t_n = 75$ $n = ?$ $S_n = ?$

$$\begin{aligned} t_n &= a + (n-1)d \\ 75 &= 1 + (n-1)(2) \\ 75 &= 1 + 2n - 2 \\ 75 &= 2n - 1 \\ 75 + 1 &= 2n \\ 76 &= 2n \\ \frac{76}{2} &= n \end{aligned}$$

$$\boxed{n = 38}$$

$$\begin{aligned} S_n &= \frac{n}{2}\left(a + t_n\right) \\ S_{38} &= \frac{38}{2}\left(1 + 75\right) \\ S_{38} &= 19(76) \end{aligned}$$

$$\boxed{S_{38} = 1444}$$

(AP) Example 6: A child has 1162 wooden blocks of equal size with each in the shape of a cube. If she stacks the blocks in a shape of a cone with the top row having one cube and each successive row having 3 more blocks than before, how many rows will this cone have if she uses all 1162 wooden blocks?

$a = 1$ $d = 3$ $n = ?$ $S_n = 1162$

$$\begin{aligned} S_n &= \frac{n}{2}\left[2a + (n-1)d\right] \\ 1162 &= \frac{n}{2}\left[2(1) + (n-1)(3)\right] \\ 1162 &= \frac{n}{2}\left[2 + 3n - 3\right] \\ 1162 &= \frac{n(3n-1)}{2} \\ 2324 &= 3n^2 - n \end{aligned}$$

$0 = 3n^2 - n - 2324$ **(Quadratic equations. we have to factor to solve for *n*)**

$0 = (3n + 83)(n - 28)$

$$\begin{aligned} 3n + 83 &= 0 \\ 3n &= -83 \\ n &= \cancel{-27.66\ldots} \end{aligned}$$

(*n* cannot be negative)

$$n - 28 = 0$$

$$\boxed{n = 28}$$

2-7 Homework Assignments

Regular: pg. 82 to 83 #1 to 21 (odd), 22 to 29, 33

AP: pg. 82 to 83 #2 to 20 (even), 22 to 33

2-8: Geometric Sequences

Geometric Sequence: - a sequence where the pattern is multiplying a fixed number (***r* = common ratio**).

Example 1: For the following sequences, find the next two terms.

a. 3, 6, 12, …

$a = 3$
$r = 2$ (multiply 2 to get the next term)

$\boxed{\mathbf{t_4 = 24 \text{ and } t_5 = 48}}$

b. 4, −12, 36, −108, …

$a = 4$
$r = -3$ (multiply −3 to get the next term)

$\boxed{\mathbf{t_5 = 324 \text{ and } t_6 = -972}}$

c. 8, 4, 2, 1, …

$a = 8$
$r = 0.5$ or $\frac{1}{2}$ (divide by 2 or multiply $\frac{1}{2}$ to get the next term)

$\boxed{t_5 = 0.5 = \frac{1}{2} \text{ and } t_6 = 0.25 = \frac{1}{4}}$

d. $90, -10, \frac{10}{9}, \ldots$

$a = 90$
$r = \frac{1}{9}$ (divide by −9 or multiply $-\frac{1}{9}$ to get the next term)

$\boxed{t_4 = -\frac{10}{81} \text{ and } t_5 = \frac{10}{729}}$

Example 2: An average home is said to increase its value by 3.5% every year. If a $200,000 house is purchased today, how much will it be worth 5 years from now?

$a = \$200{,}000$
$r = 103.5\% = 1.035$ (multiply 1.035 to get the next term)

$t_2 = \$207,000$ $t_3 = \$214{,}245$ $t_4 = \$221{,}743.58$ $\boxed{\mathbf{t_5 = \$229{,}504.60}}$

Example 3: A filter can take out 80% of the impurities in any untreated water. If there were 900 mg of impurities in 5 L of untreated water, what is the amount of impurities remaining after the water passed through 4 filters?

$a = 900$ mg
$r = 100\% - 80\% = 20\% = 0.2$ (80% of impurities is removed which means 20% is still remaining)

$t_2 = 180$ mg (after 1 filter) $t_3 = 36$ mg (after 2 filters)
$t_4 = 7.2$ mg (after 3 filters) $t_5 = 1.44$ mg (after 4 filters)

$\boxed{\mathbf{t_6 = 0.288 \text{ mg (after 5 filters)}}}$

(AP) Example 4: The length and width of a picture is to be reduced to 85% of the original dimensions every time it passes through a photocopier. If the final area of the picture is about 27% of the original area, how many times has the picture been reduced by the photocopier?

a = 100% (Original Area)
$r = 85\% \times 85\% = 0.85 \times 0.85 = 0.7225$ (Area = length × width. Both dimensions are reduced to 85% each)

$t_2 = 72.25\%$ (area after 1 copy) $t_3 = 52.2\%$ (area after 2 copies)
$t_4 = 37.715\%$ (area after 3 copies)

$t_5 = 27.249\%$ (area after 4 copies)

Geometric Means: - the terms between a pair of non-consecutive terms in a geometric sequence.

Example 5: Find two geometric means between 6 and 162.

6, ______, ______, 162

$a = 6$ $t_4 = 162$ $r = ?$

$$162 = 6(r)(r)(r)$$
$$162 = 6r^3$$
$$\frac{162}{6} = r^3$$
$$27 = r^3$$
$$\sqrt[3]{27} = r$$

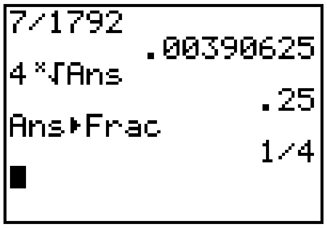
$r = 3$ **6, 18, 54, 162**

Example 6: Find three geometric means between 1792 and 7.

1792, ______, ______, ______, 7

$a = 1792$ $t_4 = 7$ $r = ?$

$$7 = 1792(r)(r)(r)(r)$$
$$7 = 1792r^4$$
$$\frac{7}{} = r^4$$
$$0.00390625 = r^4$$
$$\sqrt{0.00390625} = r$$



```
7/1792
         .00390625
4ˣ√Ans
               .25
Ans▸Frac
               1/4
█
```

$r = \pm 0.25 = \pm\frac{1}{4}$

1792, 448, 112, 28, 7 **or** **1792, –448, 112, –28, 7**

2-8 Homework Assignments

Regular: pg.84 Part 1: #1 to 14
and Part 2: #1 to 10
pg. 85 Part 3: #1 to 4
and Part 4: #2 and #3

AP: pg.84 Part 1: #1 to 14
and Part 2: #1 to 10
pg. 85 Part 3: #1 to 4
and Part 4: #2 and #3