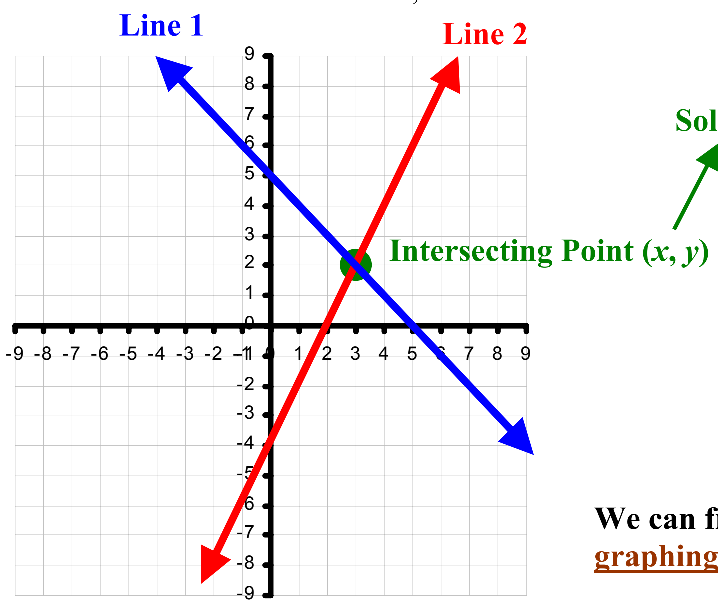
Unit 1: Linear and Non-Linear Systems

1.1: Solving Systems of Linear Equations Graphically

System of Linear Equations: - two or more linear equations on the same coordinate grid.

Solution of a System of Linear Equations:

- the **intersecting point** of two or more linear equations
- on the Cartesian Coordinate Grid, the solution contains two parts: the ***x*-coordinate** and the ***y*-coordinate** (can be expressed as an **Ordered Pair**)



Solution of the System of Linear Equations

We can find the solution of a system of linear equation graphing manually or using a graphing calculator.

Example 1: Find the solution of the following system of equations by graphing manually using

$$x + 2y = 6$$
$$x - y = 3$$

a. the slope and y-intercept form

For Line 1:

$$x + 2y = 6$$
$$2y = -x + 6$$
$$y = \frac{-x+6}{2}$$
$$y = -\frac{1}{2}x + 3$$

y-int = (0, 3)

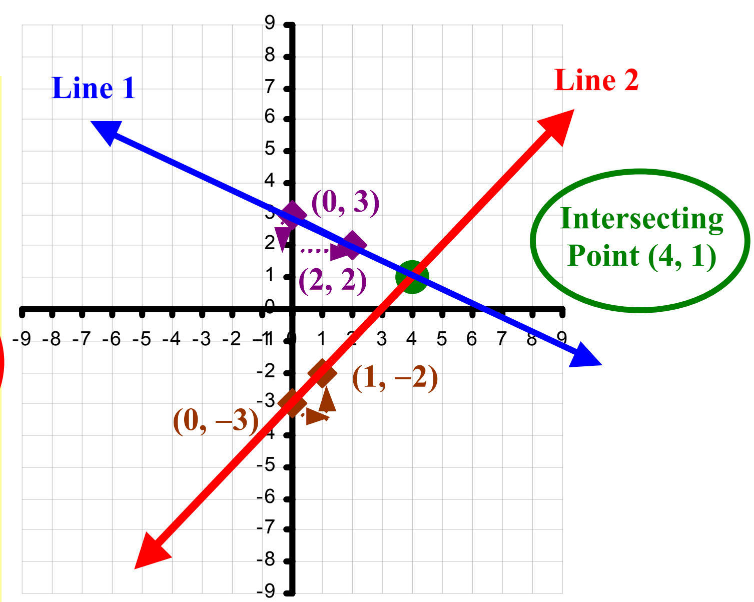
$$\text{slope} = \frac{-1}{2} = \frac{\text{1 Down}}{\text{2 Right}}$$

For Line 2:

$$x - y = 3$$
$$-y = -x + 3$$
$$y = x - 3$$

y-int = (0, –3)

$$\text{slope} = \frac{1}{1} = \frac{\text{1 Up}}{\text{1 Right}}$$



b. x and y-intercepts

Line 1:

For y-int, let $x = 0$

$x + 2y = 6$

$(0) + 2y = 6$

$y = 3$ (0, 3)

For x-int, let $y = 0$

$x + 2y = 6$

$x + 2(0) = 6$

$x = 6$ (6, 0)

Line 2:

For y-int, let $x = 0$

$x - y = 3$

$(0) - y = 3$

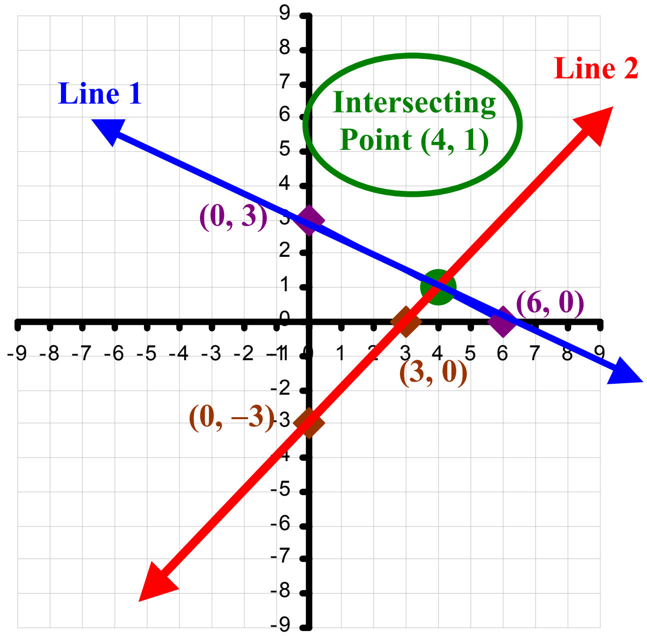
$y = -3$ (0, −3)

For x-int, let $y = 0$

$x - y = 3$

$x - (0) = 3$

$x = 3$ (3, 0)

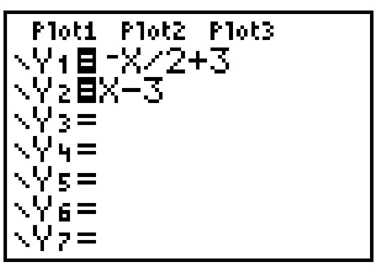


Example 2: Find the solution of the following system of equations by using the graphing calculator.

$$x + 2y = 6$$
$$x - y = 3$$

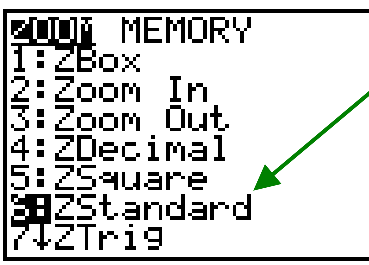
Step 1: Rearrange to Slope and y–intercept form

Y=



Step 2: Set WINDOWS to Standard Mode

ZOOM



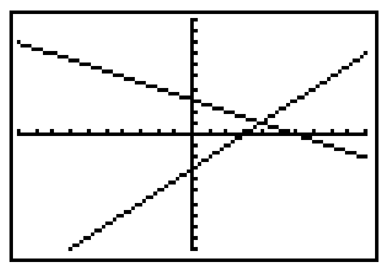
Choose Option 6 will set

x: [−10, 10, 1]

y: [−10, 10, 1]

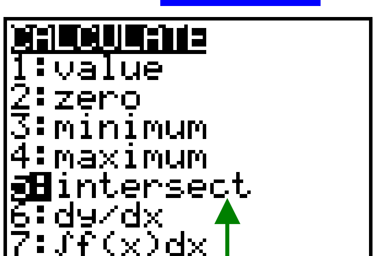
Step 3: Graph

GRAPH



Step 4: Run INTERSECT

2nd **CALC** **TRACE**



Choose Option 5

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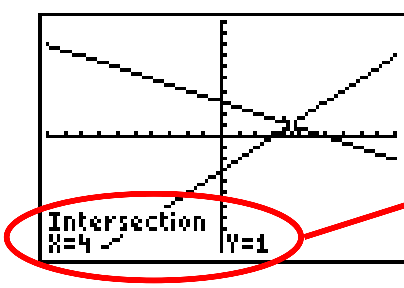


Step 5: Verify using TABLE

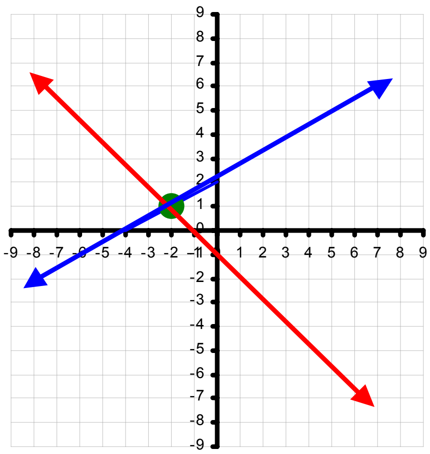
2nd **TABLE** **GRAPH**

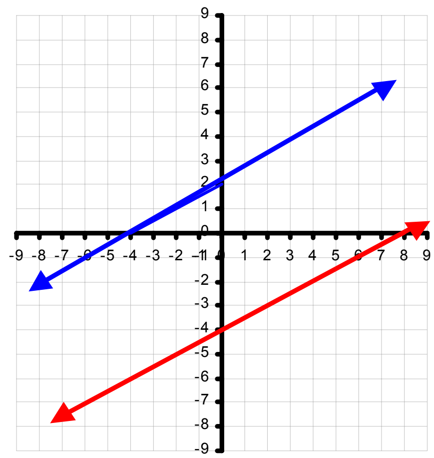
X	Y1	Y2
0	3	-3
1	2.5	-2
2	2	-1
3	1.5	0
4	1	1
5	.5	2
6	0	3

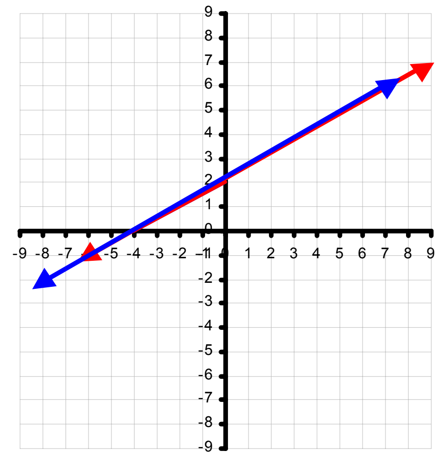
X=4

There are three types of solutions to a system of linear equations:

1. Intersecting Lines	**2. Parallel Lines**	**3. Overlapping (Coincident) Lines**
One distinct Solution	No Solution	Many (Infinite) Solutions
Different Slopes	Identical Slopes	Identical Slopes
Different y-Intercepts	Different y-Intercepts	Identical y-Intercepts







Example 3: Determine the number of solutions for the systems of equations below.

a. $x + 2y = 10$
$x + 2y = 6$

Line 1:

$$x + 2y = 10$$
$$2y = -x + 10$$
$$y = \frac{-x + 10}{2}$$
$$\boldsymbol{y = \frac{-1}{2}x + 5}$$

$\boldsymbol{m = \frac{-1}{2}}$**, $\boldsymbol{y}$-int = 5**

Line 2:

$$x + 2y = 6$$
$$2y = -x + 6$$
$$y = \frac{-x + 6}{2}$$
$$\boldsymbol{y = \frac{-1}{2}x + 3}$$

$\boldsymbol{m = \frac{-1}{2}}$**, $\boldsymbol{y}$-int = 5**

Identical slopes, but different y-intercepts mean parallel lines. Therefore, this system has NO SOLUTION.

b. $2x + 5y = 15$
$6x + 15y = 45$

Line 1:

$$2x + 5y = 15$$
$$5y = -2x + 15$$
$$y = \frac{-2x + 15}{5}$$
$$\boldsymbol{y = \frac{-2}{5}x + 3}$$

$\boldsymbol{m = \frac{-2}{5}}$**, $\boldsymbol{y}$-int = 3**

Line 2:

$$6x + 15y = 45$$
$$15y = -6x + 45$$
$$y = \frac{-6x + 45}{15}$$
$$\boldsymbol{y = \frac{-2}{5}x + 3}$$

$\boldsymbol{m = \frac{-2}{5}}$**, $\boldsymbol{y}$-int = 3**

Identical slopes and y-intercepts mean overlapping lines. Therefore, this system has MANY SOLUTIONS.

1-1 Assignment: **pg. 11 #1 to 29 (odd)**

1-3: Solving Systems of Linear Equations by Substitution

When using the **substitution method** to solve a system of linear equations:

1. **Isolate a variable from one equation. (Always pick the variable with 1 as a coefficient.)**
2. **Substitute the resulting expression into that variable of the other equation.**
3. **Solve for the other variable.**
4. **Substitute the result from the last step into one of the original equation and solve for the remaining variable.**

Example 1: Using the substitution method, solve the following systems of equations algebraically. Verify the solutions with the graphing calculator.

a. $5x + y = -17$
$3y - 4x = 6$

Isolate y from the first equation (a variable with 1 as a coefficient).

$$5x + y = -17$$
$$y = -5x - 17$$

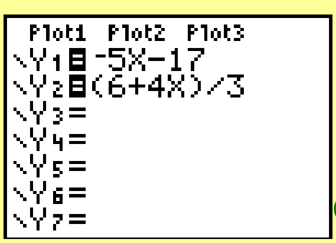
Substitute expression into y in the second equation.

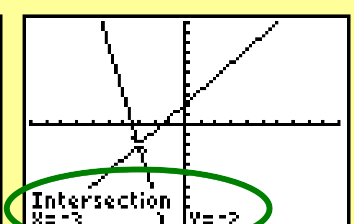
$$3y - 4x = 6$$
$$3(-5x - 17) - 4x = 6$$
$$-15x - 51 - 4x = 6$$
$$-19x = 6 + 51$$
$$-19x = 57$$
$$x = \frac{57}{-19} \qquad x = -3$$

Solve for the remaining variable. Pick the easier equation of the two.

$$5x + y = -17$$
$$5(-3) + y = -17$$
$$-15 + y = -17$$
$$y = -17 + 15 \qquad y = -2$$

Verify with graphing calculator. Rearrange equation first.





b. $3x + 6y = 5$
$x - 2y = -2$

Isolate x from the second equation (a variable with 1 as a coefficient).

$$x - 2y = -2$$
$$x = 2y - 2$$

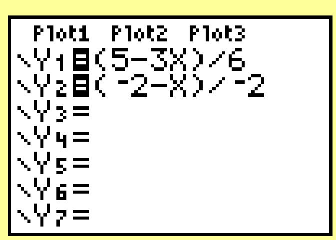
Substitute expression into x in the first equation.

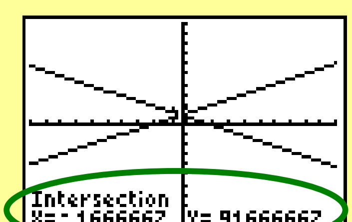
$$3x + 6y = 5$$
$$3(2y - 2) + 6y = 5$$
$$6y - 6 + 6y = 5$$
$$12y = 5 + 6$$
$$12y = 11 \qquad y = \frac{11}{12} = 0.91\overline{7}$$

Solve for the remaining variable. Pick the easier equation of the two.

$$x - 2y = -2$$
$$x - 2\left(\frac{11}{12}\right) = -2$$
$$x - \frac{11}{6} = -2$$
$$x = -2 + \frac{11}{6} \qquad x = -\frac{1}{6} = -0.1\overline{6}$$

Verify with graphing calculator. Rearrange equation first.





Example 2: Using the substitution method, algebraically solve the system of equations below.

$$2(x - 2y) = 5(5 - y)$$
$$3(y - x) = -2(y + 7)$$

Expand each equation accordingly.

Equation 1

$$2(x - 2y) = 5(5 - y)$$
$$2x - 4y = 25 - 5y$$
$$\mathbf{2x + y = 25}$$

Equation 2

$$3(y - x) = -2(y + 7)$$
$$3y - 3x = -2y - 14$$
$$\mathbf{5y - 3x = -14}$$

Solve for both variables using the substitution method.

Isolate y from the first equation.

$$2x + y = 25$$
$$\mathbf{y = 25 - 2x}$$

Substitute expression into y in the second equation.

$$5y - 3x = -14$$
$$5(25 - 2x) - 3x = -14$$
$$125 - 10x - 3x = -14$$
$$-13x = -14 - 125$$
$$-13x = -139$$
$$x = \frac{139}{13}$$

Solve for the remaining variable. Pick the easier equation of the two.

$$2x + y = 25$$
$$2\left(\frac{139}{13}\right) + y = 25$$
$$\frac{278}{13} + y = 25$$
$$y = 25 - \frac{278}{13}$$
$$y = \frac{47}{13}$$

Example 3: Three shirts and one pair of jeans cost \$155. Two shirts and three pairs of jeans cost \$220. Find the cost of a single shirt and the cost of one pair of jeans.

First, **define** the variables.

Let s = **cost of one shirt**
Let j = **cost of one pair of jeans**

Next, **set up the system of equations by translating the sentences.**

$$3s + j = 155$$
$$2s + 3j = 220$$

Solve for both variables using the substitution method.

Isolate j from the first equation.

$$3s + j = 155$$
$$\mathbf{j = 155 - 3s}$$

Substitute expression into j in the second equation.

$$2s + 3j = 220$$
$$2s + 3(155 - 3s) = 220$$
$$2s + 465 - 9s = 220$$
$$-7s = 220 - 465$$
$$-7s = -245$$
$$s = \frac{-245}{-7}$$

s = \$35 / shirt

Solve for the remaining variable. Pick the easier equation of the two.

$$3s + j = 155$$
$$3(35) + j = 155$$
$$105 + j = 155$$

j = \$50 / jeans

Example 4: Mary owes a total of \$1500 on her credit cards. One of her credit card, MasterCard, charges 1.8%/month on her outstanding balance. While her other credit card, American Express, charges 2.1% on her balance. In one month, her total interest is \$29.96. What are her balances on each of her credit cards?

First, **define** the variables.

Let m = **Balance on MasterCard**
Let a = **Balance on American Express**

Next, **set up the system of equations** by **translating the sentences.**

$$m + a = 1500 \quad \text{(Total Balance)}$$
$$0.018m + 0.021a = 29.96 \quad \text{(Total Interest)}$$

Solve for both variables using the substitution method.

Isolate m from the first equation.

$$m + a = 1500$$
$$m = 1500 - a$$

Substitute expression into m in the second equation.

$$0.018m + 0.021a = 29.96$$
$$0.018(1500 - a) + 0.021a = 29.96$$
$$27 - 0.018a + 0.021a = 29.96$$
$$0.003a = 29.96 - 27$$
$$0.003a = 2.96$$
$$a = \frac{2.96}{0.003} \qquad a = \$986.67$$

Solve for the remaining variable. Pick the easier equation of the two.

$$m + a = 1500$$
$$m + (986.67) = 1500$$
$$m = 1500 - 986.67 \qquad m = \$513.33$$

The Balance of the MasterCard is \$513.33.
The Balance of the American Express is \$986.67

1-3 Assignment: **pg. 25-27 #1 to 23 (odd), 25a, 27, 29a, 29c, 36, 37**

1-5: Solving Systems of Linear Equations by Elimination

Since substitution method is only useful when an equation has 1 or −1 as the numerical coefficient, we need another way to solve other systems of linear equations.

Elimination by Addition: - most useful when both equations has the same like terms with opposite signs.

Example 1: Solve the system of linear equations below by elimination. Verify the solution using a graphing calculator.

$$3x + 2y = 5$$
$$9x - 2y = 15$$

First, eliminate *y* by adding the equations.

$$\begin{aligned} 3x + \cancel{2y} &= 5 \\ \boxed{+}\,(9x - \cancel{2y} &= 15) \\ 12x \quad &= 20 \\ x &= \frac{20}{12} \end{aligned}$$

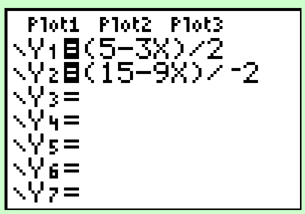
$$x = \frac{5}{3}$$

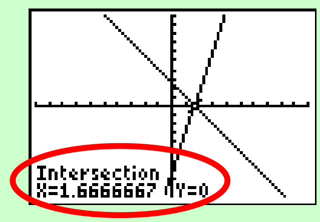
Next, substitute *x* into one of the equations to solve for *y*.

$$\begin{aligned} 3x + 2y &= 5 \\ 3\left(\frac{5}{3}\right) + 2y &= 5 \\ 5 + 2y &= 5 \\ 2y &= 5 - 5 \\ 2y &= 0 \end{aligned}$$

$$y = 0$$

Verify with graphing calculator.





Elimination by Subtraction: - most useful when both equations has exactly the same like terms.

Example 2: Solve the system of linear equations below using the elimination method. Verify the solution with a graphing calculator.

$$5x + 2y = -1$$
$$5x - 4y = -13$$

First, eliminate *x* by subtracting the equations.

$$\begin{aligned} \cancel{5x} + 2y &= -1 \\ \boxed{-}\,(\cancel{5x} - 4y &= -13) \\ 6y &= 12 \end{aligned}$$

2*y* − (−4*y*) **−1 − (−13)**

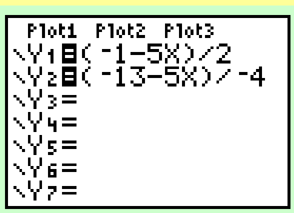
$$y = 2$$

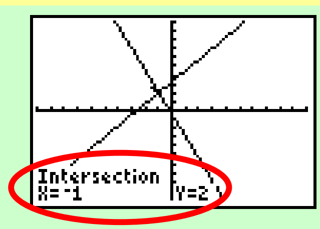
Next, substitute *y* into one of the equations to solve for *x*.

$$\begin{aligned} 5x + 2y &= -1 \\ 5x + 2(2) &= -1 \\ 5x + 4 &= -1 \\ 5x &= -1 - 4 \\ 5x &= -5 \end{aligned}$$

$$x = -1$$

Verify with graphing calculator.





Elimination by Multiplication: - most useful when neither equations has the same like terms.
- by multiplying different numbers (factors of their LCM) on each equation, we can change these equations into their equivalent form with the same like terms.

Example 3: Solve the following systems of linear equations by elimination.

a. $3x + 4y = 18$
$2x - 3y = -5$

Method 1: Eliminating *x*

LCM of $3x$ and $2x$ = $6x$

(Multiply Equation 1 by **2** to obtain $6x$)
(Multiply Equation 2 by **3** to obtain $6x$)

$2 \times (3x + 4y = 18) \rightarrow \mathbf{6x + 8y = 36}$
$3 \times (2x - 3y = -5) \rightarrow \mathbf{6x - 9y = -15}$

Eliminate *x* by subtraction.

$$\begin{aligned} \cancel{6x} + 8y &= \ \ 36 \\ -\ (\cancel{6x} - 9y &= -15) \\ \hline 17y &= 51 \\ y &= \frac{51}{17} \end{aligned}$$

$\mathbf{8y - (-9y)}$ $\qquad$ $\mathbf{36 - (-15)}$

$y = 3$

Next, substitute *y* into one of the equations to solve for *x*.

$$\begin{aligned} 3x + 4y &= 18 \\ 3x + 4(3) &= 18 \\ 3x &= 18 - 12 \\ 3x &= 6 \end{aligned}$$

$x = 2$

Method 2: Eliminating *y*

LCM of $4y$ and $3y$ = $12y$

(Multiply Equation 1 by **3** to obtain $12y$)
(Multiply Equation 2 by **4** to obtain $12y$)

$3 \times (3x + 4y = 18) \rightarrow \mathbf{9x + 12y = 54}$
$4 \times (2x - 3y = -5) \rightarrow \mathbf{8x - 12y = -20}$

Eliminate *y* by addition.

$$\begin{aligned} 9x + \cancel{12y} &= \ \ 54 \\ +\ (8x - \cancel{12y} &= -20) \\ \hline 17x \quad &= 34 \\ x &= \frac{34}{17} \end{aligned}$$

$x = 2$

Next, substitute *x* into one of the equations to solve for *y*.

$$\begin{aligned} 3x + 4y &= 18 \\ 3(2) + 4y &= 18 \\ 4y &= 18 - 6 \\ 4y &= 12 \end{aligned}$$

$y = 3$

b. $3x + 7y = 3$
$4y - 5x = 42$

First, rearrange the equations so *x* and *y* terms will line up.

$$\begin{aligned} \mathbf{3x + 7y} &= \mathbf{3} \\ \mathbf{-5x + 4y} &= \mathbf{42} \end{aligned}$$

Eliminating *x*

LCM of $3x$ and $5x$ = $15x$

(Multiply Equation 1 by **5** to obtain $15x$)
(Multiply Equation 2 by **3** to obtain $15x$)

$5 \times (3x + 7y = 3) \rightarrow \mathbf{15x + 35y = 15}$
$3 \times (-5x + 4y = 42) \rightarrow \mathbf{-15x + 12y = 126}$

Eliminate *x* by addition.

$$\begin{aligned} \cancel{15x} + 35y &= \ \ 15 \\ +\ (-\cancel{15x} + 12y &= 126) \\ \hline 47y &= 141 \\ y &= \frac{141}{47} \end{aligned}$$

$y = 3$

Next, substitute *y* into one of the equations to solve for *x*.

$$\begin{aligned} 3x + 7y &= 3 \\ 3x + 7(3) &= 3 \\ 3x &= 3 - 21 \\ 3x &= -18 \end{aligned}$$

$x = -6$

Systems of Rational Equations

When solving systems of rational equations:

1. **Convert any rational equations into linear equations by multiplying the ENTIRE equation with the LCM of the denominators.**
2. **Expand any equations if necessary.**
3. **Solve the resulting system of linear equations by substitution or elimination (as required algebraically), or by graphical method (if the question is open to any method).**

Example 4: Solve the system of rational equations algebraically.

$$\frac{x+5}{4}-\frac{2(y-2)}{3}=6$$
$$\frac{3x-1}{6}+\frac{3(y+4)}{8}=1$$

Multiply each term of the first equation by its Lowest Common Denominator 12.

$$\overset{3}{\cancel{12}}\left(\frac{x+5}{\cancel{4}}\right)-\overset{4}{\cancel{12}}\left(\frac{2(y-2)}{\cancel{3}}\right)=\mathbf{12}(6)$$
$$3(x+5)-8(y-2)=72$$
$$3x+15-8y+16=72$$
$$3x-8y=72-15-16$$
$$\mathbf{3x-8y=41}$$

Multiply each term of the second equation by its Lowest Common Denominator 24.

$$\overset{4}{\cancel{24}}\left(\frac{3x-1}{\cancel{6}}\right)+\overset{3}{\cancel{24}}\left(\frac{3(y+4)}{\cancel{8}}\right)=\mathbf{24}(1)$$
$$4(3x-1)+9(y+4)=24$$
$$12x-4+9y+36=24$$
$$12x+9y=24+4-36$$
$$\mathbf{12x+9y=-8}$$

Eliminating x

LCM of $3x$ and $12x$ = $\mathbf{12x}$

(Multiply Equation 1 by **4** to obtain $12x$)

$\boxed{4\times}\ (3x-8y=41) \rightarrow \mathbf{12x-32y=164}$

$(12x+9y=-8) \rightarrow \mathbf{12x+\ 9y=\ -8}$

Eliminate x by subtraction.

$$\cancel{12x}-32y=164$$
$$\boxed{-}\ (\cancel{12x}+9y=-8)$$
$$-41y=172$$

$\mathbf{-32y \boxed{-} (+9y)}$ $\mathbf{164 \boxed{-} (-8)}$

$$y=-\frac{172}{41}$$

Next, substitute y into one of the equations to solve for x.

$$\mathbf{3x-8y=41}$$
$$\mathbf{3x-8\left(-\frac{172}{41}\right)=41}$$
$$\mathbf{3x+\frac{1376}{41}=41}$$
$$\mathbf{3x=41-\frac{1376}{41}}$$
$$\mathbf{3x=\frac{305}{41}}$$
$$\mathbf{x=\frac{305}{41}\div 3}$$
$$x=\frac{305}{123}$$

Distance-Speed-Time Problems: - always use a table to organize the information.

$$Speed = \frac{Distance}{Time}$$

Example 5: A bus went 150 km from Calgary to Red Deer. The bus drove at 90 km/h for the most part, but due to a snowstorm near Red Deer, its speed was reduced to 30 km/h. If the entire trip took 3 hours, how far did the bus travel in the storm?

First, set up a table.

$$Time = \frac{Distance}{Speed}$$

Conditions	Distance	Speed	Time
Normal	x	90 km/h	$\frac{x}{90}$
Snowstorm	y	30 km/h	$\frac{y}{30}$
Total	150 km		3 hours

Next, set up the system of equations.

$$x + y = 150 \rightarrow x + y = 150$$

$$\frac{x}{90} + \frac{y}{30} = 3 \rightarrow x + 3y = 270$$

Multiply by LCM = 90

Eliminate x by subtraction.

$$\begin{aligned} \cancel{x} + y &= 150 \\ -(\cancel{x} + 3y &= 270) \\ -2y &= -120 \\ y &= \frac{-120}{-2} \end{aligned}$$

$y = 60$ km

The bus went 60 km in the snowstorm.

Example 6: An aircraft flew from Calgary to San Francisco, a distance of 1018 km, in 2.5 hours with the tail wind. The return trip took 30 minutes longer with the head wind. Find the speed of the aircraft in still air and the speed of the wind.

First, define the variables and set up a table.

$$Speed = \frac{Distance}{Time}$$

Let x = speed of plane **Let y = speed of wind**

	Distance	Speed	Time
Tail Wind	1018 km	$x + y$	2.5 hours
Head Wind	1018 km	$x - y$	2.5 hours + 30 min = 3 hours

Next, set up the system of equations.

$$x + y = \frac{1018}{2.5} \rightarrow x + y = 407.2$$

$$x - y = \frac{1018}{3} \rightarrow x - y = 746.5\overline{3}$$

Eliminate y by addition.

$$\begin{aligned} x + \cancel{y} &= 407.2 \\ +(x - \cancel{y} &= 339.\overline{3}) \\ 2x &= 746.5\overline{3} \\ x &= \frac{746.5\overline{3}}{2} \end{aligned}$$

$x = 373.3$ km/h

Substitute x into one of the equations to solve for y.

$$\begin{aligned} x + y &= 407.2 \\ (373.3) + y &= 407.2 \\ y &= 407.2 - 373.3 \end{aligned}$$

$y = 33.9$ km/h

The plane was flying at 373.3 km/h.
The wind had a speed of 33.9 km/h.

1-5 Assignment: **pg. 38-40 #1 to 29 (odd), 31 to 34, 35 to 41 (odd), 44, 47 to 52**

1-6: Solving Systems of Linear Equations in Three Variables

In linear algebra, we can find the solutions for *n* number of variables when there are *n* number of equations relating them.

When solving systems of **3 equations with variables**:

1. **Select a set of two equations out of the three equations given where a variable can be easily eliminated.**
2. **Select another set of two equations out of the three equations given where the same variable can be eliminated (may have to use elimination by multiplication).**
3. **Once that variable is eliminated, we will be left with a system of two equations-two variables. Solve those variables.**
4. **Substitute the solutions of the two variables found in the last step into one of the three equations given originally. Find the very first variable that was eliminated.**

Example 1: Solve the following systems of linear equations.

a.
$$\begin{aligned} x + 3y + 4z &= 19 \\ x + 2y + z &= 12 \\ x + y + z &= 8 \end{aligned}$$

Select Equations 2 and 3. (We can eliminate both *x* and *z*.)

$$\begin{aligned} \cancel{x} + 2y + \cancel{z} &= 12 \\ -\ (\cancel{x} + y + \cancel{z} &= 8) \\ \hline y &= 4 \end{aligned}$$

$y = 4$

Select Equations 1 and 2 to eliminate *x* only.

$$\begin{aligned} \cancel{x} + 3y + 4z &= 19 \\ -\ (\cancel{x} + 2y + z &= 12) \\ \hline y + 3z &= 7 \end{aligned}$$

Since we know $y = 4$, we can solve for *z*.

$$\begin{aligned} (4) + 3z &= 7 \\ 3z &= 3 \end{aligned}$$

$z = 1$

Substitute *y* and *z* into Equation 3 to solve for *x*.

$$\begin{aligned} x + y + z &= 8 \\ x + (4) + (1) &= 8 \\ x + 5 &= 8 \end{aligned}$$

$x = 3$

Verify with Equation 1.

$$\begin{aligned} x + 3y + 4z &= 19 \\ (3) + 3\,(4) + 4\,(1) &= 19 \\ 3 + 12 + 4 &= 19 \\ 19 &= 19 \end{aligned}$$

L.H.S = R.H.S.

Verify with Equation 2.

$$\begin{aligned} x + 2y + z &= 12 \\ (3) + 2\,(4) + (1) &= 12 \\ 3 + 8 + 1 &= 12 \\ 12 &= 12 \end{aligned}$$

L.H.S = R.H.S.

b. $4x + 5y - 3z = 4$
$5x + 3y - 2z = -3$
$3x + 2y - 2z = -2$

Select Equations 2 and 3 to eliminate z.

$$\begin{array}{rl} & 5x + 3y - \cancel{2z} = -3 \\ - & (3x + 2y - \cancel{2z} = -2) \\ \hline & 2x + y \quad = -1 \quad \textbf{Equation 4} \end{array}$$

Select Equations 1 and 2 to eliminate z.
(Multiply Equation 1 by **2** to obtain **$-6z$**)
(Multiply Equation 2 by **3** to obtain **$-6z$**)

$$\boxed{2\times}\,(4x + 5y - 3z = 4) \longrightarrow 8x + 10y - 6z = 8$$
$$\boxed{3\times}\,(5x + 3y - 2z = -3) \longrightarrow 15x + 9y - 6z = -9$$

$$\begin{array}{rl} & 8x + 10y - \cancel{6z} = 8 \\ - & (15x + 9y - \cancel{6z} = -9) \\ \hline & -7x + y \quad = 17 \quad \textbf{Equation 5} \end{array}$$

Subtract Equations 4 and 5 to eliminate y, and solve for x.

$$\begin{array}{rl} & 2x + y = -1 \\ - & (-7x + y = 17) \\ \hline & 9x \quad = -18 \end{array}$$

$$x = -2$$

Substitute x into Equation 4 and solve for y.

$$2x + y = -1$$
$$2(-2) + y = -1$$
$$-4 + y = -1$$
$$y = -1 + 4$$
$$y = 3$$

Substitute x and y into Equation 3 and solve for z.

$$3x + 2y - 2z = -2$$
$$3(-2) + 2(3) - 2z = -2$$
$$-6 + 6 - 2z = -2$$
$$-2z = -2$$
$$z = 1$$

Verify with Equation 1.

$$4x + 5y - 3z = 4$$
$$4(-2) + 5(3) - 3(1) = 4$$
$$-8 + 15 - 3 = 4$$
$$4 = 4$$

L.H.S = R.H.S.

Verify with Equation 2.

$$5x + 3y - 2z = -3$$
$$5(-2) + 3(3) - 2(1) = -3$$
$$-10 + 9 - 2 = -3$$
$$-3 = -3$$

L.H.S = R.H.S.

c. $0.3x + 0.4y + 0.2z = 0.3$
$0.2x - 0.1y + 0.4z = 0.4$
$0.2x + 0.3y \qquad = 0.4$

Multiply all equations by **10** to obtain **integer coefficients**.

$10 \times (0.3x + 0.4y + 0.2z = 0.3) \longrightarrow \mathbf{3x + 4y + 2z = 3}$
$10 \times (0.2x - 0.1y + 0.4z = 0.4) \longrightarrow \mathbf{2x - y + 4z = 4}$
$10 \times (0.2x + 0.3y \qquad = 0.4) \longrightarrow \mathbf{2x + 3y \qquad = 4}$

Select Equations 1 and 2 to eliminate z.
(Multiply Equation 1 by **2** to obtain **4z**)

$2 \times (3x + 4y + 2z = 3) \longrightarrow \mathbf{6x + 8y + 4z = 6}$

$$\begin{array}{rl} & 6x + 8y + \cancel{4z} = 6 \\ - & (2x - y + \cancel{4z} = 4) \\ \hline & \mathbf{4x + 9y \qquad = 2} \end{array}$$

Equation 4

Select Equations 3 and 4 to eliminate x and solve for y. (Multiply Equation 3 by **2** to obtain **4x**)

$2 \times (2x + 3y = 4) \longrightarrow \mathbf{4x + 6y = 8}$

$$\begin{array}{rl} & \cancel{4x} + 6y = 8 \\ - & (\cancel{4x} + 9y = 2) \\ \hline & -3y = 6 \end{array}$$

$y = -2$

Substitute y into Equation 3 and solve for x.

$2x + 3y = 4$
$2x + 3(-2) = 4$
$2x - 6 = 4$
$2x = 4 + 6$
$2x = 10$

$x = 5$

Substitute x and y into Equation 1 and solve for z.

$3x + 4y + 2z = 3$
$3(5) + 4(-2) + 2z = 3$
$15 - 8 + 2z = 3$
$7 + 2z = 3$
$2z = -4$

$z = -2$

Verify with Equation 2.

$2x - y + 4z = 4$
$2(5) - (-2) + 4(-2) = 4$
$10 + 2 - 8 = 4$
$4 = 4$

L.H.S = R.H.S.

d. $$\frac{x}{3}+\frac{y}{6}-\frac{z}{4}=\frac{7}{12}$$
$$\frac{x}{6}+\frac{y}{2}+\frac{z}{6}=\frac{1}{3}$$
$$\frac{x}{4}+y-\frac{z}{2}=-\frac{9}{4}$$

Multiply <u>each term</u> of the first equation by its Lowest Common Denominator 12.

$$\mathbf{12}\left(\frac{x}{3}\right)+\mathbf{12}\left(\frac{y}{6}\right)-\mathbf{12}\left(\frac{z}{4}\right)=\mathbf{12}\left(\frac{7}{12}\right)$$

Equation 1 $\qquad 4x + 2y - 3z = 7$

Multiply <u>each term</u> of the second equation by its Lowest Common Denominator 6.

$$\mathbf{6}\left(\frac{x}{6}\right)+\mathbf{6}\left(\frac{y}{2}\right)+\mathbf{6}\left(\frac{z}{6}\right)=\mathbf{6}\left(\frac{1}{3}\right)$$

Equation 2 $\qquad x + 3y + z = 2$

Multiply <u>each term</u> of the first equation by its Lowest Common Denominator 4.

$$\mathbf{4}\left(\frac{x}{4}\right)+\mathbf{4}(y)-\mathbf{4}\left(\frac{z}{2}\right)=\mathbf{4}\left(-\frac{9}{4}\right)$$

Equation 3 $\qquad x + 4y - 2z = -9$

Select Equations 2 and 3 to eliminate x.

$$\begin{array}{rl} & \cancel{x} + 3y + z = 2 \\ \boxed{-} & (\cancel{x} + 4y - 2z = -9) \\ \hline & -y + 3z = 11 \end{array}$$

Equation 4

Select Equations 1 and 2 to eliminate z.
(Multiply Equation 2 by **4** to obtain $\mathbf{4x}$)

$$\boxed{4\times}\,(x + 3y + z = 2) \longrightarrow \mathbf{4x + 12y + 4z = 8}$$

$$\begin{array}{rl} & \cancel{4x} + 2y - 3z = 7 \\ \boxed{-} & (\cancel{4x} + 12y + 4z = 8) \\ \hline & -10y - 7z = -1 \end{array}$$

Equation 5 $\qquad -10y \;\; -7z = -1$

Select Equations 4 and 5 to eliminate y and solve for z. (Multiply Equation 4 by **10** to obtain $\mathbf{-10y}$)

$$\boxed{10\times}\,(-y + 3z = 11) \longrightarrow \mathbf{-10y + 30y = 110}$$

$$\begin{array}{rl} & \cancel{-10y} + 30z = 110 \\ \boxed{-} & (\cancel{-10y} - 7z = -1) \\ \hline & 37z = 111 \end{array}$$

$$z = 3$$

Substitute z into Equation 4 and solve for y.

$$-y + 3z = 11$$
$$-y + 3\,(3) = 11$$
$$-y + 9 = 11$$
$$-y = 11 - 9$$
$$-y = 2$$
$$y = -2$$

Substitute y and z into Equation 2 and solve for x.

$$x + 3y + z = 2$$
$$x + 3\,(-2) + (3) = 2$$
$$x - 6 + 3 = 2$$
$$x - 3 = 2$$
$$x = 2 + 3$$
$$x = 5$$

Verify with Equation 1.

$$4x + 2y - 3z = 7$$
$$4\,(5) + 2\,(-2) - 3\,(3) = 7$$
$$7 = 7$$

L.H.S = R.H.S.

Verify with Equation 3.

$$x + 4y - 2z = -9$$
$$(5) + 4\,(-2) - 2\,(3) = -9$$
$$-9 = -9$$

L.H.S = R.H.S.

We can solve systems of linear equations using matrix.

<u>Matrix</u>: - a rectangular array of numbers, known as elements, which are enclosed within brackets.

A system of linear equations (2 variables with 2 equations), $\begin{aligned} 8x - 3y &= 5 \\ -2x + 7y &= 12 \end{aligned}$, can be written as a matrix below.

Name of the Matrix (in capital letter)

$$B = \begin{bmatrix} 8 & -3 & 5 \\ -2 & 7 & 12 \end{bmatrix} \begin{array}{l} \text{row 1} \\ \text{row 2} \end{array}$$

$$\begin{array}{ccc} \text{column 1} & \text{column 2} & \text{column 3} \end{array}$$

A system of linear equations (3 variables with 3 equations), $\begin{aligned} 2x + 3y + z &= 0 \\ -x + 4y + 5z &= 5 \\ 3x - 2y - 3z &= -4 \end{aligned}$, can be written as a matrix below.

Name of the Matrix (in capital letter)

$$A = \begin{bmatrix} 2 & 3 & 1 & 0 \\ -1 & 4 & 5 & 6 \\ 3 & -2 & -3 & -4 \end{bmatrix} \begin{array}{l} \text{row 1} \\ \text{row 2} \\ \text{row 3} \end{array}$$

$$\begin{array}{cccc} \text{column 1} & \text{column 2} & \text{column 3} & \text{column 4} \end{array}$$

<u>Dimension</u>: - the size of the matrix (***number of rows by number of columns***).
- the example above is a 3×4 matrix. We can write B_{34}

<u>Elements</u>: - individual numbers on the matrix.
- represented by the name of the matrix (in lower case letter), followed by row number and then column number in subscripts
- the example above has 5 as an element in row 2 and column 3. We can write $a_{23} = 5$

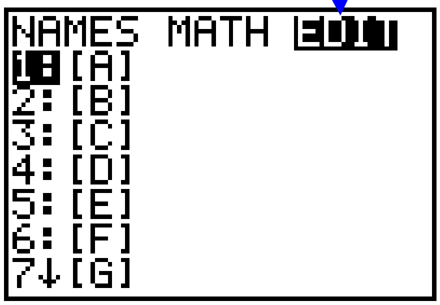
ALL Systems of Linear Equations has <u>ONE MORE COLUMN THAN THE NUMBER OF ROWS</u> as the dimension of the matrix.

Using a Graphing Calculator to Operate with Matrices

A. To Enter a Matrix:

1. Press **2nd** **MATRX** x^{-1}

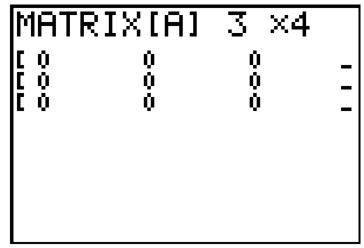
2. Use ▶ to access EDIT



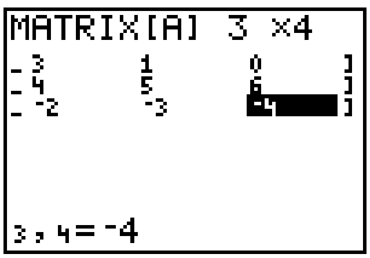
3. Select Option 1 if the desired name of the Matrix is [A]. Otherwise select other options for other names.

4. Press **ENTER**

5. Enter the dimensions of the matrix. (Using the second matrix on the previous page as example.)



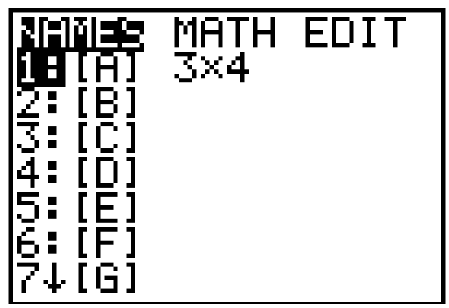
6. Enter the elements of the matrix (along each row).



7. Press **2nd** **QUIT MODE** when finished.

B. To Recall a Matrix from the Home Screen:

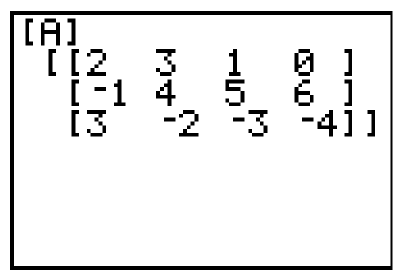
1. Press **2nd** **MATRX** x^{-1}



2. Select Option 1 if the desired matrix to be recalled is [A]. Otherwise select other options for other matrices.

3. Press **ENTER**

4. Press **ENTER** again to see the entire matrix on the home screen. (Highly recommended for matrices bigger than 3 × 3 to verify if there are any mistakes while entering elements.)



C. To Delete a Matrix:

1. Press **2nd** **MEM** **+**

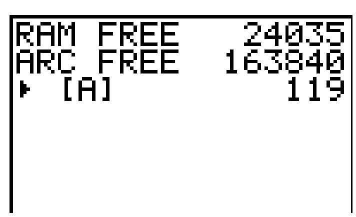
2. Select Option 2.

3. Press **ENTER**

4. Select Option 5.

5. Press **ENTER**

6. Press **DEL** next to the matrix that needs to be deleted.



Reduced Row-Echelon Form: - a matrix where the **last column indicates the solutions** of the variables of a system of linear equation.
- the diagonal elements are 1 with the rest of the elements equal to 0.

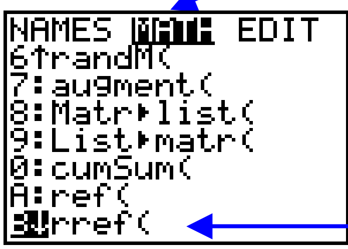
rref ([Name of the Matrix] **)**

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 \end{bmatrix} \qquad \begin{aligned} x + 0y + 0z &= 3 \\ 0x + y + 0z &= 2 \\ 0x + 0y + z &= 4 \end{aligned} \qquad \begin{aligned} x &= 3 \\ y &= 2 \\ z &= 4 \end{aligned}$$

D. To Solve a Matrix using rref:

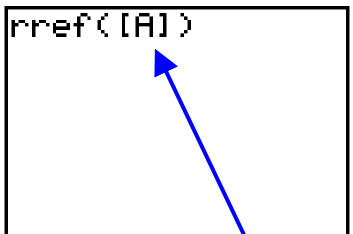
1. Press **2nd** **MATRX** x^{-1}

2. Use ▶ **to access MATH**



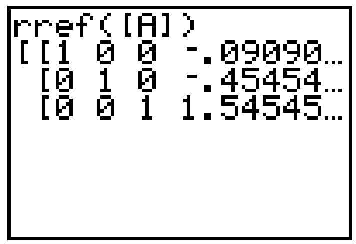
3. Select Option B

4. Press **ENTER**



5. Recall the Name of the Matrix to be Solved.

6. Read the Results

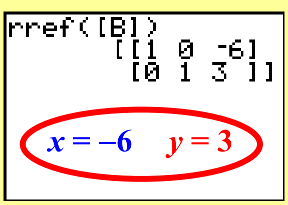


$x = –0.9090…$
$y = –0.4545…$
$z = 1.5454…$

Example 2: Solve the following system of linear equations using matrix.

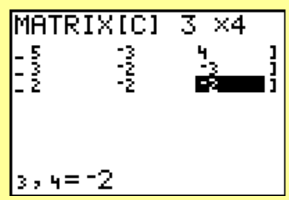
a. $3x + 7y = 3$
$4y – 5x = 42$

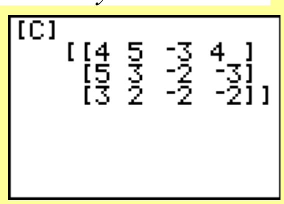
Rearrange Equations
$3x + 7y = 3$
$–5x + 4y = 42$

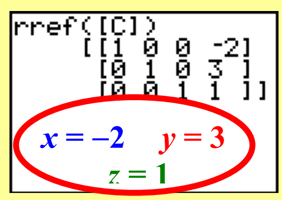


$x = –6 \quad y = 3$

b. $4x + 5y – 3z = 4$
$5x + 3y – 2z = –3$
$3x + 2y – 2z = –2$



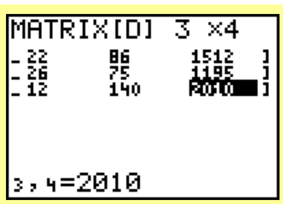


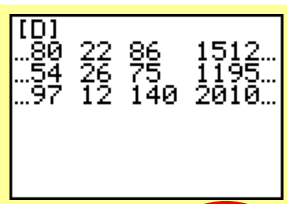


$x = –2 \quad y = 3 \quad z = 1$

Example 3: A school play generated the following results. Find the cost of each type of ticket using matrix.

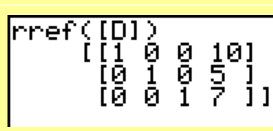
	Number of Tickets Sold			
Day	**Adult**	**Seniors**	**Students**	**Revenue**
1	80	22	86	$1512
2	54	26	75	$1195
3	97	12	140	$2010





Let x = **Price of an Adult ticket**
y = **Price of a Senior ticket**
z = **Price of a Student ticket**

$80x + 22y + 86z = 1512$
$54x + 26y + 75z = 1195$
$97x + 12y + 140z = 2010$



$x = \$10$
$y = \$5$
$z = \$7$

1-6 Assignment: **pg. 44-45 #1 to 25 (odd), 27 to 39 (odd), 48**

2-1: Reviewing Linear Inequalities in One Variable

When solving **Linear Inequalities**:

1. **Solve the inequalities like they were equations. Treat the inequality sign like it was an equal sign.**
2. **Always ISOLATE the Variable on the LEFT side of the inequality.**
3. **When Dividing or Multiplying by a negative number on BOTH sides of the inequality (bringing a negative number from one side to the other to Multiply or Divide), SWITCH the direction of the inequality sign.**

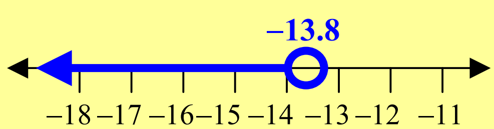
Example 1: Solve the following inequalities. Graph the solution.

a. $-5x - 7 > 62$

$$-5x - 7 > 62$$
$$-5x > 62 + 7$$
$$-5x > 69$$
$$x < \frac{69}{-5}$$

Switch Inequality Sign (Divided by a Negative)

$$x < -\frac{69}{5} \quad \text{or} \quad x < -13.8$$

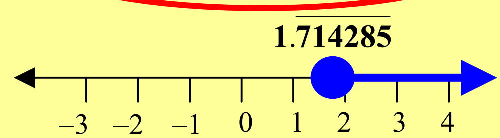


b. $3(2y + 4) \le 4(5y - 4)$

$$3(2y + 4) \le 4(5y - 4)$$
$$6y + 8 \le 20y - 16$$
$$6y - 20y \le -16 - 8$$
$$-14y \le -24$$
$$y \ge \frac{-24}{-14}$$

Switch Inequality Sign (Divided by a Negative)

$$y \ge \frac{12}{7} \quad \text{or} \quad x \ge 1.\overline{714285}$$

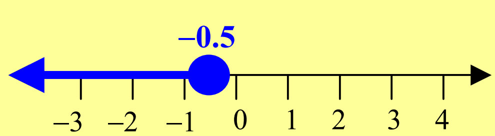


c. $3 - (2 + 4a) \ge 4 + 2(3a + 1)$

$$3 - (2 + 4a) \ge 4 + 2(3a + 1)$$
$$3 - 2 - 4a \ge 4 + 6a + 2$$
$$1 - 4a \ge 6 + 6a$$
$$-4a - 6a \ge 6 - 1$$
$$-10a \ge 5$$
$$a \le \frac{5}{-10}$$

Switch Inequality Sign (Divided by a Negative)

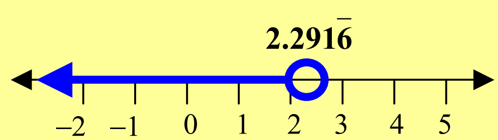
$$a \le -\frac{1}{2} \quad \text{or} \quad a \le -0.5$$



d. $\frac{6n}{5} - 1 < \frac{7}{4}$

$$\frac{6n}{5} - 1 < \frac{7}{4}$$
$$20\left(\frac{6n}{5}\right) - 20(1) < 20\left(\frac{7}{4}\right)$$
$$24n - 20 < 35$$
$$24n < 35 + 20$$
$$24n < 55$$

$$n < \frac{55}{24} \quad \text{or} \quad n < 2.291\overline{6}$$



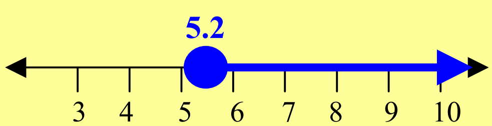
e. $\dfrac{-(1-10x)}{3} \ge 3 + \dfrac{5(x+6)}{4}$

Multiply <u>each term</u> of the inequality by its Lowest Common Denominator 12.

$$\overset{4}{\cancel{(\mathbf{12})}}\frac{-(1-10x)}{\cancel{3}} \ge (\mathbf{12})3 + \overset{3}{\cancel{(\mathbf{12})}}\frac{5(x+6)}{\cancel{4}}$$

$$-4(1 - 10x) \ge 36 + 15(x + 6)$$
$$-4 + 40x \ge 36 + 15x + 90$$
$$40x - 15x \ge 36 + 90 + 4$$
$$25x \ge 130$$
$$x \ge \frac{130}{25}$$

$$x \ge \frac{26}{5} \quad \text{or} \quad x \ge 5.2$$



Example 2: John is a car salesman. His monthly salary consists of \$400 and 3% of his sales. How much in sales must John make if he has to earn at least \$2500 in a month?

Let x = **Total Sales John must make in a month**.

$$400 + 0.03x \le 2500$$
$$0.03x \le 2500 - 400$$
$$0.03x \le 2100$$
$$x \le \frac{2100}{0.03}$$

"at least" means $\le$

$x \ge \$70{,}000$

John must make \$70,000 in sales to earn at least \$2500 in a month.

2-1 Assignment: **pg. 63 #13, 15, 19, 24, 27, 32, 33 to 36, 47 to 50, 51, 54, 55 and 56**

2-3: Graphing Linear Inequalities in Two Variables

When Graphing **Linear Inequalities in Two Variables (as in *y* versus *x*)**:

1. **ISOLATE *y* as the same in linear equation.**
2. **For inequalities with < or >, use BROKEN Line for the graph. For inequalities with ≤ or ≥, use SOLID Line for the graph.**
3. **SHADE in the Proper Region. For < or ≤, shade BELOW the Line. For > or ≥, shade ABOVE the Line.**

Example 1: For which of the given coordinates are the solution of the inequality, $5x - 4y > 12$?

a. $(3, -2)$

Substitute $x = 3$ and $y = -2$ and Evaluate the inequality.

$$5(3) - 4(-2) > 12$$
$$15 + 8 > 12$$
$$23 > 12 \quad \textbf{(TRUE)}$$

Therefore, (3, −2) is a solution.

b. $(-6, 1)$

Substitute $x = -6$ and $y = 1$ and Evaluate the inequality.

$$5(-6) - 4(1) > 12$$
$$-30 - 4 > 12$$
$$-34 > 12 \quad \textbf{(FALSE)}$$

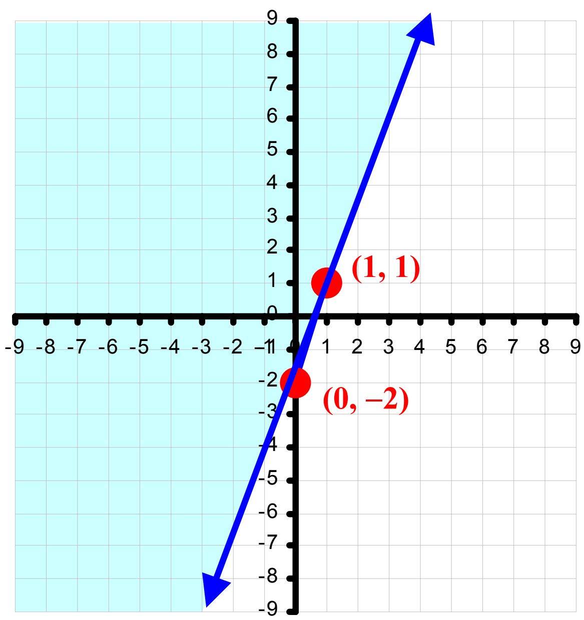
Therefore, (−6, 1) is a solution.

Example 2: Graph the following inequalities.

a. $y \geq 3x - 2$

$$\textbf{slope} = 3 = \frac{3 \text{ up}}{1 \text{ right}}$$

$$b = y\text{-intercept} = -2$$



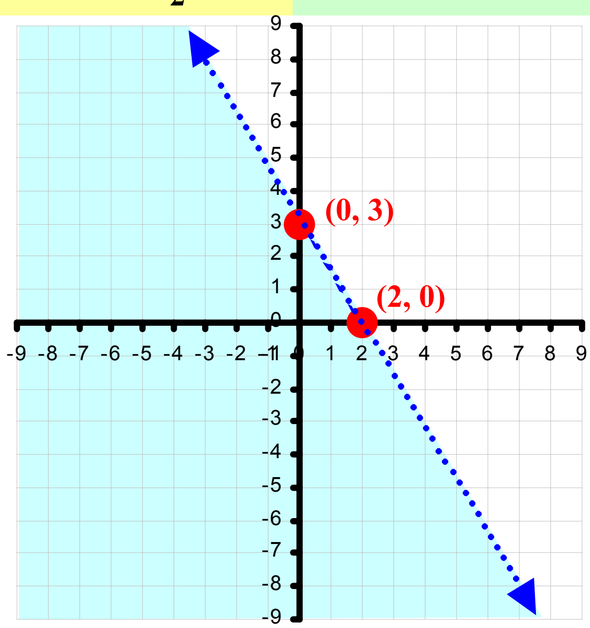
b. $3x + 2y < 6$

Rearrange to slope *y*-intercept form.

$$3x + 2y < 6$$
$$2y < -3x + 6$$
$$y < -\frac{3}{2}x + 3$$

$$\textbf{slope} = -\frac{3}{2} = \frac{3 \text{ down}}{2 \text{ right}}$$

$$b = y\text{-intercept} = 3$$



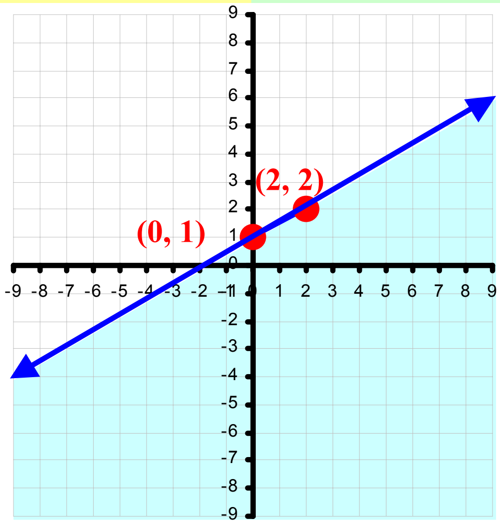
c. $x - 2y + 2 \geq 0$

Rearrange to slope y-intercept form.

$$x - 2y + 2 \geq 0$$
$$-2y \geq -x - 2$$
$$y \leq \frac{1}{2}x + 1$$

slope $= \frac{1}{2} = \frac{1 \text{ up}}{2 \text{ right}}$

$b = y\text{-intercept} = 1$



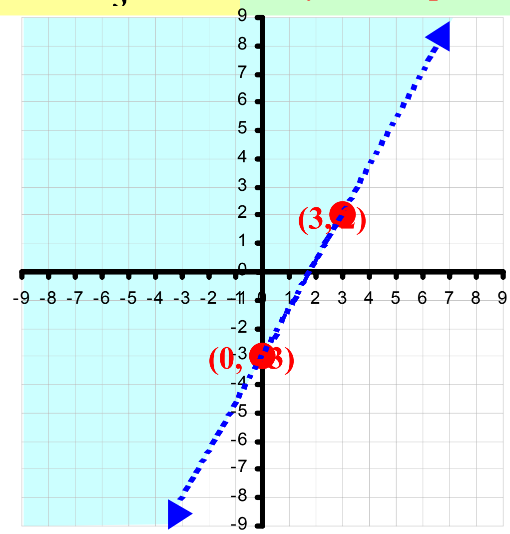
d. $5x - 3y < 9$

Rearrange to slope y-intercept form.

$$5x - 3y < 9$$
$$-3y < -5x + 9$$
$$y > \frac{5}{3}x - 3$$

slope $= \frac{5}{3} = \frac{5 \text{ up}}{3 \text{ right}}$

$b = y\text{-intercept} = -3$



Example 3: Graph the following inequalities with the given restrictions.

a. $2x + 3y - 12 < 0$, $x \geq 0$, $y \geq 0$

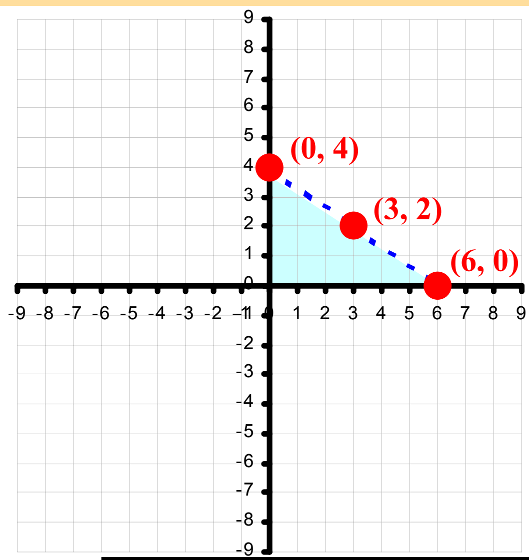
Rearrange to slope y-intercept form.

$$2x + 3y - 12 < 0$$
$$3y < -2x + 12$$
$$y < -\frac{2}{3}x + 4$$

slope $= -\frac{2}{3} = \frac{2 \text{ down}}{3 \text{ right}}$

$b = y\text{-intercept} = 4$

$x \geq 0$, $y \geq 0$ (positive x; positive y) Quadrant I



b. $2x - 7y \leq 14$, $x \geq 0$, $y \leq 0$

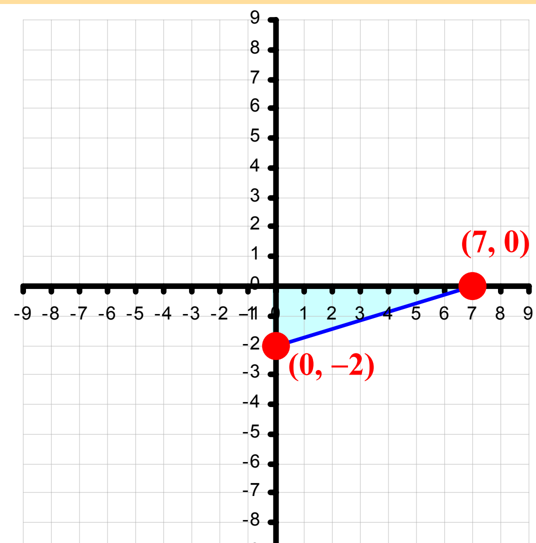
Rearrange to slope y-intercept form.

$$2x - 7y \leq 14$$
$$-7y \leq -2x + 14$$
$$y \geq \frac{2}{7}x - 2$$

slope $= \frac{2}{7} = \frac{2 \text{ up}}{7 \text{ right}}$

$b = y\text{-intercept} = -2$

$x \geq 0$, $y \leq 0$ (positive x; negative y) Quadrant IV



2-3 Assignment: **pg. 75 #1, 3, 5, 8, 13, 15, 19, 21, 27, 29, 32 and 33**