Unit 1: Polynomials

3-1: Reviewing Polynomials

Expressions: - mathematical sentences with no equal sign.  
**Example:** 3x + 2

Equations: - mathematical sentences that are equated with an equal sign.  
**Example:** 3x + 2 = 5x + 8

Terms: - are separated by an addition or subtraction sign. 
- each term begins with the sign preceding the variable or coefficient.

Monomial: - one term expression.  
**Example:** $5x^2$

Binomial: - two terms expression.  
**Example:** $5x^2 + 5x$

Trinomial: - three terms expression.  
**Example:** $x^2 + 5x + 6$

Polynomial: - many terms (more than one) expression.

All Polynomials must have **whole numbers** as exponents!!

**Example:** $9x^{-1} + 12x^{\frac{1}{2}}$ is NOT a polynomial.

Degree: - the term of a polynomial that contains the largest sum of exponents

**Example:** $9x^2y^3 + 4x^5y^2 + 3x^4$  Degree 7 ($5 + 2 = 7$)

Example 1: Fill in the table below.

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Number of Terms</th>
<th>Classification</th>
<th>Degree</th>
<th>Classified by Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>1</td>
<td>monomial</td>
<td>0</td>
<td>constant</td>
</tr>
<tr>
<td>4x</td>
<td>1</td>
<td>monomial</td>
<td>1</td>
<td>linear</td>
</tr>
<tr>
<td>9x + 2</td>
<td>2</td>
<td>binomial</td>
<td>1</td>
<td>linear</td>
</tr>
<tr>
<td>$x^2 - 4x + 2$</td>
<td>3</td>
<td>trinomial</td>
<td>2</td>
<td>quadratic</td>
</tr>
<tr>
<td>$2x^3 - 4x^2 + x + 9$</td>
<td>4</td>
<td>polynomial</td>
<td>3</td>
<td>cubic</td>
</tr>
<tr>
<td>$4x^4 - 9x + 2$</td>
<td>3</td>
<td>trinomial</td>
<td>4</td>
<td>quartic</td>
</tr>
</tbody>
</table>

Like Terms: - terms that have the same variables and exponents.

**Examples:**  
$2x^2y$ and $5x^2y$ are **like terms**  
$2x^2y$ and $5xy^2$ are **NOT** like terms
To Add and Subtract Polynomials:

Combine like terms by adding or subtracting their numerical coefficients.

Example 2: Simplify the followings.

a. \[3x^2 + 5x - x^2 + 4x - 6\]  
   \[= 3x^2 + 5x - x^2 + 4x - 6\]  
   \[= 2x^2 + 9x - 6\]

b. \[(9x^2y^3 + 4x^3y^2) + (3x^3y^2 - 10x^2y^3)\]
   \[= 9x^2y^3 + 4x^3y^2 + 3x^3y^2 - 10x^2y^3\]
   \[= -x^2y^3 + 7x^3y^2\]

c. \[(9x^2y^3 + 4x^3y^2) - (3x^3y^2 - 10x^2y^3)\]
   \[= 9x^2y^3 + 4x^3y^2 - 3x^3y^2 + 10x^2y^3\]
   \[= 19x^2y^3 + x^3y^2\]

(d. Subtract \[\frac{9x^2 + 4x}{5x^2 - 7x}\]
   This is the same as \[(9x^2 + 4x) - (5x^2 - 7x)\]
   \[= 9x^2 + 4x - 5x^2 + 7x\]
   \[= 4x^2 + 11x\]

To Multiply and Divide Monomials:

Multiply or Divide (Reduce) Numerical Coefficients.
Add or Subtract exponents of the same variable according to basic exponential laws.

Example 3: Simplify the followings.

a. \[(3x^3y^2)(7x^2y^4)\]
   \[= (3)(7)(x^3)(y^2)(y^4)\]
   \[= 21x^5y^6\]

b. \[\frac{24x^7y^4z^5}{6x^3yz^5}\]
   \[= \left(\frac{24}{6}\right)\left(x^7\right)^{-3}\left(y^4\right)^{-1}\left(z^5\right)^{-1}\]
   \[= 4x^4y^3z^0\]  \[z^0 = 1\]
   \[= 4x^4y^3\]

c. \[\frac{75a^3b^4}{25a^5b^3}\]
   \[= \left(\frac{75}{25}\right)\left(a^3\right)^{-2}\left(b^4\right)^{-1}\]
   \[= \frac{3a^{-2}b}{a^2}\] or \[\frac{3}{a^2}\]
(AP) Example 4: Find the area of the following ring.

General Formula for Area of a Circle  \[ A = \pi r^2 \]

Inner Circle Radius = 2x  
Outer Circle Radius = \((2x + 4x) = 6x\)

Inner Circle Area:  
\[ A = \pi (2x)^2 \]
\[ A = \pi (4x^2) \]
\[ A = 4\pi x^2 \]

Outer Circle Area:  
\[ A = \pi (6x)^2 \]
\[ A = \pi (36x^2) \]
\[ A = 36\pi x^2 \]

Shaded Area = \(36\pi x^2 - 4\pi x^2\)  
Shaded Area = \(32\pi x^2\)

3-1 Homework Assignment

Regular: pg. 102-103 #1 to 51, 55, 56

AP: pg. 102-103 #1 to 51, 53-57
3-3: Multiplying Polynomials

To Multiply Monomials with Polynomials

Example 1: Simplify the followings.

a. \(3 \ (2x^2 - 4x + 7)\)
   \[= 3 \ 6x^2 - 12x + 21\]

b. \(2x \ (3x^2 + 2x - 4)\)
   \[= 2x \ 6x^3 + 4x^2 - 8x\]

c. \(3x \ (5x + 4) - 4 \ (x^2 - 3x)\)
   (only multiply the brackets right after the monomial)
   \[= 3x \ 5x + 4 - 4 \ x^2 + 12x\]
   \[= 15x^2 + 12x - 4x^2 + 12x\]
   \[= 11x^2 + 24x\]

d. \(8 \ (a^2 - 2a + 3) - 4 \ (3a^2 + 7)\)
   \[= 8a^2 - 16a + 24 - 4 \ 3a^2 - 28\]
   \[= 5a^2 - 16a + 13\]

To Multiply Polynomials with Polynomials

Example 2: Simplify the followings.

a. \((3x + 2) \ (4x - 3)\)
   \[= (3x + 2) \ 12x^2 - 9x + 8x - 6\]
   \[= 12x^2 - x - 6\]

b. \((x + 3) \ (2x^2 - 5x + 3)\)
   \[= (x + 3) \ 2x^3 - 10x^2 + 3x + 6x^2 - 9x + 9\]
   \[= 2x^3 + x^2 - 12x + 9\]

c. \(3 \ (x + 2) \ (2x + 3) - (2x - 1) \ (x + 3)\)
   \[= 3 \ (x + 2) \ (2x^2 + 3x + 6) - (2x - 1) \ (x^2 + 3x - 3)\]
   \[= 3 \ (2x^2 + 3x - 6) \ (2x^2 + 5x - 3)\]
   \[= 6x^2 + 3x - 18 \ 2x^2 + 5x + 3\]
   \[= 4x^2 - 2x - 15\]

d. \((x^2 - 2x + 1) \ (3x^2 + x - 4)\)
   \[= (x^2 - 2x + 1) \ 3x^4 + x^3 - 4x^2 - 6x^3 - 2x^2 + 8x + 3x^2 + x - 4\]
   \[= 3x^4 - 5x^3 - 3x^2 + 9x - 4\]
Example 3: Find the shaded area of each of the followings.

a. \[ \text{Shaded Area} = \text{Big Rectangle} - \text{Small Square} \]
   \[ = (5x + 4)(2x - 1) - (x + 1)(x + 1) \]
   \[ = (10x^2 - 5x + 8x - 4) - (x^2 + 2x + 1) \]
   \[ = 10x^2 + 3x - 4 \]

\[ \text{Shaded Area} = 9x^2 + x - 5 \]

b. \[ \text{Total Area} = \text{Top Rectangle} + \text{Bottom Rectangle} \]
   \[ = (7x - 2)(x + 2) + (2x - 1)(x + 5) \]
   \[ = (7x^2 + 14x - 2x - 4) + (2x^2 + 10x - x - 5) \]
   \[ = 7x^2 + 12x - 4 + 2x^2 + 9x - 5 \]

\[ \text{Total Area} = 9x^2 + 21x - 9 \]

3-3 Homework Assignment

Regular: pg. 107-109 #1 to 77 (odd), 87, 88

AP: pg. 107-109 #2 to 84 (even), 85, 87, 88, 91
3-4: Special Products

\[(x + y)^2 = (x + y) (x + y)\]
\[(x + y)^3 = (x + y) (x + y) (x + y)\]
\((x + y)^2\) is NOT \(x^2 + y^2\)

Example 1: Simplify the followings.

a. \((2x + 3)^2\)
   \[
   = (2x + 3) (2x + 3) \\
   = 4x^2 + 6x + 6x + 9 \\
   = 4x^2 + 12x + 9
   \]

b. \((x - 2)^3\)
   \[
   = (x - 2) (x - 2) (x - 2) \\
   = (x - 2) (x^2 - 2x - 2x + 4) \\
   = (x - 2) (x^2 - 4x + 4) \\
   = x^3 - 4x^2 + 4x - 2x^2 + 8x - 8 \\
   = x^3 - 6x^2 + 12x - 8
   \]

c. \((3x + 2)^2 - (2x - 1)^2\)
   \[
   = (3x + 2) (3x + 2) - (2x - 1) (2x - 1) \\
   = (9x^2 + 6x + 6x + 4) - (4x^2 - 2x - 2x + 1) \\
   = (9x^2 + 12x + 4) - 4x^2 + 4x - 1 \\
   = 5x^2 + 16x + 3
   \]

Example 2: Find the volume of the box below.

\[Volume = Length \times Width \times Height\]
\[
V = (3x - 1)^2 (2x + 1) \\
V = (9x^2 - 3x - 3x + 1) (2x + 1) \\
V = (9x^2 - 6x + 1) (2x + 1) \\
V = 18x^3 + 9x^2 - 12x^2 - 6x + 2x + 1 \\
\]

3-4 Homework Assignment

Regular: pg.112-113 #1 to 47 (odd), 49, 51, 54 (a, c, e, g), 55 (a, c, e, g), 56

AP: pg.112-113 #2 to 48 (even), 49 to 53, 54 (b, d, f, h), 55 (b, d, f, h), 56, 57
### 3-6: Common Factors

Common Factors can consist of two parts:

a. **Numerical GCF**: - Greatest Common Factor of all numerical coefficients and constant.

b. **Variable GCF**: - the lowest exponent of a particular variable.

After obtaining the GCF, use it to divide each term of the polynomial for the remaining factor.

**Example 1**: Factor the followings

a. $3x^2 + 6x + 12$

   GCF = 3

   $= 3 (x^2 + 2x + 4)$

b. $4a^2b − 8ab^2 + 6ab$

   GCF = $2ab$

   $= 2ab (2a − 4b + 3)$

---

#### Factor by Grouping (Common Brackets as GCF)

\[
a (c + d) + b (c + d) = (c + d) (a + b)
\]

**Example 2**: Factor.

a. $3x (2x − 1) + 4 (2x − 1)$

   $= (2x − 1) (3x + 4)$

b. $2ab + 3ac + 4b^2 + 6bc$

   $= (2ab + 3ac) + (4b^2 + 6bc)$

   $= a (2b + 3c) + 2b (2b + 3c)$

   $= (2b + 3c) (a + 2b)$

c. $3x^2 − 6y^2 + 9x − 2xy^2$

   $= (3x^2 − 6y^2) + (9x − 2xy^2)$

   $= 3 (x^2 − 2y^2) + x (9 − 2y^2)$

   **Brackets are NOT the same! We might have to first rearrange terms.**

Try again after rearranging terms!

\[
3x^2 + 9x − 2xy^2 − 6y^2
\]

\[
= (3x^2 + 9x) − (2xy^2 + 6y^2)
\]

\[
= 3x (x + 3) − 2y^2 (x + 3)
\]

\[
= (x + 3) (3x − 2y^2)
\]
Example 3: Find the area of the shaded region in factored form and as a polynomial.

\[
\text{Shaded Area} = \text{Area of Square} - \text{Area of Circle}
\]

\[
\text{Shaded Area} = x^2 - \pi \left( \frac{x}{2} \right)^2
\]

\[
\text{Shaded Area} = x^2 - \frac{\pi x^2}{4} \quad \text{(Polynomial Form)}
\]

\[
\text{Shaded Area} = x^2 \left( 1 - \frac{\pi}{4} \right) \quad \text{(Factored Form)}
\]

Area of a Circle \( A = \pi r^2 \)

Radius of Circle = \( \frac{x}{2} \)

3-6 Homework Assignments

Regular: pg. 120 #1 to 35 (odd), 38 to 43

AP: pg.120 #2 to 36 (even), 37 to 44
3-8: Factoring \( x^2 + bx + c \)

(Leading Coefficient is 1)

What two numbers multiply to give \( c \), but add up to be \( b \)?

Example 1: Completely factor the followings.

a. \( x^2 + 5x + 6 \)  
   \[ \begin{array}{c|cccc} \text{Product of 6} & 1 & 6 & -1 & -6 \\ \hline 2 & 3 & -2 & -3 \end{array} \]  
   \[ \begin{array}{c} \text{sum of 5} \end{array} \]
   \[ = (x + 2) (x + 3) \]

b. \( x^2 - 3x - 10 \)  
   \[ \begin{array}{c|cccc} \text{Product of -10} & -1 & 10 & 1 & -10 \\ \hline -2 & 5 & 2 & -5 \end{array} \]  
   \[ \begin{array}{c} \text{sum of -3} \end{array} \]
   \[ = (x + 2) (x -5) \]

c. \( a^2 - 8a + 15 \)  
   \[ \begin{array}{c|cccc} \text{Product of 15} & 1 & 15 & -1 & -15 \\ \hline 3 & 5 & -3 & -5 \end{array} \]  
   \[ \begin{array}{c} \text{sum of -8} \end{array} \]
   \[ = (a - 3) (a - 5) \]

d. \( x^2 - 7xy + 12y^2 \)  
   \[ \begin{array}{c|cccc} \text{Product of 12} & 1 & 12 & -1 & -12 \\ \hline 2 & 6 & -2 & -6 \\ 3 & 4 & -3 & -4 \end{array} \]  
   \[ \begin{array}{c} \text{sum of -7} \end{array} \]
   \[ = (x - 3y) (x -4y) \]

e. \( x^2y^2 - 6xy - 16 \)  
   \[ \begin{array}{c|cccc} \text{Product of -16} & -1 & 16 & 1 & -16 \\ \hline -2 & 8 & 2 & -8 \\ -4 & 4 & 4 & -4 \end{array} \]  
   \[ \begin{array}{c} \text{sum of -6} \end{array} \]
   \[ = (xy + 2) (xy - 8) \]

f. \( 14 - 5w - w^2 \)  
   \[ \begin{array}{c} \text{Rearrange in Descending Degree.} \end{array} \]
   \[ = -w^2 - 5w + 14 \]
   \[ = - (w^2 + 5w - 14) \]
   \[ = - (w + 7) (w - 2) \]
   \[ \begin{array}{c} \text{Take out -1 as common factor.} \end{array} \]
   \[ (+7) (-2) = -14 \]
   \[ (+7) + (-2) = 5 \]

g. \( 3ab^2 - 3ab - 60a \)  
   \[ \begin{array}{c} \text{Take out GCF} \end{array} \]
   \[ = 3a (b^2 - b - 20) \]
   \[ \begin{array}{c} \text{GCF} \end{array} \]
   \[ (+4) (-5) = -20 \]
   \[ = 3a (b + 4) (b - 5) \]
   \[ \begin{array}{c} \text{GCF} \end{array} \]
   \[ (+4) + (-5) = -1 \]
Example 2: List all values of \( k \) such that the trinomial \( x^2 + kx - 24 \) can be factored.

<table>
<thead>
<tr>
<th>Product of (-24)</th>
<th>Possible Sums for ( k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1) ( 24)</td>
<td>( 1 ) (-24)</td>
</tr>
<tr>
<td>(-2) ( 12)</td>
<td>( 2 ) (-12)</td>
</tr>
<tr>
<td>(-3) ( 8)</td>
<td>( 3 ) (-8)</td>
</tr>
<tr>
<td>(-4) ( 6)</td>
<td>( 4 ) (-6)</td>
</tr>
<tr>
<td>23</td>
<td>(-23)</td>
</tr>
<tr>
<td>10</td>
<td>(-10)</td>
</tr>
<tr>
<td>5</td>
<td>(-5)</td>
</tr>
<tr>
<td>2</td>
<td>(-2)</td>
</tr>
</tbody>
</table>

Example 3: A rectangular has an area of \( x^2 + 9x - 10 \).

a. What are the dimensions of the rectangle?
b. If \( x = 5 \) cm, what are the actual dimensions?

\[ \text{Area} = x^2 + 9x - 10 \]

\[ \text{Dimensions} = (x + 10)(x - 1) \]

\[ \text{Dimensions} = 15 \text{ cm} \times 4 \text{ cm} \]

(AP) Example 4: Factor the followings.

a. \( x^4 + 14x^2 - 32 \)

\[ = (x^2 + 16)(x^2 - 2) \]

b. \( (x + 3)^2 + 6(x + 3) + 8 \)

\[ = (y^2 + 6y + 8) \quad \text{Do NOT Expand!!} \]

\[ = (y + 4)(y + 2) \]

Assume \( x^4 + bx^2 + c \) as the same as \( x^2 + bx + c \) and factor. The answer will be \((x^2 - ) (x^2 - )\).

\[ = (x + 3 + 4)(x + 3 + 2) \]

\[ = (x + 7)(x + 5) \]

3-8 Homework Assignments

Regular: pg. 127 #19 to 59 (odd), 61, 65, 66

AP: pg. 127 #20 to 60 (even), 61, 65-68
3-9: Factoring \( ax^2 + bx + c \) (Leading Coefficient is not 1, \( a \neq 1 \))

For factoring trinomial with the form \( ax^2 + bx + c \), we will have to factor by grouping.

Example 1: Factor \( 6x^2 + 11x + 4 \)

First, we look for GCF. But there is no GCF!

\[
\begin{align*}
6x^2 + 11x + 4 &= 6x^2 + 3x + 8x + 4 \\
&= (6x^2 + 3x) + (8x + 4) \\
&= 3x(2x + 1) + 4(2x + 1) \\
&= (3x + 4)(2x + 1)
\end{align*}
\]

Example 2: Completely factor the followings.

\[
\begin{align*}
a. \quad 2y^2 - 3y - 9 &= (2)(-9) = -18 \\
&= 2y^2 + 3y - 6y - 9 \\
&= (2y^2 + 3y) - (6y - 9) \\
&= y(2y + 3) - 3(2y + 3) \\
&= (2y + 3)(y - 3) \\
&= (2y + 3)(y - 3)
\end{align*}
\]

\[
\begin{align*}
b. \quad 8d^2 - 2d - 3 &= (8)(-3) = -24 \\
&= 8d^2 + 4d - 6d - 3 \\
&= (8d^2 + 4d) - (6d + 3) \\
&= 4d(2d + 1) - 3(2d + 1) \\
&= (2d + 1)(4d - 3)
\end{align*}
\]

\[
\begin{align*}
c. \quad 6x^3 - 14x^2 + 4x &= GCF = 2x \\
&= 2x(3x^2 - 7x + 2) \\
&= 2x[(3x^2 - x) - 6x + 2] \\
&= 2x[(3x - 1)(x - 2)] \\
&= 2x(3x - 1)(x - 2)
\end{align*}
\]

\[
\begin{align*}
d. \quad 8m^2 - 6mn - 9n^2 &= 8 \times -9 = -72 \\
&= 8m^2 + 6mn - 12mn - 9n^2 \\
&= (8m^2 + 6mn) - (12mn + 9n^2) \\
&= 2m(4m + 3n) - 3n(4m + 3n) \\
&= (4m + 3n)(2m - 3n)
\end{align*}
\]
Example 3: List all possible values for \( k \) in \( 5x^2 + kx - 4 \) so it could be factored.

<table>
<thead>
<tr>
<th>Product of ((5)(-4)) = -20</th>
<th>Possible Sums for ( k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1  20</td>
<td>1  -20</td>
</tr>
<tr>
<td>-2  10</td>
<td>2  -10</td>
</tr>
<tr>
<td>-4  5</td>
<td>4  -5</td>
</tr>
</tbody>
</table>

(AP) Example 4: Factor the followings.

a. \[ 4x^4 + 13x^2 + 9 \]
   \[ = 4x^4 + 4x^2 + 9x^2 + 9 \]
   \[ = (4x^4 + 4x^2) + (9x^2 + 9) \]
   \[ = x^2(x^2 + 1) + 9(x^2 + 1) \]
   \[ = (x^2 + 1)(x^2 + 9) \]

\[ 4 \times 9 = 36 \]
\[ (4)(9) = 36 \]
\[ (4) + (9) = 13 \]

b. \[ 18x^4 - 27x^2y + 4y^2 \]
   \[ = 18x^4 - 3x^2y - 24x^2y + 4y^2 \]
   \[ = (18x^4 - 3x^2y) - (24x^2y - 4y^2) \]
   \[ = 3x^2(6x^2 - y) - 4y(6x^2 - y) \]
   \[ = (6x^2 - y)(3x^2 - 4y) \]

\[ 18 \times 4 = 72 \]
\[ (-3)(-24) = 72 \]
\[ (-3) + (-24) = 72 \]

switch sign!
(= sign in front of brackets)

3-9 Homework Assignments

Regular: pg. 130 - 131 #7 to 49 (odd), 51, 54, 55
AP: pg. 130 - 131 #8 to 50 (even), 51, 54-56
3-10: Factoring Special Quadratics

**Difference of Squares** (Square – Square) \( x^2 − y^2 = (x − y) (x + y) \)

**Example 1:** Completely factor the followings.

a. \( x^2 − 25 \)  
   \[= (x − 5) (x + 5)\]

b. \( x^2 + 9 \)  
   (NOT Factorable – Sum of Squares)

c. \( 3x^2 − 300 \)  
   \[= 3 (x^2 − 100)\]  
   \[= 3 (x − 10) (x + 10)\]

d. \( x^4 − 81 \)
   \[= (x^2 − 9) (x^2 + 9)\]
   \[= (x − 3) (x + 3) (x^2 + 9)\]

e. \( 9x^2 − 64y^2 \)
   \[= (3x − 8y) (3x + 8y)\]

(AP) **Example 2:** Completely factor the followings.

a. \( (x − 4)^2 − 49 \)  
   Look at \((x − 4)\) as a single item!
   \[= [(x − 4) − 7] [(x − 4) + 7]\]
   \[= (x − 11) (x + 3)\]

b. \( (2x + 3)^2 − (3x − 1)^2 \)  
   Look at \((x − 2)\) and \((3x + 1)\) as individual items!
   \[= [(2x + 3) − (3x + 1)] [(2x + 3) + (3x + 1)]\]
   \[= [−x + 4] [5x + 2]\]
   \[= −(x − 4) (5x + 2)\]

**Perfect Trinomial Square**  
\[ a^2 + bx + c = \left(\sqrt{a}x + \sqrt{c}\right)^2 \]

where \(a, c\) are square numbers, and \(b = 2\sqrt{a}\sqrt{c}\)

**Example 3:** Expand \((3x + 2)^2\).

\[= (3x + 2) (3x + 2)\]
\[= 9x^2 + 12x + 4\]
Example 4:  Completely factor the followings.

a. \(9x^2 + 30x + 25\)

\[
9x^2 + 30x + 25 = 9x^2 + 2 \cdot 30x + 25 = (3x)^2 + 2 \cdot 30x + 5^2 = (3x + 5)^2
\]

b. \(4x^2 - 28x + 49\)

\[
4x^2 - 28x + 49 = 4x^2 - 2 \cdot 14x + 49 = (2x)^2 - 2 \cdot 14x + 7^2 = (2x - 7)^2
\]

(AP) Example 5: Factor \(x^6 - 20x^3 + 100\).

Assumes \(x^6 + bx^3 + c\) is the same as \(x^2 + bx + c\).

But the answer will be in the form of \((x^3 + \_)(x^3 + \_)\).

\[
x^6 - 20x^3 + 100 = (x^3)^2 - 20x^3 + 10^2 = (x^3 - 10)^2
\]

Example 6:  List all possible values for \(k\) that can make the following polynomials as perfect squares.

a. \(x^2 + kx + 64\)

\[
k = 2(8) = 16 \quad \text{and} \quad -16
\]

The middle term of any perfect trinomial squares can have a positive or a negative numerical coefficient!

b. \(kx^2 + 20x + 25\)

\[
k = \frac{20}{k} = \frac{20}{5} = 4
\]

c. \(49x^2 + 56xy + ky^2\)

\[
k = 16
\]

3-10 Homework Assignments

Regular: pg. 133 - 134 #13 to 43 (odd), 54 to 56

AP: pg. 133 - 134 #14 to 44 (even), 46 to 57, 59, 61, 63
4-1A: Dividing Polynomials

Consider \(562 \div 3\).

\[
\begin{array}{c|c|c|c}
\text{Quotient} & \text{Divisor} & \text{Remainder} \\
\hline
187 & 3 & 1 \\
26 & 3 \text{ (3)} & 22 \\
24 & 3 & 21 \\
\hline
\text{Original Number} & \text{Divisor} & \text{Quotient} & \text{Remainder} \\
562 & 3 & 187 & 1 \\
\end{array}
\]

We can say that \( \frac{562}{3} = 187 + \frac{1}{3} \)

Or, we can say \(562 = (3)(187) + 1\)

In general, for \(P(x) \div D(x)\), we can write

\[
\frac{P(x)}{D(x)} = \frac{Q(x)}{D(x)} + \frac{R}{D(x)}
\]

or \(P(x) = D(x)Q(x) + R\)

**Restriction:** \(D(x) \neq 0\)

**Non-Permissible Value (NPV):** - restriction on what the variable CANNOT be equal to due to the fact that the Denominator CANNOT be 0.

(You can never divide by 0!)

**Division with Monomials**

**Example 1:** Simplify the followings

a. \(\frac{21x^2y^2}{3x} = 7xy^2\)

\(3x \neq 0\) \(\Rightarrow\) \(x \neq 0\) \(\Rightarrow\) NPV = 0

b. \(\frac{(4x^3)(6x^2)}{3x} = 24x^7\)

\(6x^3 \neq 0\) \(\Rightarrow\) \(x \neq 0\) \(\Rightarrow\) NPV = 0

c. \(\frac{6x^3 + 9x^2 + 15x}{3x} = 2x^2 + 3x + 5\)

Divide each term of the polynomial by the monomial.
Long Division to Divide Polynomials

Example 2: Divide $\frac{6x^3 + 9x^2 + 15x + 21}{2x + 1}$

\[
\begin{array}{c|cccc}
& 3x^2 & +3x & +6 \\
\hline
(2x+1) & 6x^3 & +9x^2 & +15x & +21 \\
& - (6x^3 + 3x^2) & & & \\
\hline
& 6x^2 & +15x & & \\
& - (6x^2 + 3x) & & & \\
\hline
& 12x & +21 & & \\
& - (12x + 6) & & & \\
\hline
& R & = & 15 & \\
\end{array}
\]

You cannot divide monomial by polynomial!

Dividing by Polynomial is only possible by Long Division!

\[
\frac{6x^3 + 9x^2 + 15x + 21}{2x + 1} = \frac{6x^3 + 9x^2}{2x + 1} + \frac{15x + 21}{2x + 1}
\]

OR

\[
6x^3 + 9x^2 + 15x + 21 = (2x + 1)(3x^2 + 3x + 6) + 15
\]

For NPV, we let $2x + 1 = 0$

\[2x = -1 \quad \text{NPV: } x = -\frac{1}{2}\]

Example 3: Divide $\frac{3x^3 - 4x^2 + 5x - 8}{x - 2}$

\[
\begin{array}{c|cccc}
& 3x^2 & +2x & +9 \\
\hline
(x - 2) & 3x^3 & -4x^2 & +5x & -8 \\
& - (3x^3 - 6x^2) & & & \\
\hline
& 2x^2 & +5x & & \\
& - (2x^2 - 4x) & & & \\
\hline
& 9x & -8 & & \\
& - (9x - 18) & & & \\
\hline
& R & = & 10 & \\
\end{array}
\]

\[
\frac{3x^3 - 4x^2 + 5x - 8}{x - 2} = (3x^2 + 2x + 9) + \frac{10}{x - 2}
\]

OR

\[
3x^3 - 4x^2 + 5x - 8 = (x - 2)(3x^2 + 2x + 9) + 10
\]

For NPV, we let $x - 2 = 0$

\[x = 2 \quad \text{NPV: } x = 2\]
Example 4: Divide \( \frac{2x^3 - 7x + 6}{x - 3} \)

\[
\begin{array}{c}
2x^2 + 6x + 11 \\
-(2x^3 - 6x^2)
\end{array}
\]

\[
\begin{array}{c}
6x^2 - 7x \\
-(6x^2 - 18x)
\end{array}
\]

\[
\begin{array}{c}
11x + 6 \\
-(11x - 33)
\end{array}
\]

\[
R = 39
\]

Example 5: Divide \( \frac{4x^3 - 8x^2 + 7x - 1}{2x^2 + 3} \)

\[
\begin{array}{c}
2x - 4 \\
-(2x^2 + 0x + 3)
\end{array}
\]

\[
\begin{array}{c}
-8x^2 + x - 1 \\
-(-8x^2 + 0x - 12)
\end{array}
\]

\[
R = x + 11
\]

4-1A Homework Assignments

Regular: pg. 152 - 153 #1 to 63 (odd), 65 (a, c, e), 68, 70

AP: pg. 152 - 153 #2 to 64 (even), 65 (a, c, e), 68, 71, 74, 75, 76
4-1B: Synthetic Division

Only works well on divisor that is in a form of $x + a$, where Leading coefficient of Divisor is 1 on the divisor.

Example 1: Divide $\frac{3x^3 - 4x^2 + 5x - 8}{x - 2}$

Example 2: Divide $\frac{2x^3 - 7x + 6}{x - 3}$

Example 3: Divide $\frac{2x^3 - 3x^2 - 5x + 6}{x + 2}$
The Remainder Theorem

**If you want to find only the remainder, you can simply substitute** \( a \) **from the Divisor, \((x - a)\), into the original Polynomial, \( P(x) \).**

**In general, when** \( \frac{P(x)}{x - a} \), \( P(a) = \text{Remainder} \)

Example 4: Find the remainder of the followings.

a. \[
\frac{3x^3 - 4x^2 + 5x - 8}{x - 2}
\]
\[
x - 2 = 0 \\
x = 2
\]
\[
P(2) = 3(2)^3 - 4(2)^2 + 5(2) - 8
\]
\[
= 24 - 16 + 10 - 8
\]
\[
\text{Remainder} = 10
\]

b. \[
\frac{2x^3 - 7x + 6}{x - 3}
\]
\[
x - 3 = 0 \\
x = 3
\]
\[
P(2) = 2(3)^3 - 7(3) + 6
\]
\[
= 54 - 21 + 6
\]
\[
\text{Remainder} = 39
\]

c. \[
\frac{2x^3 - 3x^2 - 5x + 6}{x + 2}
\]
\[
x + 2 = 0 \\
x = -2
\]
\[
P(2) = 2(-2)^3 - 3(-2)^2 - 5(-2) + 6
\]
\[
= -16 - 12 + 10 + 6
\]
\[
\text{Remainder} = -12
\]

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4-1B Homework Assignments

Regular: pg. 154 #1 to 17; pg. 155 Section 3 #5 (a, b, c)

AP: pg. 154 #1 to 17; pg. 155 Section 3 #5 (a, b, c)